EXPRESSWALA ACADEMY

Maths 1A: Determinants

Question Paper, Set 02

Time: 1 hour 30 minutes Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Find the value of $\begin{vmatrix} 7 & 9 \\ -3 & -4 \end{vmatrix}$. [1 mark]

2. Find x if $\begin{vmatrix} x+1 & -1 \\ -6 & x-2 \end{vmatrix} = 4$. [2 marks]

3. For $A = \begin{bmatrix} -2 & 3 & 5 \\ 0 & -3 & -2 \\ -6 & 8 & -5 \end{bmatrix}$, find M_{32} and A_{32}. [2 marks]

4. Find the value of $\begin{vmatrix} 0 & -2 & 0 \\ 7 & -3 & -3 \\ 6 & -5 & 5 \end{vmatrix}$ by expanding along a suitable row or column. [3 marks]

5. If $\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix} = 3$, find $\begin{vmatrix} a & b & c \\ p & q & r \\ -2x & -2y & -2z \end{vmatrix}$. [1 mark]

6. Write the value of $\begin{vmatrix} 8 & 4 & 5 \\ 16 & 8 & 10 \\ 6 & 2 & 6 \end{vmatrix}$, giving a reason. [1 mark]

7. Simplify $\begin{vmatrix} 31 & 5 & 0 \\ 125 & 20 & 2 \\ 13 & 1 & 4 \end{vmatrix}$ by row and column operations and find its value. [3 marks]

8. For $A = \begin{bmatrix} 2 & -3 & 5 \\ -5 & 3 & 0 \\ -4 & 5 & -3 \end{bmatrix}$, with A_{ij} the cofactor of a_{ij}, find $a_{11}A_{13} + a_{21}A_{23} + a_{31}A_{33}$. [1 mark]

9. Find the values of x for which $\begin{vmatrix} x-5 & -1 & -3 \\ -5 & x-1 & -3 \\ -5 & -1 & x-3 \end{vmatrix} = 0$. [5 marks]

10. A and B are square matrices of order 2 with $|A| = 2$ and $|B| = 1$. Find $|AB|$ and $|-3A^T|$. [2 marks]

11. Write the value of $\begin{vmatrix} 6 & 0 & 0 & 0 \\ 5 & 2 & 0 & 0 \\ 5 & -1 & -2 & 0 \\ -3 & 10 & 2 & 2 \end{vmatrix}$. [1 mark]

12. Evaluate $\begin{vmatrix} 3 & 8 & 4 \\ 8 & 4 & 3 \\ 4 & 3 & 8 \end{vmatrix}$ by a standard result. [2 marks]

13. Evaluate $\begin{vmatrix} 8 & 1 & 9 \\ 7 & 2 & 9 \\ 7 & 1 & 10 \end{vmatrix}$. [3 marks]

14. Show that $\begin{vmatrix} 1 & a & bc \\ 1 & b & ac \\ 1 & c & ab \end{vmatrix} = (a-b)(b-c)(c-a)$. [5 marks]

15. Express $\begin{vmatrix} (4-2)^2 & (4-1)^2 & (4-5)^2 \\ (1-2)^2 & (1-1)^2 & (1-5)^2 \\ (2-2)^2 & (2-1)^2 & (2-5)^2 \end{vmatrix}$ as the product of two determinants and hence find its value. [5 marks]

16. Differentiate $\Delta(x) = \begin{vmatrix} \cos x & x^2 \\ -4 & 4 \end{vmatrix}$ by differentiating one row at a time. [2 marks]

17. If $\Delta(x) = \begin{vmatrix} x & x^3 + x & 2x \\ 4x + 2 & 3x + 2 & -2 \\ 4 & -4 & 5 \end{vmatrix}$, find $\Delta'(x)$ and $\Delta'(-1)$. [5 marks]

