EXPRESSWALA ACADEMY

Maths 1A: Determinants

Question Paper, Set 06

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Find $\det A$ if $A = \begin{bmatrix} 5\sqrt{3} & 2 \\ -5 & -4\sqrt{3} \end{bmatrix}$. [1 mark]

2. Find x if $\begin{vmatrix} x+5 & -5 \\ 6 & x-3 \end{vmatrix} = 30$. [2 marks]

3. Write the minor and the cofactor of the element in row 1, column 3 of $\begin{bmatrix} -2 & -7 & 4 \\ 8 & 9 & 5 \\ 0 & -5 & 7 \end{bmatrix}$. [2 marks]

4. Find $\det A$ for $A = \begin{bmatrix} 8 & 7 & 1 \\ 0 & -4 & 6 \\ 3 & 8 & 0 \end{bmatrix}$. [3 marks]

5. If $\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix} = -4$, find $\begin{vmatrix} x & y & z \\ p & q & r \\ 4a & 4b & 4c \end{vmatrix}$. [1 mark]

6. Write the value of $\begin{vmatrix} 7 & 7 & 5 \\ -21 & -21 & -15 \\ -3 & 9 & -1 \end{vmatrix}$, giving a reason. [1 mark]

7. Simplify $\begin{vmatrix} -10 & -4 & 6 \\ -14 & -3 & 2 \\ -14 & -2 & 4 \end{vmatrix}$ by row and column operations and find its value. [3 marks]

8. For $A = \begin{bmatrix} -1 & 1 & 1 \\ -2 & 0 & -4 \\ 1 & 2 & -4 \end{bmatrix}$, with A_{ij} the cofactor of a_{ij}, find $a_{31}A_{21} + a_{32}A_{22} + a_{33}A_{23}$. [1 mark]

9. Using properties of determinants, solve $\begin{vmatrix} x & 3 & -5 \\ 3 & x & -5 \\ 3 & -5 & x \end{vmatrix} = 0$ for x. [5 marks]

10. A and B are square matrices of order 3 with $|A| = -6$ and $|B| = -1$. Find $|AB|$ and $|-2A^T|$. [2 marks]

11. Find the determinant of $\begin{bmatrix} 0 & 8 & -3 \\ -8 & 0 & -1 \\ 3 & 1 & 0 \end{bmatrix}$. [1 mark]

12. Evaluate $\begin{vmatrix} 1 & 2 & 8 \\ 1 & -1 & -1 \\ 1 & -3 & -27 \end{vmatrix}$ by a standard result. [2 marks]

13. Evaluate $\begin{vmatrix} 10 & 1 & 1 \\ 1 & 6 & 1 \\ 1 & 1 & 5 \end{vmatrix}$. [3 marks]

14. Show that $\begin{vmatrix} 1 & a & a^4 \\ 1 & b & b^4 \\ 1 & c & c^4 \end{vmatrix} = (a-b)(b-c)(c-a)(a^2+b^2+c^2+ab+bc+ca)$. [5 marks]

15. Express $\begin{vmatrix} (2-5)^2 & (2-2)^2 & (2-3)^2 \\ (5-5)^2 & (5-2)^2 & (5-3)^2 \\ (4-5)^2 & (4-2)^2 & (4-3)^2 \end{vmatrix}$ as the product of two determinants and hence find its value. [5 marks]

16. Differentiate $\Delta(x) = \begin{vmatrix} e^{2x} & \sin x \\ x^3 & x^2 \end{vmatrix}$ by differentiating one row at a time. [2 marks]

17. If $\Delta(x) = \begin{vmatrix} 4x-1 & x^3+x & x \\ x+4 & -2x-1 & -3x \\ -3 & -4 & 3 \end{vmatrix}$, find $\Delta'(x)$ and $\Delta'(-1)$. [5 marks]

18. Evaluate $\int_0^1 \begin{vmatrix} e^x & 4 \\ 2x+1 & 5 \end{vmatrix} dx$. [3 marks]

19. Is $\begin{bmatrix} 6 & -5 & 4 \\ 7 & 7 & 4 \\ 3 & 1 & 2 \end{bmatrix}$ singular or non-singular? [1 mark]

20. Find p if the matrix $\begin{bmatrix} 0 & 2 & 2 \\ p & 3 & 3 \\ -1 & 2 & -3 \end{bmatrix}$ is singular. [2 marks]

Key sheet

Q	Answer
1	-50
2	$x = -5$ or 3
3	Minor -40, cofactor -40
4	-246
5	16
6	0
7	-116
8	0
9	$x = -5,\ 2,\ 3$
10	6 and 48
11	0
12	60
13	281
14	$(a-b)(b-c)(c-a)(a^2+b^2+c^2+ab+bc+ca)$
15	-72
16	$\Delta'(x) = -x^3\cos x + 2x^2e^{2x} - 3x^2\sin x + 2xe^{2x}$
17	$\Delta'(x) = 24x^3 - 36x^2 - 152x - 25$, $\Delta'(-1) = 67$
18	$-13 + 5e$
19	Non-singular ($\lvert A\rvert = 14$)
20	$p = 0$

