EXPRESSWALA ACADEMY

Maths 1A: Determinants

Question Paper, Set 07

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Evaluate $\begin{vmatrix} -8 & -8 \\ -3 & -2 \end{vmatrix}$. [1 mark]

2. Solve $\begin{vmatrix} x+5 & 2 \\ -3 & x+8 \end{vmatrix} = 4$. [2 marks]

3. Find the minor and the cofactor of the element 2 in $\begin{bmatrix} -6 & 0 & -6 \\ 1 & 2 & -6 \\ 6 & -2 & 5 \end{bmatrix}$. [2 marks]

4. Evaluate $\begin{vmatrix} 7 & 1 & 6 \\ -2 & 0 & 8 \\ -2 & 2 & -5 \end{vmatrix}$. [3 marks]

5. If $\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix} = -6$, find $\begin{vmatrix} 3a & 3b & 3c \\ 3p & 3q & 3r \\ 3x & 3y & 3z \end{vmatrix}$. [1 mark]

6. Without expanding, find the value of $\begin{vmatrix} 5 & 8 & 9 \\ -3 & 8 & -6 \\ 2 & 16 & 3 \end{vmatrix}$. [1 mark]

7. Using the properties of determinants, evaluate $\begin{vmatrix} -4 & -42 & 4 \\ -8 & -86 & 6 \\ -3 & -23 & 7 \end{vmatrix}$. [3 marks]

8. For $A = \begin{bmatrix} -5 & 8 & 6 \\ -2 & 6 & 8 \\ -3 & 8 & 0 \end{bmatrix}$, with A_{ij} the cofactor of a_{ij}, find $a_{21}A_{11} + a_{22}A_{12} + a_{23}A_{13}$. [1 mark]

9. Solve $\begin{vmatrix} x+4 & -2 & 3 \\ 4 & x-2 & 3 \\ 4 & -2 & x+3 \end{vmatrix} = 0$. [5 marks]

10. If A is a 2×2 matrix and $|A| = 5$, find $|A^2|$ and $|-A|$. [2 marks]

11. Find the determinant of $\begin{bmatrix} 3 & 0 & 0 \\ -8 & 1 & 0 \\ 4 & -4 & 3 \end{bmatrix}$. [1 mark]

12. Using a standard result, find $\begin{vmatrix} 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & -1 & 1 \end{vmatrix}$. [2 marks]

13. Find the value of $\begin{vmatrix} 7 & 7 & 4 \\ 6 & 8 & 4 \\ 6 & 7 & 5 \end{vmatrix}$ using a standard result. [3 marks]

14. Show that $\begin{vmatrix} a+1 & 1 & 1 \\ 1 & b+1 & 1 \\ 1 & 1 & c+1 \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$. [5 marks]

15. By writing it as a product of two determinants, evaluate $\begin{vmatrix} (1+1)^2 & (1+3)^2 & (1+4)^2 \\ (5+1)^2 & (5+3)^2 & (5+4)^2 \\ (3+1)^2 & (3+3)^2 & (3+4)^2 \end{vmatrix}$. [5 marks]

16. If $\Delta(x) = \begin{vmatrix} x^2+1 & \cos x \\ -4 & -3 \end{vmatrix}$, find $\Delta'(x)$. [2 marks]

17. If $\Delta(x) = \begin{vmatrix} x^2+2x & x^3+x & x^3 \\ 4x+4 & 4 & 2-2x \\ -2 & -2 & -2 \end{vmatrix}$, find $\Delta'(x)$ and $\Delta'(2)$. [5 marks]

18. Evaluate $\int_0^2 \begin{vmatrix} x & x^3 \\ 1 & -1 \end{vmatrix} dx$. [3 marks]

19. Is $\begin{bmatrix} -6 & -2 & 4 \\ 4 & 7 & 1 \\ -16 & -11 & 7 \end{bmatrix}$ singular or non-singular? [1 mark]

20. For what value of p is $\begin{bmatrix} -2 & -6 \\ -5 & p \end{bmatrix}$ singular? [2 marks]

Key sheet

Q	Answer
1	-8
2	$x = -7$ or -6
3	Minor 6, cofactor 6
4	-162
5	-162
6	0
7	-36
8	0
9	$x = -5,\ 0$
10	25 and 5
11	9
12	12
13	18
14	$abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$
15	192
16	$\Delta'(x) = -6x - 4\sin x$
17	$\Delta'(x) = 16x^3 - 4$, $\Delta'(2) = 124$
18	-6
19	Singular ($\lvert A\rvert = 0$)
20	$p = -15$

