EXPRESSWALA ACADEMY

Maths 1A: Functions

Question Paper, Set 04

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. If $A = \{2, 6, 7\}$ and $B = \{r, u, v, w\}$, how many functions are there from B to A? [1 mark]

2. If $f : A \to \mathbb{Z}$ is defined by $f(x) = x^2 + 2$ where $A = \{-3, -2, -1, 0, 3\}$, find the range of f. [1 mark]

3. How many constant functions are there from a set with 4 elements to a set with 7 elements? [1 mark]

4. Find the minimum value of $f(x) = 3x^2 + 24x + 48$. [2 marks]

5. Solve $2x^2 + 6x - 20 < 0$. [2 marks]

6. Solve $|2x - 7| > 1$. [2 marks]

7. Find $[7.4] + [-7.4]$. [1 mark]

8. Solve $[x]^2 + 2[x] - 8 = 0$, where $[x]$ is the greatest integer function. [3 marks]

9. Find the value of $5^{3\log_5 35}$. [1 mark]

10. If $f(x) = 3 - 3x$, find $(f \circ f)(x)$. [2 marks]

11. If $f(x) = \dfrac{4x - 1}{2x - 4}$, show that $(f \circ f)(x) = x$ and hence find $(f \circ f \circ f)(-2)$. [5 marks]

12. Write $f(x) = 3x^4 + 4x^3 - 3x^2 - 3x + 3$ as the sum of an even function and an odd function. [3 marks]

13. If $f(x) = x + 1$ and $g(x) = x^2 - 4x + 3$, find the domain of $\dfrac{f}{g}$. [2 marks]

14. f satisfies $f(x + y) = f(x) + f(y)$ for all real x, y, and $f(3) = -6$. Find $f(5)$. [3 marks]

15. How many onto functions are there from $A = \{2, 4, 10, 12\}$ to $B = \{e, f, h\}$? [2 marks]

16. Show that $f : \mathbb{R} \to \mathbb{R}$, $f(x) = \frac{-2x-9}{5}$ is a bijection and find f^{-1}. [5 marks]

17. Find the domain of $f(x) = \sqrt{x^2 + 7x + 10} + \dfrac{1}{\sqrt{-x - 7}}$. [5 marks]

18. Find the range of $f(x) = \dfrac{x^2 + x + 2}{x^2 - x + 4}$, $x \in \mathbb{R}$. [5 marks]

19. The point $(-7, 6)$ lies on $y = f(x)$. Which point must lie on $y = -f(x)$? [1 mark]

20. Where does the graph of $y = |x^2 - 6x + 5|$ meet the x-axis, and what is the highest point of the arch between these points? [3 marks]

Key sheet

Q	Answer
1	81
2	$\text{Range} = \{2, 3, 6, 11\}$
3	7
4	0
5	$x \in (-5, 2)$
6	$x \in (-\infty, 3) \cup (4, \infty)$
7	-1
8	$x \in [-4, -3) \cup [2, 3)$
9	42875
10	$(f \circ f)(x) = 9x - 6$
11	$(f \circ f)(x) = x$; $(f \circ f \circ f)(-2) = \frac{9}{8}$
12	Even part $3x^4 - 3x^2 + 3$, odd part $4x^3 - 3x$
13	$\mathbb{R} - \{1, 3\}$
14	$f(5) = -10$
15	36
16	$f^{-1}(x) = -\frac{5x}{2} - \frac{9}{2}$
17	$(-\infty, -7)$
18	$[\frac{1}{3}, \frac{7}{5}]$
19	$(-7, -6)$
20	At $x = 1$ and $x = 5$; the top of the arch is $(3, 4)$

Solutions

1. If $A = \{2, 6, 7\}$ and $B = \{r, u, v, w\}$, how many functions are there from B to A? [1]

$n(B) = 4,\ n(A) = 3$

Number of functions from B to $A = \{n(A)\}^{n(B)} = 3^4 = 81$

Ans: 81

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2. If $f : A \to \mathbb{Z}$ is defined by $f(x) = x^2 + 2$ where $A = \{-3, -2, -1, 0, 3\}$, find the range of f. [1]

$f(-3) = 11,\ f(-2) = 6,\ f(-1) = 3,\ f(0) = 2,\ f(3) = 11$

Range $= \{2, 3, 6, 11\}$

Ans: $\text{Range} = \{2, 3, 6, 11\}$

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3. How many constant functions are there from a set with 4 elements to a set with 7 elements? [1]

A constant function sends every element to the same value k.