Solutions

1. Find $\det A$ if $A = \begin{bmatrix} 5\sqrt{3} & 2 \\ -5 & -4\sqrt{3} \end{bmatrix}$. [1]

$\begin{vmatrix} 5\sqrt{3} & 2 \\ -5 & -4\sqrt{3} \end{vmatrix} = 5\sqrt{3} \cdot (-4\sqrt{3}) - 2 \cdot (-5)$

$= -60 - (-10) = -50$

Ans: -50

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2. Find x if $\begin{vmatrix} x+5 & -5 \\ 6 & x-3 \end{vmatrix} = 30$. [2]

$(x+5)(x-3) - (-5) \cdot 6 = 30$

$\Rightarrow x^2 + 2x - 15 = 0$

$\Rightarrow (x+5)(x-3) = 0$

$\therefore x = -5 \text{ or } x = 3$

Ans: $x = -5$ or 3

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3. Write the minor and the cofactor of the element in row 1, column 3 of $\begin{bmatrix} -2 & -7 & 4 \\ 8 & 9 & 5 \\ 0 & -5 & 7 \end{bmatrix}$. [2]

Delete row 1 and column 3.

$M_{13} = \begin{vmatrix} 8 & 9 \\ 0 & -5 \end{vmatrix} = -40 - 0 = -40$

$A_{13} = (-1)^{1+3} M_{13} = -40$

Ans: Minor -40**, cofactor** -40

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4. Find $\det A$ for $A = \begin{bmatrix} 8 & 7 & 1 \\ 0 & -4 & 6 \\ 3 & 8 & 0 \end{bmatrix}$. [3]

Expand along the second row.

$\Delta = -4 \begin{vmatrix} 8 & 1 \\ 3 & 0 \end{vmatrix} - 6 \begin{vmatrix} 8 & 7 \\ 3 & 8 \end{vmatrix}$

$= -4(0 - 3) - 6(64 - 21)$

$= -4(-3) - 6(43) = -246$

Ans: -246

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5. If $\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix} = -4$, find $\begin{vmatrix} x & y & z \\ p & q & r \\ 4a & 4b & 4c \end{vmatrix}$. [1]

Two rows are interchanged (sign changes) and one row is multiplied by 4.

$\therefore \text{value} = (-4) \times (-4) = 16$

Ans: 16

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6. Write the value of $\begin{vmatrix} 7 & 7 & 5 \\ -21 & -21 & -15 \\ -3 & 9 & -1 \end{vmatrix}$, giving a reason. [1]

$R_2 = -3R_1$: two rows are proportional.

$\therefore \Delta = 0$

Ans: 0

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7. Simplify $\begin{vmatrix} -10 & -4 & 6 \\ -14 & -3 & 2 \\ -14 & -2 & 4 \end{vmatrix}$ by row and column operations and find its value. [3]

Apply $R_1 \to R_1 - 3R_2$ (the value does not change):

$$\Delta = \begin{vmatrix} 32 & 5 & 0 \\ -14 & -3 & 2 \\ -14 & -2 & 4 \end{vmatrix}$$

Apply $C_1 \to C_1 - 5C_2$:

$$\Delta = \begin{vmatrix} 7 & 5 & 0 \\ 1 & -3 & 2 \\ -4 & -2 & 4 \end{vmatrix}$$

$$\Delta = 7\begin{vmatrix} -3 & 2 \\ -2 & 4 \end{vmatrix} - 5\begin{vmatrix} 1 & 2 \\ -4 & 4 \end{vmatrix}$$

$$= 7(-12 - (-4)) - 5(4 - (-8))$$

$$= 7(-8) - 5(12) = -116$$

Ans: -116

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8. For $A = \begin{bmatrix} -1 & 1 & 1 \\ -2 & 0 & -4 \\ 1 & 2 & -4 \end{bmatrix}$, with A_{ij} the cofactor of a_{ij}, find $a_{31}A_{21} + a_{32}A_{22} + a_{33}A_{23}$. [1]

This is the elements of row 3 with the cofactors of row 2.

