EXPRESSWALA ACADEMY

Maths 1A: Determinants

Question Paper, Set 09

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Find $\det A$ if $A = \begin{bmatrix} 5\sqrt{7} & 2 \\ -3 & -\sqrt{7} \end{bmatrix}$. [1 mark]

2. Solve $\begin{vmatrix} x+3 & -1 \\ 3 & x-6 \end{vmatrix} = 3$. [2 marks]

3. Write the minor and the cofactor of the element in row 2, column 1 of $\begin{bmatrix} -3 & -7 & 0 \\ 3 & 6 & 1 \\ 5 & -3 & -3 \end{bmatrix}$. [2 marks]

4. Find $\det A$ for $A = \begin{bmatrix} -3 & 3 & 4 \\ 0 & 8 & 5 \\ -1 & -4 & -3 \end{bmatrix}$. [3 marks]

5. If $\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix} = 6$, find $\begin{vmatrix} x & y & z \\ p & q & r \\ a & b & c \end{vmatrix}$. [1 mark]

6. Without expanding, find the value of $\begin{vmatrix} 7 & -4 & 7 \\ 2 & 1 & 9 \\ -21 & 12 & -21 \end{vmatrix}$. [1 mark]

7. Using the properties of determinants, evaluate $\begin{vmatrix} -3 & -8 & -1 \\ 7 & 11 & 1 \\ -7 & 53 & 5 \end{vmatrix}$. [3 marks]

8. For $A = \begin{bmatrix} 8 & 7 & 4 \\ 0 & 1 & 2 \\ -4 & -1 & 8 \end{bmatrix}$, with A_{ij} the cofactor of a_{ij}, find $a_{11}A_{12} + a_{21}A_{22} + a_{31}A_{32}$. [1 mark]

9. Using properties of determinants, solve $\begin{vmatrix} x & 7 & 7 \\ 7 & x & 7 \\ 7 & 7 & x \end{vmatrix} = 0$ for x. [5 marks]

10. A is a square matrix of order 3 with $|A| = 4$. Find $|3A|$ and the determinant of the cofactor matrix of A. [2 marks]

11. Find the determinant of $\begin{bmatrix} 0 & 5 & 1 \\ -5 & 0 & 5 \\ -1 & -5 & 0 \end{bmatrix}$. [1 mark]

12. Using a standard result, find $\begin{vmatrix} 1 & 2 & 8 \\ 1 & -4 & -64 \\ 1 & 3 & 27 \end{vmatrix}$. [2 marks]

13. Find the value of $\begin{vmatrix} 25 & 36 & 49 \\ 36 & 49 & 64 \\ 49 & 64 & 81 \end{vmatrix}$ using a standard result. [3 marks]

14. Show that $\begin{vmatrix} n^2 & (n+1)^2 & (n+2)^2 \\ (n+1)^2 & (n+2)^2 & (n+3)^2 \\ (n+2)^2 & (n+3)^2 & (n+4)^2 \end{vmatrix} = -8$. [5 marks]

15. By writing it as a product of two determinants, evaluate $\begin{vmatrix} (3+2)^2 & (3+5)^2 & (3+3)^2 \\ (2+2)^2 & (2+5)^2 & (2+3)^2 \\ (5+2)^2 & (5+5)^2 & (5+3)^2 \end{vmatrix}$. [5 marks]

16. If $\Delta(x) = \begin{vmatrix} 3x-2 & 2x+1 \\ -3 & 4 \end{vmatrix}$, find $\Delta'(x)$. [2 marks]

17. If $\Delta(x) = \begin{vmatrix} 4x-1 & x^2+2x & 1 \\ 2 & 1-2x & 2x+1 \\ 1 & -2 & -3 \end{vmatrix}$, find $\Delta'(x)$ and $\Delta'(-1)$. [5 marks]