Solutions

1. Evaluate $\begin{vmatrix} -8 & -8 \\ -3 & -2 \end{vmatrix}$. [1]

$$\begin{vmatrix} -8 & -8 \\ -3 & -2 \end{vmatrix} = (-8)\cdot(-2) - (-8)\cdot(-3)$$

$$= 16 - 24 = -8$$

Ans: -8

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2. Solve $\begin{vmatrix} x+5 & 2 \\ -3 & x+8 \end{vmatrix} = 4$. [2]

$$(x+5)(x+8) - 2\cdot(-3) = 4$$

$$\Rightarrow x^2 + 13x + 42 = 0$$

$$\Rightarrow (x+7)(x+6) = 0$$

$$\therefore x = -7 \text{ or } x = -6$$

Ans: $x = -7$ or -6

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3. Find the minor and the cofactor of the element 2 in $\begin{bmatrix} -6 & 0 & -6 \\ 1 & 2 & -6 \\ 6 & -2 & 5 \end{bmatrix}$. [2]

Delete row 2 and column 2.

$$M_{22} = \begin{vmatrix} -6 & -6 \\ 6 & 5 \end{vmatrix} = -30 - (-36) = 6$$

$$A_{22} = (-1)^{2+2} M_{22} = 6$$

Ans: Minor 6**, cofactor** 6

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4. Evaluate $\begin{vmatrix} 7 & 1 & 6 \\ -2 & 0 & 8 \\ -2 & 2 & -5 \end{vmatrix}$. [3]

Expand along the second row.

$$\Delta = 2\begin{vmatrix} 1 & 6 \\ 2 & -5 \end{vmatrix} - 8\begin{vmatrix} 7 & 1 \\ -2 & 2 \end{vmatrix}$$

$$= 2(-5-12) - 8(14-(-2))$$

$$= 2(-17) - 8(16) = -162$$

Ans: -162

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5. If $\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix} = -6$, find $\begin{vmatrix} 3a & 3b & 3c \\ 3p & 3q & 3r \\ 3x & 3y & 3z \end{vmatrix}$. [1]

Every element is multiplied by 3: each of the 3 rows gives a factor 3.

$$\therefore \text{value} = 27 \times (-6) = -162$$

Ans: -162

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6. Without expanding, find the value of $\begin{vmatrix} 5 & 8 & 9 \\ -3 & 8 & -6 \\ 2 & 16 & 3 \end{vmatrix}$. [1]

$R_3 \to R_3 - R_1 - R_2$ gives a row of zeros (value unchanged).

$\therefore \Delta = 0$

Ans: 0

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7. Using the properties of determinants, evaluate $\begin{vmatrix} -4 & -42 & 4 \\ -8 & -86 & 6 \\ -3 & -23 & 7 \end{vmatrix}$. [3]

Apply $R_2 \to R_2 - 2R_1$ (the value does not change):

$$\Delta = \begin{vmatrix} -4 & -42 & 4 \\ 0 & -2 & -2 \\ -3 & -23 & 7 \end{vmatrix}$$

Apply $C_2 \to C_2 - 10C_1$:

$$\Delta = \begin{vmatrix} -4 & -2 & 4 \\ 0 & -2 & -2 \\ -3 & 7 & 7 \end{vmatrix}$$

$$\Delta = -2\begin{vmatrix} -4 & 4 \\ -3 & 7 \end{vmatrix} + 2\begin{vmatrix} -4 & -2 \\ -3 & 7 \end{vmatrix}$$

$$= -2(-28 - (-12)) + 2(-28 - 6)$$

$$= -2(-16) + 2(-34) = -36$$

Ans: -36

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8. For $A = \begin{bmatrix} -5 & 8 & 6 \\ -2 & 6 & 8 \\ -3 & 8 & 0 \end{bmatrix}$, with A_{ij} the cofactor of a_{ij}, find $a_{21}A_{11} + a_{22}A_{12} + a_{23}A_{13}$. [1]

This is the elements of row 2 with the cofactors of row 1.