18. Evaluate $\int_0^1 \begin{vmatrix} 4x^3 & x^2 + x \\ 3 & 2 \end{vmatrix} dx$. [3 marks]

19. Is $\begin{bmatrix} -1 & 2 & -5 \\ -4 & -4 & -2 \\ 7 & -4 & 5 \end{bmatrix}$ singular or non-singular? [1 mark]

20. Find p if the matrix $\begin{bmatrix} p & 0 & -1 \\ 4 & 6 & 2 \\ -4 & -3 & -3 \end{bmatrix}$ is singular. [2 marks]

Key sheet

Q	Answer
1	-1
2	$x = -3$ or 4
3	Minor 4, cofactor -4
4	106
5	-6
6	0
7	48
8	0
9	$x = 0,\ 9$
10	2 and 18
11	-48
12	-315
13	18
14	$(a-b)(b-c)(c-a)$
15	-144
16	$\Delta'(x) = 8x - 4\sin x$
17	$\Delta'(x) = -80x^3 - 54x^2 - 122x - 48$, $\Delta'(-1) = 100$
18	$-\frac{1}{2}$
19	Non-singular ($\lvert A\rvert = -180$)
20	$p = -1$

Solutions

1. Find the value of $\begin{vmatrix} 7 & 9 \\ -3 & -4 \end{vmatrix}$. [1]

$\begin{vmatrix} 7 & 9 \\ -3 & -4 \end{vmatrix} = 7 \cdot (-4) - 9 \cdot (-3)$

$= -28 - (-27) = -1$

Ans: -1

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2. Find x if $\begin{vmatrix} x+1 & -1 \\ -6 & x-2 \end{vmatrix} = 4$. [2]

$(x+1)(x-2) - (-1) \cdot (-6) = 4$

$\Rightarrow x^2 - x - 12 = 0$

$\Rightarrow (x+3)(x-4) = 0$

$\therefore x = -3$ or $x = 4$

Ans: $x = -3$ or 4

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3. For $A = \begin{bmatrix} -2 & 3 & 5 \\ 0 & -3 & -2 \\ -6 & 8 & -5 \end{bmatrix}$, find M_{32} and A_{32}. [2]

Delete row 3 and column 2.

$M_{32} = \begin{vmatrix} -2 & 5 \\ 0 & -2 \end{vmatrix} = 4 - 0 = 4$

$A_{32} = (-1)^{3+2} M_{32} = -4$

Ans: Minor 4**, cofactor** -4

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4. Find the value of $\begin{vmatrix} 0 & -2 & 0 \\ 7 & -3 & -3 \\ 6 & -5 & 5 \end{vmatrix}$ by expanding along a suitable row or column. [3]

Expand along the first row.

$\Delta = 2 \begin{vmatrix} 7 & -3 \\ 6 & 5 \end{vmatrix}$

$= 2(35 - (-18))$

$= 2(53) = 106$

Ans: 106

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5. If $\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix} = 3$, find $\begin{vmatrix} a & b & c \\ p & q & r \\ -2x & -2y & -2z \end{vmatrix}$. [1]

One row is multiplied by -2, so the value is multiplied by -2.

$\therefore$ value $= (-2) \times 3 = -6$

Ans: -6

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6. Write the value of $\begin{vmatrix} 8 & 4 & 5 \\ 16 & 8 & 10 \\ 6 & 2 & 6 \end{vmatrix}$, giving a reason. [1]

$R_2 = 2R_1$: two rows are proportional.

$\therefore \Delta = 0$

Ans: 0

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7. Simplify $\begin{vmatrix} 31 & 5 & 0 \\ 125 & 20 & 2 \\ 13 & 1 & 4 \end{vmatrix}$ by row and column operations and find its value. [3]

Apply $R_2 \to R_2 - 3R_1$ (the value does not change):

$$\Delta = \begin{vmatrix} 31 & 5 & 0 \\ 32 & 5 & 2 \\ 13 & 1 & 4 \end{vmatrix}$$

