EXPRESSWALA ACADEMY

Maths 1A: Determinants

Question Paper, Set 03

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Find $\det A$ if $A = \begin{bmatrix} 5\sqrt{7} & -5 \\ 1 & -2\sqrt{7} \end{bmatrix}$. [1 mark]

2. Solve $\begin{vmatrix} x+5 & 6 \\ 1 & x-14 \end{vmatrix} = -84$. [2 marks]

3. Write the minor and the cofactor of the element in row 1, column 3 of $\begin{bmatrix} -5 & -4 & 8 \\ 5 & -1 & 0 \\ -3 & 6 & 2 \end{bmatrix}$. [2 marks]

4. Find $\det A$ for $A = \begin{bmatrix} -2 & 0 & 6 \\ 2 & 3 & 1 \\ -2 & 4 & -4 \end{bmatrix}$. [3 marks]

5. If $\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix} = 8$, find $\begin{vmatrix} b & a & c \\ q & p & r \\ y & x & z \end{vmatrix}$. [1 mark]

6. Without expanding, find the value of $\begin{vmatrix} -3 & -6 & 2 \\ -5 & 9 & -1 \\ -8 & 3 & 1 \end{vmatrix}$. [1 mark]

7. Using the properties of determinants, evaluate $\begin{vmatrix} 22 & 7 & 2 \\ 128 & 20 & 11 \\ -46 & 5 & -4 \end{vmatrix}$. [3 marks]

8. For $A = \begin{bmatrix} 6 & 1 & 0 \\ -1 & 6 & 8 \\ 2 & 4 & 7 \end{bmatrix}$, with A_{ij} the cofactor of a_{ij}, find $a_{31}A_{21} + a_{32}A_{22} + a_{33}A_{23}$. [1 mark]

9. Using properties of determinants, solve $\begin{vmatrix} x & -2 & -1 \\ -2 & x & -1 \\ -2 & -1 & x \end{vmatrix} = 0$ for x. [5 marks]

10. If A is a 3×3 matrix and $|A| = 7$, find $|A^2|$ and $|-A|$. [2 marks]

11. Find the determinant of $\begin{bmatrix} 0 & -3 & -3 \\ 3 & 0 & 3 \\ 3 & -3 & 0 \end{bmatrix}$. [1 mark]

12. Using a standard result, find $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & 3 & 27 \end{vmatrix}$. [2 marks]

13. Find the value of $\begin{vmatrix} 2 & -3 & -4 \\ -3 & 10 & 12 \\ -4 & 12 & 17 \end{vmatrix}$ using a standard result. [3 marks]

14. Show that $\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = (a+b+c)(a-b)(b-c)(c-a)$. [5 marks]

15. By writing it as a product of two determinants, evaluate $\begin{vmatrix} (4+3)^2 & (4+4)^2 & (4+1)^2 \\ (3+3)^2 & (3+4)^2 & (3+1)^2 \\ (1+3)^2 & (1+4)^2 & (1+1)^2 \end{vmatrix}$. [5 marks]

16. If $\Delta(x) = \begin{vmatrix} e^{2x} & x^3 \\ \cos x & 2x+1 \end{vmatrix}$, find $\Delta'(x)$. [2 marks]

17. If $\Delta(x) = \begin{vmatrix} 4x-1 & x^3 & x \\ 2x-2 & x+1 & 2x-1 \\ -4 & 3 & -1 \end{vmatrix}$, find $\Delta'(x)$ and $\Delta'(1)$. [5 marks]

18. Evaluate $\int_0^1 \begin{vmatrix} x^2+x & 3 \\ 4x^3 & 4 \end{vmatrix} dx$. [3 marks]

19. Is $\begin{bmatrix} 7 & -4 & -4 \\ -1 & -6 & -1 \\ 20 & -18 & -13 \end{bmatrix}$ singular or non-singular? [1 mark]

20. For what value of p is $\begin{bmatrix} -4 & -4 \\ 7 & p \end{bmatrix}$ singular? [2 marks]

Key sheet

Q	Answer
1	-65
2	$x = 1$ or 8
3	Minor 27, cofactor 27
4	116
5	-8
6	0
7	192
8	0
9	$x = -2,\ -1,\ 3$
10	49 and -7
11	0
12	-48
13	27
14	$(a+b+c)(a-b)(b-c)(c-a)$
15	72
16	$\Delta'(x) = x^3 \sin x - 3x^2 \cos x + 4xe^{2x} + 4e^{2x}$
17	$\Delta'(x) = -24x^3 + 6x^2 - 36x + 13$, $\Delta'(1) = -41$
18	$\frac{1}{3}$
19	Singular ($\lvert A\rvert = 0$)
20	$p = 7$

