EXPRESSWALA ACADEMY

Maths 1A: Functions

Question Paper, Set 10

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. If $A = \{6, 9\}$ and $B = \{q, s, t, w\}$, how many functions are there from B to A? [1 mark]

2. For $f(x) = -3x - 2$ on the domain $A = \{-3, -2, -1, 4\}$, find the pre-image of 7. [1 mark]

3. How many constant functions are there from a set with 5 elements to a set with 6 elements? [1 mark]

4. Find the minimum value of $f(x) = x^2 - 8x + 24$. [2 marks]

5. Solve $4x^2 - 16x + 12 \geq 0$. [2 marks]

6. Solve $|3x - 3| \geq 5$. [2 marks]

7. Find the value of $[-3.7] + [-8.1]$, where $[x]$ is the greatest integer function. [1 mark]

8. Using $[x] + \left[x + \frac{1}{n}\right] + \cdots + \left[x + \frac{n-1}{n}\right] = [nx]$, evaluate $[0.06] + \left[0.06 + \frac{1}{3}\right] + \left[0.06 + \frac{2}{3}\right]$. [3 marks]

9. Find the value of $2^{3\log_2 15}$. [1 mark]

10. If $f(x) = 5 - 2x$ and $g(x) = 2x^2 + 2$, find $(g \circ f)(x)$. [2 marks]

11. If $f(x) = \dfrac{7 - 5x}{3x + 5}$, show that $(f \circ f)(x) = x$ and hence find $(f \circ f \circ f)(-1)$. [5 marks]

12. Write $f(x) = 2x^4 - 5x^3 + 5x^2 + 4x + 5$ as the sum of an even function and an odd function. [3 marks]

13. If $f(x) = x - 4$ and $g(x) = x^2 + 3x - 4$, find the domain of $\dfrac{f}{g}$. [2 marks]

14. f satisfies $f(x + y) = f(x) + f(y)$ for all real x, y, and $f(6) = -36$. Find $f(3)$. [3 marks]

15. How many many-one functions are there from $A = \{6, 8, 10, 11\}$ to $B = \{d, e, f, g, h\}$? [2 marks]

16. Let $A = \mathbb{R} - \{1\}$ and $B = \mathbb{R} - \left\{\frac{5}{2}\right\}$. Show that $f : A \to B$, $f(x) = \frac{5x-6}{2x-2}$ is a bijection and find f^{-1}. [5 marks]

17. Find the domain of $f(x) = \sqrt{x^2 - 36} + \log_e(x - 8)$. [5 marks]

18. Find the range of $f(x) = \dfrac{x^2 - x + 3}{x^2 + 2x + 2}$, $x \in \mathbb{R}$. [5 marks]

19. The point $(8, 4)$ lies on $y = f(x)$. Which point must lie on $y = f^{-1}(x)$? [1 mark]

20. The graph of $y = -x^2$ is shifted 5 units to the right and 1 units up. Write the equation of the new graph. [3 marks]

Key sheet

Q	Answer
1	16
2	$x = -3$
3	6
4	8
5	$x \in (-\infty, 1] \cup [3, \infty)$
6	$x \in (-\infty, -\frac{2}{3}] \cup [\frac{8}{3}, \infty)$
7	-13
8	0
9	3375
10	$(g \circ f)(x) = 8x^2 - 40x + 52$
11	$(f \circ f)(x) = x$; $(f \circ f \circ f)(-1) = 6$
12	Even part $2x^4 + 5x^2 + 5$, odd part $-5x^3 + 4x$
13	$\mathbb{R} - \{-4, 1\}$
14	$f(3) = -18$
15	505
16	$f^{-1}(x) = \frac{2x-6}{2x-5}$
17	$(8, \infty)$
18	$[\frac{1}{2}, \frac{11}{2}]$
19	$(4, 8)$
20	$y = -x^2 + 10x - 24$

Solutions

1. If $A = \{6, 9\}$ and $B = \{q, s, t, w\}$, how many functions are there from B to A? [1]

$n(B) = 4,\ n(A) = 2$

Number of functions from B to $A = \{n(A)\}^{n(B)} = 2^4 = 16$

Ans: 16

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2. For $f(x) = -3x - 2$ on the domain $A = \{-3, -2, -1, 4\}$, find the pre-image of 7. [1]

$f(x) = 7 \Rightarrow -3x - 2 = 7$

$\Rightarrow x = -3$

$-3 \in A$, so the pre-image is -3.