There is one constant function for each element of the co-domain: 7.

Ans: 7

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4. Find the minimum value of $f(x) = 3x^2 + 24x + 48$. [2]

$\frac{4ac - b^2}{4a} = \frac{4(3)(48) - (24)^2}{4(3)} = 0$

It occurs at $x = -\frac{b}{2a} = -4$.

Ans: 0

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5. Solve $2x^2 + 6x - 20 < 0$. [2]

$2x^2 + 6x - 20 = 2\,(x - 2)\,(x + 5)$

The roots are -5 and 2, and $a = 2 > 0$.

The expression has the sign opposite to a between the roots.

Solution set: $(-5, 2)$

Ans: $x \in (-5, 2)$

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6. Solve $|2x - 7| > 1$. [2]

$2x - 7 > 1$ or $2x - 7 < -1$

Solution set: $(-\infty, 3) \cup (4, \infty)$

Ans: $x \in (-\infty, 3) \cup (4, \infty)$

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7. Find $[7.4] + [-7.4]$. [1]

$[7.4] = 7,\ [-7.4] = -8$

The sum is -1 (for a non-integer x, $[x] + [-x] = -1$).

Ans: -1

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8. Solve $[x]^2 + 2[x] - 8 = 0$, where $[x]$ is the greatest integer function. [3]

Put $t = [x]$: $t^2 + 2t - 8 = 0$

$(t+4)(t-2) = 0 \Rightarrow t = -4$ or 2

$[x] = -4 \Rightarrow x \in [-4, -3)$, $[x] = 2 \Rightarrow x \in [2, 3)$

So $x \in [-4, -3) \cup [2, 3)$.

Ans: $x \in [-4, -3) \cup [2, 3)$

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9. Find the value of $5^{3\log_5 35}$. [1]

$5^{3\log_5 35} = 5^{\log_5 35^3}$

$= 35^3 = 42875$ (using $a^{\log_a N} = N$)

Ans: 42875

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10. If $f(x) = 3 - 3x$, find $(f \circ f)(x)$. [2]

$(f \circ f)(x) = f(3 - 3x) = -3(3 - 3x) + 3$

$= 9x - 6$

Ans: $(f \circ f)(x) = 9x - 6$

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11. If $f(x) = \dfrac{4x-1}{2x-4}$, show that $(f \circ f)(x) = x$ and hence find $(f \circ f \circ f)(-2)$. [5]

$(f \circ f)(x) = f\left(\frac{4x-1}{2x-4}\right) = \dfrac{4\left(\frac{4x-1}{2x-4}\right) - 1}{2\left(\frac{4x-1}{2x-4}\right) - 4}$

Multiply the numerator and the denominator by $2x - 4$:

$= \dfrac{(4-2x) + (16x-4)}{(16-8x) + (8x-2)} = \dfrac{14x}{14}$

$= x$

So $f \circ f = I$, and $(f \circ f \circ f)(x) = f(x)$.

$(f \circ f \circ f)(-2) = f(-2) = \frac{-9}{-8} = \frac{9}{8}$

Ans: $(f \circ f)(x) = x$; $(f \circ f \circ f)(-2) = \frac{9}{8}$

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12. Write $f(x) = 3x^4 + 4x^3 - 3x^2 - 3x + 3$ as the sum of an even function and an odd function. [3]

$f(x) = \frac{1}{2}[f(x) + f(-x)] + \frac{1}{2}[f(x) - f(-x)]$

$f(-x) = 3x^4 - 4x^3 - 3x^2 + 3x + 3$

$\frac{1}{2}[f(x) + f(-x)] = 3x^4 - 3x^2 + 3$ (even)

$\frac{1}{2}[f(x) - f(-x)] = 4x^3 - 3x$ (odd)

$f(x) = \left(3x^4 - 3x^2 + 3\right) + \left(4x^3 - 3x\right)$

Ans: Even part $3x^4 - 3x^2 + 3$, odd part $4x^3 - 3x$

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13. If $f(x) = x + 1$ and $g(x) = x^2 - 4x + 3$, find the domain of $\dfrac{f}{g}$. [2]

$D_f = D_g = \mathbb{R}$, and $\frac{f}{g}$ needs $g(x) \neq 0$.