Solutions

1. Find $\det A$ if $A = \begin{bmatrix} 5\sqrt{7} & -5 \\ 1 & -2\sqrt{7} \end{bmatrix}$. [1]

$\begin{vmatrix} 5\sqrt{7} & -5 \\ 1 & -2\sqrt{7} \end{vmatrix} = 5\sqrt{7} \cdot (-2\sqrt{7}) - (-5) \cdot 1$

$= -70 - (-5) = -65$

Ans: -65

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2. Solve $\begin{vmatrix} x+5 & 6 \\ 1 & x-14 \end{vmatrix} = -84$. [2]

$(x+5)(x-14) - 6 \cdot 1 = -84$

$\Rightarrow x^2 - 9x + 8 = 0$

$\Rightarrow (x-1)(x-8) = 0$

$\therefore x = 1 \text{ or } x = 8$

Ans: $x = 1$ or 8

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3. Write the minor and the cofactor of the element in row 1, column 3 of $\begin{bmatrix} -5 & -4 & 8 \\ 5 & -1 & 0 \\ -3 & 6 & 2 \end{bmatrix}$. [2]

Delete row 1 and column 3.

$M_{13} = \begin{vmatrix} 5 & -1 \\ -3 & 6 \end{vmatrix} = 30 - 3 = 27$

$A_{13} = (-1)^{1+3} M_{13} = 27$

Ans: Minor 27**, cofactor** 27

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4. Find $\det A$ for $A = \begin{bmatrix} -2 & 0 & 6 \\ 2 & 3 & 1 \\ -2 & 4 & -4 \end{bmatrix}$. [3]

Expand along the first row.

$\Delta = -2\begin{vmatrix} 3 & 1 \\ 4 & -4 \end{vmatrix} + 6\begin{vmatrix} 2 & 3 \\ -2 & 4 \end{vmatrix}$

$= -2(-12 - 4) + 6(8 - (-6))$

$= -2(-16) + 6(14) = 116$

Ans: 116

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5. If $\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix} = 8$, find $\begin{vmatrix} b & a & c \\ q & p & r \\ y & x & z \end{vmatrix}$. [1]

Interchanging two columns changes the sign.

$\therefore \text{value} = (-1) \times 8 = -8$

Ans: -8

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6. Without expanding, find the value of $\begin{vmatrix} -3 & -6 & 2 \\ -5 & 9 & -1 \\ -8 & 3 & 1 \end{vmatrix}$. [1]

$R_3 \to R_3 - R_1 - R_2$ gives a row of zeros (value unchanged).

$\therefore \Delta = 0$

Ans: 0

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7. Using the properties of determinants, evaluate $\begin{vmatrix} 22 & 7 & 2 \\ 128 & 20 & 11 \\ -46 & 5 & -4 \end{vmatrix}$. [3]

Apply $R_2 \to R_2 - 2R_1$ (the value does not change):

$$\Delta = \begin{vmatrix} 22 & 7 & 2 \\ 84 & 6 & 7 \\ -46 & 5 & -4 \end{vmatrix}$$

Apply $C_1 \to C_1 - 12C_3$:

$$\Delta = \begin{vmatrix} -2 & 7 & 2 \\ 0 & 6 & 7 \\ 2 & 5 & -4 \end{vmatrix}$$

$$\Delta = 6\begin{vmatrix} -2 & 2 \\ 2 & -4 \end{vmatrix} - 7\begin{vmatrix} -2 & 7 \\ 2 & 5 \end{vmatrix}$$

$$= 6(8-4) - 7(-10-14)$$

$$= 6(4) - 7(-24) = 192$$

Ans: 192

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8. For $A = \begin{bmatrix} 6 & 1 & 0 \\ -1 & 6 & 8 \\ 2 & 4 & 7 \end{bmatrix}$, with A_{ij} the cofactor of a_{ij}, find $a_{31}A_{21} + a_{32}A_{22} + a_{33}A_{23}$. [1]

This is the elements of row 3 with the cofactors of row 2.