The sum of products of the elements of one line with the cofactors of a different line is 0.

$\therefore a_{21}A_{11} + a_{22}A_{12} + a_{23}A_{13} = 0$

Ans: 0

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9. Solve $\begin{vmatrix} x+4 & -2 & 3 \\ 4 & x-2 & 3 \\ 4 & -2 & x+3 \end{vmatrix} = 0$. [5]

$C_1 \to C_1 + C_2 + C_3$: every entry of C_1 becomes $x + 5$.

$$\Delta = (x+5)\begin{vmatrix} 1 & -2 & 3 \\ 1 & x-2 & 3 \\ 1 & -2 & x+3 \end{vmatrix}$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$:

$$\Delta = (x+5)\begin{vmatrix} 1 & -2 & 3 \\ 0 & x & 0 \\ 0 & 0 & x \end{vmatrix} = (x+5)x^2$$

$\Delta = 0 \Rightarrow x = 0$ or $x = -5$

Ans: $x = -5,\ 0$

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10. If A is a 2×2 matrix and $|A| = 5$, find $|A^2|$ and $|-A|$. [2]

$|A^2| = |A|\,|A| = 25$

$|-A| = (-1)^2|A| = 5$

Ans: 25 **and** 5

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11. Find the determinant of $\begin{bmatrix} 3 & 0 & 0 \\ -8 & 1 & 0 \\ 4 & -4 & 3 \end{bmatrix}$. [1]

The matrix is triangular: the determinant is the product of the diagonal elements.

$3 \times 1 \times 3 = 9$

Ans: 9

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12. Using a standard result, find $\begin{vmatrix} 1 & 2 & 4 \\ 1 & 3 & 9 \\ 1 & -1 & 1 \end{vmatrix}$. [2]

Use result 3, $(a-b)(b-c)(c-a)$ with $a, b, c = 2, 3, -1$.

$(2-3)(3-(-1))(-1-2) = 12$

Ans: 12

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13. Find the value of $\begin{vmatrix} 7 & 7 & 4 \\ 6 & 8 & 4 \\ 6 & 7 & 5 \end{vmatrix}$ using a standard result. [3]

Result 8 with $a, b, c = 6, 7, 4$:

$1 + a + b + c = 18$

Ans: 18

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14. Show that $\begin{vmatrix} a+1 & 1 & 1 \\ 1 & b+1 & 1 \\ 1 & 1 & c+1 \end{vmatrix} = abc\left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$. [5]

Take a, b, c out of R_1, R_2, R_3:

$\Delta = abc \begin{vmatrix} 1+\frac{1}{a} & \frac{1}{a} & \frac{1}{a} \\ \frac{1}{b} & 1+\frac{1}{b} & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & 1+\frac{1}{c} \end{vmatrix}$

$R_1 \to R_1 + R_2 + R_3$; every entry of R_1 becomes $1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$:

$\Delta = abc\left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) \begin{vmatrix} 1 & 1 & 1 \\ \frac{1}{b} & 1+\frac{1}{b} & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & 1+\frac{1}{c} \end{vmatrix}$

$C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$:

$= abc\left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) \begin{vmatrix} 1 & 0 & 0 \\ \frac{1}{b} & 1 & 0 \\ \frac{1}{c} & 0 & 1 \end{vmatrix} = abc\left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$

Ans: $abc\left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$

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15. By writing it as a product of two determinants, evaluate $\begin{vmatrix}(1+1)^2 & (1+3)^2 & (1+4)^2\\(5+1)^2 & (5+3)^2 & (5+4)^2\\(3+1)^2 & (3+3)^2 & (3+4)^2\end{vmatrix}$. [5]