18. Evaluate $\int_0^2 \begin{vmatrix} x^3 & 2 \\ x & 6 \end{vmatrix} \, dx$. [3 marks]

19. Is $\begin{bmatrix} 3 & -1 & -2 \\ -4 & -5 & 8 \\ 13 & 2 & -14 \end{bmatrix}$ singular or non-singular? [1 mark]

20. For what value of p is $\begin{bmatrix} p & -2 \\ -9 & 3 \end{bmatrix}$ singular? [2 marks]

Key sheet

Q	Answer
1	-29
2	$x = -3$ or 6
3	Minor 21, cofactor -21
4	29
5	-6
6	0
7	-118
8	0
9	$x = -14,\ 7$
10	108 and 16
11	0
12	-42
13	-8
14	-8
15	-72
16	$\Delta'(x) = 18$
17	$\Delta'(x) = 6x^2 + 102x + 2$, $\Delta'(-1) = -94$
18	20
19	Singular ($\lvert A\rvert = 0$)
20	$p = 6$

Solutions

1. Find $\det A$ if $A = \begin{bmatrix} 5\sqrt{7} & 2 \\ -3 & -\sqrt{7} \end{bmatrix}$. [1]

$\begin{vmatrix} 5\sqrt{7} & 2 \\ -3 & -\sqrt{7} \end{vmatrix} = 5\sqrt{7} \cdot (-\sqrt{7}) - 2 \cdot (-3)$

$= -35 - (-6) = -29$

Ans: -29

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2. Solve $\begin{vmatrix} x+3 & -1 \\ 3 & x-6 \end{vmatrix} = 3$. [2]

$(x+3)(x-6) - (-1) \cdot 3 = 3$

$\Rightarrow x^2 - 3x - 18 = 0$

$\Rightarrow (x+3)(x-6) = 0$

$\therefore x = -3 \text{ or } x = 6$

Ans: $x = -3$ or 6

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3. Write the minor and the cofactor of the element in row 2, column 1 of $\begin{bmatrix} -3 & -7 & 0 \\ 3 & 6 & 1 \\ 5 & -3 & -3 \end{bmatrix}$. [2]

Delete row 2 and column 1.

$M_{21} = \begin{vmatrix} -7 & 0 \\ -3 & -3 \end{vmatrix} = 21 - 0 = 21$

$A_{21} = (-1)^{2+1} M_{21} = -21$

Ans: Minor 21**, cofactor** -21

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4. Find $\det A$ for $A = \begin{bmatrix} -3 & 3 & 4 \\ 0 & 8 & 5 \\ -1 & -4 & -3 \end{bmatrix}$. [3]

Expand along the second row.

$\Delta = 8 \begin{vmatrix} -3 & 4 \\ -1 & -3 \end{vmatrix} - 5 \begin{vmatrix} -3 & 3 \\ -1 & -4 \end{vmatrix}$

$= 8(9 - (-4)) - 5(12 - (-3))$

$= 8(13) - 5(15) = 29$

Ans: 29

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5. If $\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix} = 6$, find $\begin{vmatrix} x & y & z \\ p & q & r \\ a & b & c \end{vmatrix}$. [1]

Interchanging two rows changes the sign.

$\therefore \text{value} = (-1) \times 6 = -6$

Ans: -6

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6. Without expanding, find the value of $\begin{vmatrix} 7 & -4 & 7 \\ 2 & 1 & 9 \\ -21 & 12 & -21 \end{vmatrix}$. [1]

$R_3 = -3R_1$: two rows are proportional.

$\therefore \Delta = 0$

Ans: 0

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7. Using the properties of determinants, evaluate $\begin{vmatrix} -3 & -8 & -1 \\ 7 & 11 & 1 \\ -7 & 53 & 5 \end{vmatrix}$. [3]

Apply $R_3 \to R_3 + 2R_2$ (the value does not change):

$$\Delta = \begin{vmatrix} -3 & -8 & -1 \\ 7 & 11 & 1 \\ 7 & 75 & 7 \end{vmatrix}$$

Apply $C_2 \to C_2 - 11C_3$:

$$\Delta = \begin{vmatrix} -3 & 3 & -1 \\ 7 & 0 & 1 \\ 7 & -2 & 7 \end{vmatrix}$$

$$\Delta = -7\begin{vmatrix} 3 & -1 \\ -2 & 7 \end{vmatrix} - \begin{vmatrix} -3 & 3 \\ 7 & -2 \end{vmatrix}$$

$$= -7(21 - 2) - (6 - 21)$$

$$= -7(19) - (-15) = -118$$

Ans: -118

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8. For $A = \begin{bmatrix} 8 & 7 & 4 \\ 0 & 1 & 2 \\ -4 & -1 & 8 \end{bmatrix}$, with A_{ij} the cofactor of a_{ij}, find $a_{11}A_{12} + a_{21}A_{22} + a_{31}A_{32}$. [1]

This is the elements of column 1 with the cofactors of column 2.