The sum of products of the elements of one line with the cofactors of a different line is 0.

$\therefore a_{31}A_{21} + a_{32}A_{22} + a_{33}A_{23} = 0$

Ans: 0

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9. Using properties of determinants, solve $\begin{vmatrix} x & -2 & -1 \\ -2 & x & -1 \\ -2 & -1 & x \end{vmatrix} = 0$ for x. [5]

$C_1 \to C_1 + C_2 + C_3$: every entry of C_1 becomes $x - 3$.

$$\Delta = (x-3)\begin{vmatrix} 1 & -2 & -1 \\ 1 & x & -1 \\ 1 & -1 & x \end{vmatrix}$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$:

$$\Delta = (x-3)\begin{vmatrix} 1 & -2 & -1 \\ 0 & x+2 & 0 \\ 0 & 1 & x+1 \end{vmatrix}$$

$$= (x-3)(x+2)(x+1)$$

$\Delta = 0 \Rightarrow x = 3,\ -2,\ -1$

Ans: $x = -2,\ -1,\ 3$

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10. If A is a 3×3 matrix and $|A| = 7$, find $|A^2|$ and $|-A|$. [2]

$|A^2| = |A|\,|A| = 49$

$|-A| = (-1)^3|A| = -7$

Ans: 49 **and** -7

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11. Find the determinant of $\begin{bmatrix} 0 & -3 & -3 \\ 3 & 0 & 3 \\ 3 & -3 & 0 \end{bmatrix}$. [1]

$A^T = -A$: the matrix is skew-symmetric of odd order 3.

The determinant of a skew-symmetric matrix of odd order is 0.

Ans: 0

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12. Using a standard result, find $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & 3 & 27 \end{vmatrix}$. [2]

Use result 4, $(a+b+c)(a-b)(b-c)(c-a)$ with $a, b, c = 1, -1, 3$.

$(3)(1-(-1))(-1-3)(3-1) = -48$

Ans: -48

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13. Find the value of $\begin{vmatrix} 2 & -3 & -4 \\ -3 & 10 & 12 \\ -4 & 12 & 17 \end{vmatrix}$ using a standard result. [3]

Result 10 with $a, b, c = -1, 3, 4$:

$1 + a^2 + b^2 + c^2 = 1 + 1 + 9 + 16 = 27$

Ans: 27

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14. Show that $\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = (a+b+c)(a-b)(b-c)(c-a)$. [5]

$R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$, and take out $a-b$, $b-c$:

$$\Delta = (a-b)(b-c)\begin{vmatrix} 0 & 1 & a^2+ab+b^2 \\ 0 & 1 & b^2+bc+c^2 \\ 1 & c & c^3 \end{vmatrix}$$

Expand along C_1:

$= (a-b)(b-c)\left[(b^2+bc+c^2) - (a^2+ab+b^2)\right]$

$= (a-b)(b-c)\left[(c^2-a^2) + b(c-a)\right]$

$= (a+b+c)(a-b)(b-c)(c-a)$

Ans: $(a+b+c)(a-b)(b-c)(c-a)$

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15. By writing it as a product of two determinants, evaluate $\begin{vmatrix}(4+3)^2 & (4+4)^2 & (4+1)^2\\(3+3)^2 & (3+4)^2 & (3+1)^2\\(1+3)^2 & (1+4)^2 & (1+1)^2\end{vmatrix}$. [5]