The sum of products of the elements of one line with the cofactors of a different line is 0.

$\therefore a_{31}A_{21} + a_{32}A_{22} + a_{33}A_{23} = 0$

Ans: 0

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9. Using properties of determinants, solve $\begin{vmatrix} x & 3 & -5 \\ 3 & x & -5 \\ 3 & -5 & x \end{vmatrix} = 0$ for x. [5]

$C_1 \to C_1 + C_2 + C_3$: every entry of C_1 becomes $x - 2$.

$$\Delta = (x-2)\begin{vmatrix} 1 & 3 & -5 \\ 1 & x & -5 \\ 1 & -5 & x \end{vmatrix}$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$:

$$\Delta = (x-2)\begin{vmatrix} 1 & 3 & -5 \\ 0 & x-3 & 0 \\ 0 & -8 & x+5 \end{vmatrix}$$

$$= (x-2)(x-3)(x+5)$$

$\Delta = 0 \Rightarrow x = 2,\ 3,\ -5$

Ans: $x = -5,\ 2,\ 3$

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10. A and B are square matrices of order 3 with $|A| = -6$ and $|B| = -1$. Find $|AB|$ and $|-2A^T|$. [2]

$|AB| = |A|\,|B| = (-6) \times (-1) = 6$

$|-2A^T| = (-2)^3|A^T| = -8 \times (-6) = 48$

Ans: 6 **and** 48

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11. Find the determinant of $\begin{bmatrix} 0 & 8 & -3 \\ -8 & 0 & -1 \\ 3 & 1 & 0 \end{bmatrix}$. [1]

$A^T = -A$: the matrix is skew-symmetric of odd order 3.

The determinant of a skew-symmetric matrix of odd order is 0.

Ans: 0

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12. Evaluate $\begin{vmatrix} 1 & 2 & 8 \\ 1 & -1 & -1 \\ 1 & -3 & -27 \end{vmatrix}$ by a standard result. [2]

Use result 4, $(a+b+c)(a-b)(b-c)(c-a)$ with $a, b, c = 2, -1, -3$.

$(-2)(2-(-1))(-1-(-3))(-3-2) = 60$

Ans: 60

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13. Evaluate $\begin{vmatrix} 10 & 1 & 1 \\ 1 & 6 & 1 \\ 1 & 1 & 5 \end{vmatrix}$. [3]

Result 9 with $a, b, c = 9, 5, 4$:

$abc\left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) = 180 + 20 + 36 + 45 = 281$

Ans: 281

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14. Show that $\begin{vmatrix} 1 & a & a^4 \\ 1 & b & b^4 \\ 1 & c & c^4 \end{vmatrix} = (a-b)(b-c)(c-a)(a^2+b^2+c^2+ab+bc+ca)$. [5]

$R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$, and take out $a-b$, $b-c$ (using $a^4 - b^4 = (a-b)(a+b)(a^2+b^2)$):

$$\Delta = (a-b)(b-c)\begin{vmatrix} 0 & 1 & (a+b)(a^2+b^2) \\ 0 & 1 & (b+c)(b^2+c^2) \\ 1 & c & c^4 \end{vmatrix}$$

Expand along C_1:

$= (a-b)(b-c)\left[(b+c)(b^2+c^2) - (a+b)(a^2+b^2)\right]$

$= (a-b)(b-c)\left[(c^3-a^3) + b(c^2-a^2) + b^2(c-a)\right]$

$= (a-b)(b-c)(c-a)(a^2+b^2+c^2+ab+bc+ca)$

Ans: $(a-b)(b-c)(c-a)(a^2+b^2+c^2+ab+bc+ca)$

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15. Express $\begin{vmatrix}(2-5)^2 & (2-2)^2 & (2-3)^2\\(5-5)^2 & (5-2)^2 & (5-3)^2\\(4-5)^2 & (4-2)^2 & (4-3)^2\end{vmatrix}$ as the product of two determinants and hence find its value. [5]