$(u+v)^2 = u^2\cdot 1 + 1\cdot v^2 + 2u\cdot v$

So, multiplying row by column,

$$\Delta = \begin{vmatrix}1 & 1 & 2\\25 & 1 & 10\\9 & 1 & 6\end{vmatrix}\begin{vmatrix}1 & 1 & 1\\1 & 9 & 16\\1 & 3 & 4\end{vmatrix}$$

$$\begin{vmatrix}1 & 1 & 2\\25 & 1 & 10\\9 & 1 & 6\end{vmatrix} = -32$$

$$\begin{vmatrix}1 & 1 & 1\\1 & 9 & 16\\1 & 3 & 4\end{vmatrix} = -6$$

$\therefore \Delta = (-32)\times(-6) = 192$

Ans: 192

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16. If $\Delta(x) = \begin{vmatrix}x^2+1 & \cos x\\ -4 & -3\end{vmatrix}$, find $\Delta'(x)$. [2]

$$\Delta'(x) = \begin{vmatrix}2x & -\sin x\\ -4 & -3\end{vmatrix} + \begin{vmatrix}x^2+1 & \cos x\\ 0 & 0\end{vmatrix}$$

$= (-6x - 4\sin x) + (0)$

$= -6x - 4\sin x$

Ans: $\Delta'(x) = -6x - 4\sin x$

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17. If $\Delta(x) = \begin{vmatrix}x^2+2x & x^3+x & x^3\\ 4x+4 & 4 & 2-2x\\ -2 & -2 & -2\end{vmatrix}$, find $\Delta'(x)$ and $\Delta'(2)$. [5]

Differentiate one row at a time and add:

$$\Delta'(x) = \begin{vmatrix}2x+2 & 3x^2+1 & 3x^2\\ 4x+4 & 4 & 2-2x\\ -2 & -2 & -2\end{vmatrix}$$

$$+\begin{vmatrix}x^2+2x & x^3+x & x^3\\ 4 & 0 & -2\\ -2 & -2 & -2\end{vmatrix}$$

$$+\begin{vmatrix}x^2+2x & x^3+x & x^3\\ 4x+4 & 4 & 2-2x\\ 0 & 0 & 0\end{vmatrix}$$

The third determinant has a row of zeros, so it is 0.

$$\begin{vmatrix}2x+2 & 3x^2+1 & 3x^2\\ 4x+4 & 4 & 2-2x\\ -2 & -2 & -2\end{vmatrix} = 12x^3 + 4x^2 - 4x - 4$$

$$\begin{vmatrix}x^2+2x & x^3+x & x^3\\ 4 & 0 & -2\\ -2 & -2 & -2\end{vmatrix} = 4x^3 - 4x^2 + 4x$$

$\therefore \Delta'(x) = 16x^3 - 4$

$\Delta'(2) = 124$

Ans: $\Delta'(x) = 16x^3 - 4$, $\Delta'(2) = 124$

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18. Evaluate $\int_0^2 \begin{vmatrix} x & x^3 \\ 1 & -1 \end{vmatrix} dx$. [3]

Only one row contains x, so integrate that row:

$\begin{vmatrix} \int_0^2 x\,dx & \int_0^2 x^3\,dx \\ 1 & -1 \end{vmatrix} = \begin{vmatrix} 2 & 4 \\ 1 & -1 \end{vmatrix}$

$= -6$

Ans: -6

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19. Is $\begin{bmatrix} -6 & -2 & 4 \\ 4 & 7 & 1 \\ -16 & -11 & 7 \end{bmatrix}$ singular or non-singular? [1]

$R_3 = 2R_1 - R_2$, so $R_3 \to R_3 - (2R_1 - R_2)$ gives a row of zeros.

$|A| = 0$

So the matrix is singular.

Ans: Singular ($|A| = 0$)

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20. For what value of p is $\begin{bmatrix} -2 & -6 \\ -5 & p \end{bmatrix}$ singular? [2]

$\begin{vmatrix} -2 & -6 \\ -5 & p \end{vmatrix} = -2p - 30 = 0$

$\Rightarrow p = -15$

Ans: $p = -15$