Ans: $x = -3$

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3. How many constant functions are there from a set with 5 elements to a set with 6 elements? [1]

A constant function sends every element to the same value k.

There is one constant function for each element of the co-domain: 6.

Ans: 6

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4. Find the minimum value of $f(x) = x^2 - 8x + 24$. [2]

$\frac{4ac - b^2}{4a} = \frac{4(1)(24) - (-8)^2}{4(1)} = 8$

It occurs at $x = -\frac{b}{2a} = 4$.

Ans: 8

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5. Solve $4x^2 - 16x + 12 \geq 0$. [2]

$4x^2 - 16x + 12 = 4\,(x - 3)\,(x - 1)$

The roots are 1 and 3, and $a = 4 > 0$.

The expression has the sign of a outside the roots.

Solution set: $(-\infty, 1] \cup [3, \infty)$

Ans: $x \in (-\infty, 1] \cup [3, \infty)$

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6. Solve $|3x - 3| \geq 5$. [2]

$3x - 3 \geq 5$ or $3x - 3 \leq -5$

Solution set: $(-\infty, -\frac{2}{3}] \cup [\frac{8}{3}, \infty)$

Ans: $x \in (-\infty, -\frac{2}{3}] \cup [\frac{8}{3}, \infty)$

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7. Find the value of $[-3.7] + [-8.1]$, where $[x]$ is the greatest integer function. [1]

$[-3.7] = -4,\ [-8.1] = -9$

$[-3.7] + [-8.1] = -13$

Ans: -13

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8. Using $[x] + \left[x + \frac{1}{n}\right] + \cdots + \left[x + \frac{n-1}{n}\right] = [nx]$, evaluate $[0.06] + \left[0.06 + \frac{1}{3}\right] + \left[0.06 + \frac{2}{3}\right]$. [3]

Here $x = 0.06$, $n = 3$.

$\text{Sum} = [3 \times 0.06] = [0.18] = 0$

Check: the terms are 0, 0, 0.

Ans: 0

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9. Find the value of $2^{3\log_2 15}$. [1]

$2^{3\log_2 15} = 2^{\log_2 15^3}$

$= 15^3 = 3375$ (using $a^{\log_a N} = N$)

Ans: 3375

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10. If $f(x) = 5 - 2x$ and $g(x) = 2x^2 + 2$, find $(g \circ f)(x)$. [2]

$(g \circ f)(x) = g(f(x)) = g(5 - 2x)$

$= 2(5 - 2x)^2 + 2$

$= 8x^2 - 40x + 52$

Ans: $(g \circ f)(x) = 8x^2 - 40x + 52$

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11. If $f(x) = \dfrac{7 - 5x}{3x + 5}$, show that $(f \circ f)(x) = x$ and hence find $(f \circ f \circ f)(-1)$. [5]

$(f \circ f)(x) = f\left(\frac{7-5x}{3x+5}\right) = \dfrac{-5\left(\frac{7-5x}{3x+5}\right) + 7}{3\left(\frac{7-5x}{3x+5}\right) + 5}$

Multiply the numerator and the denominator by $3x + 5$:

$= \dfrac{(21x + 35) + (25x - 35)}{(21 - 15x) + (15x + 25)} = \dfrac{46x}{46}$

$= x$

So $f \circ f = I$, and $(f \circ f \circ f)(x) = f(x)$.

$(f \circ f \circ f)(-1) = f(-1) = \frac{12}{2} = 6$

Ans: $(f \circ f)(x) = x$; $(f \circ f \circ f)(-1) = 6$

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12. Write $f(x) = 2x^4 - 5x^3 + 5x^2 + 4x + 5$ as the sum of an even function and an odd function. [3]

$f(x) = \frac{1}{2}[f(x) + f(-x)] + \frac{1}{2}[f(x) - f(-x)]$

$f(-x) = 2x^4 + 5x^3 + 5x^2 - 4x + 5$

$\frac{1}{2}[f(x) + f(-x)] = 2x^4 + 5x^2 + 5$ (even)

$\frac{1}{2}[f(x) - f(-x)] = -5x^3 + 4x$ (odd)

$f(x) = \left(2x^4 + 5x^2 + 5\right) + \left(-5x^3 + 4x\right)$

Ans: Even part $2x^4 + 5x^2 + 5$, odd part $-5x^3 + 4x$

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13. If $f(x) = x - 4$ and $g(x) = x^2 + 3x - 4$, find the domain of $\dfrac{f}{g}$. [2]

$D_f = D_g = \mathbb{R}$, and $\frac{f}{g}$ needs $g(x) \neq 0$.