$(u-v)^2 = u^2\cdot 1 + 1\cdot v^2 - 2u\cdot v$

So, multiplying row by column,

$$\Delta = \begin{vmatrix}4 & 1 & -4\\25 & 1 & -10\\16 & 1 & -8\end{vmatrix}\begin{vmatrix}1 & 1 & 1\\25 & 4 & 9\\5 & 2 & 3\end{vmatrix}$$

$$\begin{vmatrix}4 & 1 & -4\\25 & 1 & -10\\16 & 1 & -8\end{vmatrix} = 12$$

$$\begin{vmatrix}1 & 1 & 1\\25 & 4 & 9\\5 & 2 & 3\end{vmatrix} = -6$$

$\therefore \Delta = 12\times(-6) = -72$

Ans: -72

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16. Differentiate $\Delta(x) = \begin{vmatrix}e^{2x} & \sin x\\ x^3 & x^2\end{vmatrix}$ by differentiating one row at a time. [2]

$$\Delta'(x) = \begin{vmatrix}2e^{2x} & \cos x\\ x^3 & x^2\end{vmatrix} + \begin{vmatrix}e^{2x} & \sin x\\ 3x^2 & 2x\end{vmatrix}$$

$$= (-x^3\cos x + 2x^2e^{2x}) + (-3x^2\sin x + 2xe^{2x})$$

$$= -x^3\cos x + 2x^2e^{2x} - 3x^2\sin x + 2xe^{2x}$$

Ans: $\Delta'(x) = -x^3\cos x + 2x^2e^{2x} - 3x^2\sin x + 2xe^{2x}$

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17. If $\Delta(x) = \begin{vmatrix}4x-1 & x^3+x & x\\ x+4 & -2x-1 & -3x\\ -3 & -4 & 3\end{vmatrix}$, find $\Delta'(x)$ and $\Delta'(-1)$. [5]

Differentiate one row at a time and add:

$$\Delta'(x) = \begin{vmatrix}4 & 3x^2+1 & 1\\ x+4 & -2x-1 & -3x\\ -3 & -4 & 3\end{vmatrix}$$
$$+\begin{vmatrix}4x-1 & x^3+x & x\\ 1 & -2 & -3\\ -3 & -4 & 3\end{vmatrix}$$
$$+\begin{vmatrix}4x-1 & x^3+x & x\\ x+4 & -2x-1 & -3x\\ 0 & 0 & 0\end{vmatrix}$$

The third determinant has a row of zeros, so it is 0.

$$\begin{vmatrix}4 & 3x^2+1 & 1\\ x+4 & -2x-1 & -3x\\ -3 & -4 & 3\end{vmatrix} = 18x^3 - 36x^2 - 76x - 43$$

$$\begin{vmatrix}4x-1 & x^3+x & x\\ 1 & -2 & -3\\ -3 & -4 & 3\end{vmatrix} = 6x^3 - 76x + 18$$

$\therefore \Delta'(x) = 24x^3 - 36x^2 - 152x - 25$

$\Delta'(-1) = 67$

Ans: $\Delta'(x) = 24x^3 - 36x^2 - 152x - 25$, $\Delta'(-1) = 67$

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18. Evaluate $\int_0^1 \begin{vmatrix} e^x & 4 \\ 2x+1 & 5 \end{vmatrix} dx$. [3]

Only one column contains x, so integrate that column:

$$\begin{vmatrix} \int_0^1 e^x\,dx & 4 \\ \int_0^1 (2x+1)\,dx & 5 \end{vmatrix} = \begin{vmatrix} -1+e & 4 \\ 2 & 5 \end{vmatrix}$$

$$= -13 + 5e$$

Ans: $-13 + 5e$

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19. Is $\begin{bmatrix} 6 & -5 & 4 \\ 7 & 7 & 4 \\ 3 & 1 & 2 \end{bmatrix}$ singular or non-singular? [1]

Expand along the first row.

$$|A| = 6\begin{vmatrix} 7 & 4 \\ 1 & 2 \end{vmatrix} + 5\begin{vmatrix} 7 & 4 \\ 3 & 2 \end{vmatrix} + 4\begin{vmatrix} 7 & 7 \\ 3 & 1 \end{vmatrix}$$

$$= 6(14-4) + 5(14-12) + 4(7-21)$$

$$= 6(10) + 5(2) + 4(-14) = 14$$

$|A| \neq 0$, so the matrix is non-singular.

Ans: Non-singular ($|A| = 14$)

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20. Find p if the matrix $\begin{bmatrix} 0 & 2 & 2 \\ p & 3 & 3 \\ -1 & 2 & -3 \end{bmatrix}$ is singular. [2]

Expand along any row; $|A|$ is linear in p:

$$|A| = 10p$$

$$|A| = 0 \Rightarrow p = 0$$

Ans: $p = 0$