$(u+v)^2 = u^2\cdot 1 + 1\cdot v^2 + 2u\cdot v$

So, multiplying row by column,

$$\Delta = \begin{vmatrix}16 & 1 & 8\\9 & 1 & 6\\1 & 1 & 2\end{vmatrix}\begin{vmatrix}1 & 1 & 1\\9 & 16 & 1\\3 & 4 & 1\end{vmatrix}$$

$$\begin{vmatrix}16 & 1 & 8\\9 & 1 & 6\\1 & 1 & 2\end{vmatrix} = -12$$

$$\begin{vmatrix}1 & 1 & 1\\9 & 16 & 1\\3 & 4 & 1\end{vmatrix} = -6$$

$\therefore \Delta = (-12)\times(-6) = 72$

Ans: 72

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16. If $\Delta(x) = \begin{vmatrix}e^{2x} & x^3\\\cos x & 2x+1\end{vmatrix}$, find $\Delta'(x)$. [2]

$$\Delta'(x) = \begin{vmatrix}2e^{2x} & 3x^2\\\cos x & 2x+1\end{vmatrix} + \begin{vmatrix}e^{2x} & x^3\\-\sin x & 2\end{vmatrix}$$

$$= (-3x^2\cos x + 4xe^{2x} + 2e^{2x}) + (x^3\sin x + 2e^{2x})$$

$$= x^3\sin x - 3x^2\cos x + 4xe^{2x} + 4e^{2x}$$

Ans: $\Delta'(x) = x^3\sin x - 3x^2\cos x + 4xe^{2x} + 4e^{2x}$

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17. If $\Delta(x) = \begin{vmatrix}4x-1 & x^3 & x\\2x-2 & x+1 & 2x-1\\-4 & 3 & -1\end{vmatrix}$, find $\Delta'(x)$ and $\Delta'(1)$. [5]

Differentiate one row at a time and add:

$$\Delta'(x) = \begin{vmatrix}4 & 3x^2 & 1\\2x-2 & x+1 & 2x-1\\-4 & 3 & -1\end{vmatrix}$$

$$+\begin{vmatrix}4x-1 & x^3 & x\\2 & 1 & 2\\-4 & 3 & -1\end{vmatrix}$$

$$+\begin{vmatrix}4x-1 & x^3 & x\\2x-2 & x+1 & 2x-1\\0 & 0 & 0\end{vmatrix}$$

The third determinant has a row of zeros, so it is 0.

$$\begin{vmatrix}4 & 3x^2 & 1\\2x-2 & x+1 & 2x-1\\-4 & 3 & -1\end{vmatrix} = -18x^3 + 6x^2 - 18x + 6$$

$$\begin{vmatrix}4x-1 & x^3 & x\\2 & 1 & 2\\-4 & 3 & -1\end{vmatrix} = -6x^3 - 18x + 7$$

$\therefore \Delta'(x) = -24x^3 + 6x^2 - 36x + 13$

$\Delta'(1) = -41$

Ans: $\Delta'(x) = -24x^3 + 6x^2 - 36x + 13$, $\Delta'(1) = -41$

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18. Evaluate $\int_0^1 \begin{vmatrix} x^2+x & 3 \\ 4x^3 & 4 \end{vmatrix} dx$. [3]

Only one column contains x, so integrate that column:

$\begin{vmatrix} \int_0^1 (x^2+x)\,dx & 3 \\ \int_0^1 4x^3\,dx & 4 \end{vmatrix} = \begin{vmatrix} \frac{5}{6} & 3 \\ 1 & 4 \end{vmatrix}$

$= \frac{1}{3}$

Ans: $\frac{1}{3}$

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19. Is $\begin{bmatrix} 7 & -4 & -4 \\ -1 & -6 & -1 \\ 20 & -18 & -13 \end{bmatrix}$ singular or non-singular? [1]

$R_3 = 3R_1 + R_2$, so $R_3 \to R_3 - (3R_1 + R_2)$ gives a row of zeros.

$|A| = 0$

So the matrix is singular.

Ans: Singular ($|A| = 0$)

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20. For what value of p is $\begin{bmatrix} -4 & -4 \\ 7 & p \end{bmatrix}$ singular? [2]

$\begin{vmatrix} -4 & -4 \\ 7 & p \end{vmatrix} = 28 - 4p = 0$

$\Rightarrow p = 7$

Ans: $p = 7$