$(x - 1)(x + 4) = 0 \Rightarrow x = -4, 1$

Domain $= \mathbb{R} - \{-4, 1\}$

Ans: $\mathbb{R} - \{-4, 1\}$

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14. f satisfies $f(x+y) = f(x) + f(y)$ for all real x, y, and $f(6) = -36$. Find $f(3)$. [3]

$f(x+y) = f(x) + f(y) \Rightarrow f(x) = kx$.

$f(6) = -36 \Rightarrow 6k = -36 \Rightarrow k = -6$

$f(3) = -6 \times 3 = -18$

Ans: $f(3) = -18$

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15. How many many-one functions are there from $A = \{6, 8, 10, 11\}$ to $B = \{d, e, f, g, h\}$? [2]

$\text{All functions} = 5^4 = 625$

$\text{One-one} = {}^5P_4 = 120$

$\text{Many-one} = 625 - 120 = 505$

Ans: 505

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16. Let $A = \mathbb{R} - \{1\}$ and $B = \mathbb{R} - \left\{\frac{5}{2}\right\}$. Show that $f : A \to B$, $f(x) = \frac{5x-6}{2x-2}$ is a bijection and find f^{-1}. [5]

One-one: $f(x_1) = f(x_2) \Rightarrow (5x_1 - 6)(2x_2 - 2) = (5x_2 - 6)(2x_1 - 2)$

$\Rightarrow 2(x_1 - x_2) = 0 \Rightarrow x_1 = x_2$, so f is one-one.

Onto: for $y \in B$, $y = \frac{5x-6}{2x-2} \Rightarrow y(2x-2) = 5x - 6$

$\Rightarrow x = \frac{2y-6}{2y-5}$, defined since $y \neq \frac{5}{2}$, and $x \neq 1$ (else $-10 = -12$).

So $x \in A$ and $f(x) = y$: f is onto. Hence f is a bijection.

$f^{-1}(x) = \frac{2x-6}{2x-5}$

Ans: $f^{-1}(x) = \frac{2x-6}{2x-5}$

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17. Find the domain of $f(x) = \sqrt{x^2 - 36} + \log_e(x - 8)$. [5]

We need $x^2 - 36 \geq 0$ and $x - 8 > 0$.

$x^2 - 36 = (x-6)(x+6) \geq 0$: the roots are $-6 < 6$ and $a > 0$.

$\Rightarrow x \in (-\infty, -6] \cup [6, \infty)$

$x - 8 > 0 \Rightarrow x > 8$

Common part: domain $= (8, \infty)$

Ans: $(8, \infty)$

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18. Find the range of $f(x) = \dfrac{x^2 - x + 3}{x^2 + 2x + 2}$, $x \in \mathbb{R}$. [5]

The denominator $x^2 + 2x + 2$ has $\Delta = -4 < 0$, so it is never zero: the domain is $\mathbb{R}$.

$y = \frac{x^2-x+3}{x^2+2x+2} \Rightarrow y(x^2 + 2x + 2) = x^2 - x + 3$

$\Rightarrow (y-1)x^2 + (2y+1)x + (2y-3) = 0$

For $y \neq 1$, x is real, so $\Delta = (2y+1)^2 - 4(y-1)(2y-3) \geq 0$

$\Rightarrow -4y^2 + 24y - 11 \geq 0$, $\Rightarrow \frac{1}{2} \leq y \leq \frac{11}{2}$

The value $y = 1$ is taken at $x = \frac{1}{3}$, and 1 lies in this interval.

Range $= [\frac{1}{2}, \frac{11}{2}]$

Ans: $[\frac{1}{2}, \frac{11}{2}]$

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19. The point $(8, 4)$ lies on $y = f(x)$. Which point must lie on $y = f^{-1}(x)$? [1]

The graph of $y = f^{-1}(x)$ is obtained by reflection in $y = x$: $(x, y) \to (y, x)$.

So $(8, 4) \to (4, 8)$.

Ans: $(4, 8)$

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20. The graph of $y = -x^2$ is shifted 5 units to the right and 1 units up. Write the equation of the new graph. [3]

Shifting right by 5 replaces x by $x - 5$.

Shifting up by 1 adds 1 to y.

$y = -(x - 5)^2 + 1 = -x^2 + 10x - 24$

Ans: $y = -x^2 + 10x - 24$