The sum of products of the elements of one line with the cofactors of a different line is 0.

$\therefore a_{11}A_{12} + a_{21}A_{22} + a_{31}A_{32} = 0$

Ans: 0

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9. Using properties of determinants, solve $\begin{vmatrix} x & 7 & 7 \\ 7 & x & 7 \\ 7 & 7 & x \end{vmatrix} = 0$ for x. [5]

At $x = 7$ all three rows are equal, so $(x - 7)^2$ is a factor (factor theorem).

$C_1 \to C_1 + C_2 + C_3$:

$$\Delta = (x + 14)\begin{vmatrix} 1 & 7 & 7 \\ 1 & x & 7 \\ 1 & 7 & x \end{vmatrix}$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$:

$$\Delta = (x + 14)\begin{vmatrix} 1 & 7 & 7 \\ 0 & x - 7 & 0 \\ 0 & 0 & x - 7 \end{vmatrix} = (x + 14)(x - 7)^2$$

$\Delta = 0 \Rightarrow x = 7$ or $x = -14$

Ans: $x = -14,\ 7$

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10. A is a square matrix of order 3 with $|A| = 4$. Find $|3A|$ and the determinant of the cofactor matrix of A. [2]

$|3A| = 3^3|A| = 27 \times 4 = 108$

$\det \text{ of cofactor matrix} = |A|^{3-1} = 4^2 = 16$

Ans: 108 **and** 16

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11. Find the determinant of $\begin{bmatrix} 0 & 5 & 1 \\ -5 & 0 & 5 \\ -1 & -5 & 0 \end{bmatrix}$. [1]

$A^T = -A$: the matrix is skew-symmetric of odd order 3.

The determinant of a skew-symmetric matrix of odd order is 0.

Ans: 0

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12. Using a standard result, find $\begin{vmatrix} 1 & 2 & 8 \\ 1 & -4 & -64 \\ 1 & 3 & 27 \end{vmatrix}$. [2]

Use result 4, $(a+b+c)(a-b)(b-c)(c-a)$ with $a, b, c = 2, -4, 3$.

$(1)(2-(-4))(-4-3)(3-2) = -42$

Ans: -42

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13. Find the value of $\begin{vmatrix} 25 & 36 & 49 \\ 36 & 49 & 64 \\ 49 & 64 & 81 \end{vmatrix}$ using a standard result. [3]

The entries are squares of consecutive numbers, each row starting one later; this is result 11 with $n = 5$.

$\Delta = -8$

Ans: -8

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14. Show that $\begin{vmatrix} n^2 & (n+1)^2 & (n+2)^2 \\ (n+1)^2 & (n+2)^2 & (n+3)^2 \\ (n+2)^2 & (n+3)^2 & (n+4)^2 \end{vmatrix} = -8$. [5]

$C_3 \to C_3 - C_2$, $C_2 \to C_2 - C_1$ (using $(k+1)^2 - k^2 = 2k+1$):

$$\Delta = \begin{vmatrix} n^2 & 2n+1 & 2n+3 \\ (n+1)^2 & 2n+3 & 2n+5 \\ (n+2)^2 & 2n+5 & 2n+7 \end{vmatrix}$$

$C_3 \to C_3 - C_2$:

$$\Delta = \begin{vmatrix} n^2 & 2n+1 & 2 \\ (n+1)^2 & 2n+3 & 2 \\ (n+2)^2 & 2n+5 & 2 \end{vmatrix}$$

$R_3 \to R_3 - R_2$, $R_2 \to R_2 - R_1$:

$$\Delta = \begin{vmatrix} n^2 & 2n+1 & 2 \\ 2n+1 & 2 & 0 \\ 2n+3 & 2 & 0 \end{vmatrix}$$

Expand along C_3:

$= 2\,[2(2n+1) - 2(2n+3)] = 2(-4) = -8$

Ans: -8

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15. By writing it as a product of two determinants, evaluate $\begin{vmatrix}(3+2)^2 & (3+5)^2 & (3+3)^2\\(2+2)^2 & (2+5)^2 & (2+3)^2\\(5+2)^2 & (5+5)^2 & (5+3)^2\end{vmatrix}$. [5]

$(u+v)^2 = u^2\cdot 1 + 1\cdot v^2 + 2u\cdot v$

So, multiplying row by column,

$$\Delta = \begin{vmatrix}9 & 1 & 6\\4 & 1 & 4\\25 & 1 & 10\end{vmatrix}\begin{vmatrix}1 & 1 & 1\\4 & 25 & 9\\2 & 5 & 3\end{vmatrix}$$

$$\begin{vmatrix}9 & 1 & 6\\4 & 1 & 4\\25 & 1 & 10\end{vmatrix} = -12$$

$$\begin{vmatrix}1 & 1 & 1\\4 & 25 & 9\\2 & 5 & 3\end{vmatrix} = 6$$

$\therefore \Delta = (-12)\times 6 = -72$

Ans: -72

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16. If $\Delta(x) = \begin{vmatrix}3x-2 & 2x+1\\-3 & 4\end{vmatrix}$, find $\Delta'(x)$. [2]

$$\Delta'(x) = \begin{vmatrix}3 & 2\\-3 & 4\end{vmatrix} + \begin{vmatrix}3x-2 & 2x+1\\0 & 0\end{vmatrix}$$

$= (18) + (0)$

$= 18$

Ans: $\Delta'(x) = 18$

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17. If $\Delta(x) = \begin{vmatrix}4x-1 & x^2+2x & 1\\2 & 1-2x & 2x+1\\1 & -2 & -3\end{vmatrix}$, find $\Delta'(x)$ and $\Delta'(-1)$. [5]

Differentiate one row at a time and add:

$$\Delta'(x) = \begin{vmatrix}4 & 2x+2 & 0\\2 & 1-2x & 2x+1\\1 & -2 & -3\end{vmatrix}$$

$$+\begin{vmatrix}4x-1 & x^2+2x & 1\\0 & -2 & 2\\1 & -2 & -3\end{vmatrix}$$

$$+\begin{vmatrix}4x-1 & x^2+2x & 1\\2 & 1-2x & 2x+1\\0 & 0 & 0\end{vmatrix}$$

The third determinant has a row of zeros, so it is 0.

$$\begin{vmatrix}4 & 2x+2 & 0\\2 & 1-2x & 2x+1\\1 & -2 & -3\end{vmatrix} = 4x^2+58x+10$$

$$\begin{vmatrix}4x-1 & x^2+2x & 1\\0 & -2 & 2\\1 & -2 & -3\end{vmatrix} = 2x^2+44x-8$$

$\therefore \Delta'(x) = 6x^2+102x+2$

$\Delta'(-1) = -94$

Ans: $\Delta'(x) = 6x^2+102x+2$, $\Delta'(-1) = -94$

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18. Evaluate $\int_0^2 \begin{vmatrix} x^3 & 2 \\ x & 6 \end{vmatrix} dx$. [3]

Only one column contains x, so integrate that column:

$$\begin{vmatrix} \int_0^2 x^3\,dx & 2 \\ \int_0^2 x\,dx & 6 \end{vmatrix} = \begin{vmatrix} 4 & 2 \\ 2 & 6 \end{vmatrix}$$

$= 20$

Ans: 20

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19. Is $\begin{bmatrix} 3 & -1 & -2 \\ -4 & -5 & 8 \\ 13 & 2 & -14 \end{bmatrix}$ singular or non-singular? [1]

$R_3 = 3R_1 - R_2$, so $R_3 \to R_3 - (3R_1 - R_2)$ gives a row of zeros.

$|A| = 0$

So the matrix is singular.

Ans: Singular ($|A| = 0$)

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20. For what value of p is $\begin{bmatrix} p & -2 \\ -9 & 3 \end{bmatrix}$ singular? [2]

$$\begin{vmatrix} p & -2 \\ -9 & 3 \end{vmatrix} = 3p - 18 = 0$$

$\Rightarrow p = 6$

Ans: $p = 6$