$(x-3)(x-1) = 0 \Rightarrow x = 1, 3$

Domain $= \mathbb{R} - \{1, 3\}$

Ans: $\mathbb{R} - \{1, 3\}$

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14. f satisfies $f(x+y) = f(x) + f(y)$ for all real x, y, and $f(3) = -6$. Find $f(5)$. [3]

$f(x+y) = f(x) + f(y) \Rightarrow f(x) = kx$.

$f(3) = -6 \Rightarrow 3k = -6 \Rightarrow k = -2$

$f(5) = -2 \times 5 = -10$

Ans: $f(5) = -10$

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15. How many onto functions are there from $A = \{2, 4, 10, 12\}$ to $B = \{e, f, h\}$? [2]

$3^4 - {}^3C_1\, 2^4 + {}^3C_2\, 1^4$

$= 81 - 48 + 3 = 36$

Ans: 36

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16. Show that $f : \mathbb{R} \to \mathbb{R}$, $f(x) = \frac{-2x-9}{5}$ is a bijection and find f^{-1}. [5]

One-one: $f(x_1) = f(x_2) \Rightarrow \frac{-2x_1-9}{5} = \frac{-2x_2-9}{5}$

$\Rightarrow -2x_1 = -2x_2 \Rightarrow x_1 = x_2$, so f is one-one.

Onto: let $y \in \mathbb{R}$ and $y = \frac{-2x-9}{5}$.

$\Rightarrow x = -\frac{5y}{2} - \frac{9}{2} \in \mathbb{R}$

and $f(x) = y$, so every y has a pre-image: f is onto.

Hence f is a bijection, and $f^{-1}(x) = -\frac{5x}{2} - \frac{9}{2}$

Ans: $f^{-1}(x) = -\frac{5x}{2} - \frac{9}{2}$

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17. Find the domain of $f(x) = \sqrt{x^2 + 7x + 10} + \dfrac{1}{\sqrt{-x-7}}$. [5]

We need $x^2 + 7x + 10 \geq 0$ and $-x - 7 > 0$.

$x^2 + 7x + 10 = (x+2)(x+5) \geq 0$: the roots are $-5 < -2$ and $a > 0$.

$\Rightarrow x \in (-\infty, -5] \cup [-2, \infty)$

$-x - 7 > 0 \Rightarrow x < -7$

Common part: domain $= (-\infty, -7)$

Ans: $(-\infty, -7)$

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18. Find the range of $f(x) = \dfrac{x^2 + x + 2}{x^2 - x + 4}$, $x \in \mathbb{R}$. [5]

The denominator $x^2 - x + 4$ has $\Delta = -15 < 0$, so it is never zero: the domain is $\mathbb{R}$.

$y = \frac{x^2+x+2}{x^2-x+4} \Rightarrow y(x^2 - x + 4) = x^2 + x + 2$

$\Rightarrow (y-1)x^2 + (-y-1)x + (4y-2) = 0$

For $y \neq 1$, x is real, so $\Delta = (-y-1)^2 - 4(y-1)(4y-2) \geq 0$

$\Rightarrow -15y^2 + 26y - 7 \geq 0$, $\Rightarrow \frac{1}{3} \leq y \leq \frac{7}{5}$

The value $y = 1$ is taken at $x = 1$, and 1 lies in this interval.

Range $= [\frac{1}{3}, \frac{7}{5}]$

Ans: $[\frac{1}{3}, \frac{7}{5}]$

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19. The point $(-7, 6)$ lies on $y = f(x)$. Which point must lie on $y = -f(x)$? [1]

The graph of $y = -f(x)$ is obtained by reflection in the x-axis: $(x, y) \to (x, -y)$.

So $(-7, 6) \to (-7, -6)$.

Ans: $(-7, -6)$

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20. Where does the graph of $y = |x^2 - 6x + 5|$ meet the x-axis, and what is the highest point of the arch between these points? [3]

$y = x^2 - 6x + 5$ cuts the x-axis where $(x - 5)(x - 1) = 0$: $x = 1, 5$.

Between them it dips to -4 at $x = 3$.

Taking $|f(x)|$ flips this part up: the arch has top $(3, 4)$.

Ans: At $x = 1$ and $x = 5$; the top of the arch is $(3, 4)$