Apply $C_1 \to C_1 - 7C_2$:

$$\Delta = \begin{vmatrix} -4 & 5 & 0 \\ -3 & 5 & 2 \\ 6 & 1 & 4 \end{vmatrix}$$

$$\Delta = -4\begin{vmatrix} 5 & 2 \\ 1 & 4 \end{vmatrix} - 5\begin{vmatrix} -3 & 2 \\ 6 & 4 \end{vmatrix}$$

$$= -4(20 - 2) - 5(-12 - 12)$$

$$= -4(18) - 5(-24) = 48$$

Ans: 48

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8. For $A = \begin{bmatrix} 2 & -3 & 5 \\ -5 & 3 & 0 \\ -4 & 5 & -3 \end{bmatrix}$, with A_{ij} the cofactor of a_{ij}, find $a_{11}A_{13} + a_{21}A_{23} + a_{31}A_{33}$. [1]

This is the elements of column 1 with the cofactors of column 3.

The sum of products of the elements of one line with the cofactors of a different line is 0.

$\therefore a_{11}A_{13} + a_{21}A_{23} + a_{31}A_{33} = 0$

Ans: 0

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9. Find the values of x for which $\begin{vmatrix} x-5 & -1 & -3 \\ -5 & x-1 & -3 \\ -5 & -1 & x-3 \end{vmatrix} = 0$. [5]

$C_1 \to C_1 + C_2 + C_3$: every entry of C_1 becomes $x - 9$.

$$\Delta = (x-9)\begin{vmatrix} 1 & -1 & -3 \\ 1 & x-1 & -3 \\ 1 & -1 & x-3 \end{vmatrix}$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$:

$$\Delta = (x-9)\begin{vmatrix} 1 & -1 & -3 \\ 0 & x & 0 \\ 0 & 0 & x \end{vmatrix} = (x-9)x^2$$

$\Delta = 0 \Rightarrow x = 0$ or $x = 9$

Ans: $x = 0,\ 9$

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10. A and B are square matrices of order 2 with $|A| = 2$ and $|B| = 1$. Find $|AB|$ and $|-3A^T|$. [2]

$|AB| = |A|\,|B| = 2 \times 1 = 2$

$|-3A^T| = (-3)^2|A^T| = 9 \times 2 = 18$

Ans: 2 **and** 18

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11. Write the value of $\begin{vmatrix} 6 & 0 & 0 & 0 \\ 5 & 2 & 0 & 0 \\ 5 & -1 & -2 & 0 \\ -3 & 10 & 2 & 2 \end{vmatrix}$. [1]

The matrix is triangular: the determinant is the product of the diagonal elements.

$6 \times 2 \times (-2) \times 2 = -48$

Ans: -48

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12. Evaluate $\begin{vmatrix} 3 & 8 & 4 \\ 8 & 4 & 3 \\ 4 & 3 & 8 \end{vmatrix}$ by a standard result. [2]

Use result 1, $-(a^3 + b^3 + c^3 - 3abc)$ with $a, b, c = 3, 8, 4$.

$-(27 + 512 + 64 - 288) = -315$

Ans: -315

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13. Evaluate $\begin{vmatrix} 8 & 1 & 9 \\ 7 & 2 & 9 \\ 7 & 1 & 10 \end{vmatrix}$. [3]

Result 8 with $a, b, c = 7, 1, 9$:

$1 + a + b + c = 18$

Ans: 18

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14. Show that $\begin{vmatrix} 1 & a & bc \\ 1 & b & ac \\ 1 & c & ab \end{vmatrix} = (a-b)(b-c)(c-a)$. [5]

$R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$:

$$\Delta = \begin{vmatrix} 0 & a-b & -c(a-b) \\ 0 & b-c & -a(b-c) \\ 1 & c & ab \end{vmatrix}$$

Take $a-b$ out of R_1 and $b-c$ out of R_2:

$$\Delta = (a-b)(b-c)\begin{vmatrix} 0 & 1 & -c \\ 0 & 1 & -a \\ 1 & c & ab \end{vmatrix}$$

Expand along C_1:

$= (a-b)(b-c)\,[(1)(-a) - (-c)(1)] = (a-b)(b-c)(c-a)$

Ans: $(a-b)(b-c)(c-a)$

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15. Express $\begin{vmatrix}(4-2)^2 & (4-1)^2 & (4-5)^2\\ (1-2)^2 & (1-1)^2 & (1-5)^2\\ (2-2)^2 & (2-1)^2 & (2-5)^2\end{vmatrix}$ as the product of two determinants and hence find its value. [5]

$(u-v)^2 = u^2\cdot 1 + 1\cdot v^2 - 2u\cdot v$

So, multiplying row by column,

$$\Delta = \begin{vmatrix}16 & 1 & -8\\ 1 & 1 & -2\\ 4 & 1 & -4\end{vmatrix}\begin{vmatrix}1 & 1 & 1\\ 4 & 1 & 25\\ 2 & 1 & 5\end{vmatrix}$$

$$\begin{vmatrix}16 & 1 & -8\\ 1 & 1 & -2\\ 4 & 1 & -4\end{vmatrix} = -12$$

$$\begin{vmatrix}1 & 1 & 1\\ 4 & 1 & 25\\ 2 & 1 & 5\end{vmatrix} = 12$$

$\therefore \Delta = (-12)\times 12 = -144$

Ans: -144

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16. Differentiate $\Delta(x) = \begin{vmatrix}\cos x & x^2\\ -4 & 4\end{vmatrix}$ by differentiating one row at a time. [2]

$$\Delta'(x) = \begin{vmatrix}-\sin x & 2x\\ -4 & 4\end{vmatrix} + \begin{vmatrix}\cos x & x^2\\ 0 & 0\end{vmatrix}$$

$= (8x - 4\sin x) + (0)$

$= 8x - 4\sin x$

Ans: $\Delta'(x) = 8x - 4\sin x$

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17. If $\Delta(x) = \begin{vmatrix}x & x^3+x & 2x\\ 4x+2 & 3x+2 & -2\\ 4 & -4 & 5\end{vmatrix}$, find $\Delta'(x)$ and $\Delta'(-1)$. [5]

Differentiate one row at a time and add:

$$\Delta'(x) = \begin{vmatrix}1 & 3x^2+1 & 2\\ 4x+2 & 3x+2 & -2\\ 4 & -4 & 5\end{vmatrix}$$

$$+\begin{vmatrix}x & x^3+x & 2x\\ 4 & 3 & 0\\ 4 & -4 & 5\end{vmatrix}$$

$$+\begin{vmatrix}x & x^3+x & 2x\\ 4x+2 & 3x+2 & -2\\ 0 & 0 & 0\end{vmatrix}$$

The third determinant has a row of zeros, so it is 0.

$$\begin{vmatrix}1 & 3x^2+1 & 2\\ 4x+2 & 3x+2 & -2\\ 4 & -4 & 5\end{vmatrix} = -60x^3 - 54x^2 - 61x - 48$$

$$\begin{vmatrix}x & x^3+x & 2x\\ 4 & 3 & 0\\ 4 & -4 & 5\end{vmatrix} = -20x^3 - 61x$$

$\therefore \Delta'(x) = -80x^3 - 54x^2 - 122x - 48$

$\Delta'(-1) = 100$

Ans: $\Delta'(x) = -80x^3 - 54x^2 - 122x - 48$, $\Delta'(-1) = 100$

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18. Evaluate $\int_0^1 \begin{vmatrix} 4x^3 & x^2+x \\ 3 & 2 \end{vmatrix} dx$. [3]

Only one row contains x, so integrate that row:

$\begin{vmatrix} \int_0^1 4x^3\,dx & \int_0^1 (x^2+x)\,dx \\ 3 & 2 \end{vmatrix} = \begin{vmatrix} 1 & \frac{5}{6} \\ 3 & 2 \end{vmatrix}$

$= -\frac{1}{2}$

Ans: $-\frac{1}{2}$

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19. Is $\begin{bmatrix} -1 & 2 & -5 \\ -4 & -4 & -2 \\ 7 & -4 & 5 \end{bmatrix}$ singular or non-singular? [1]

Expand along the first row.

$|A| = -\begin{vmatrix} -4 & -2 \\ -4 & 5 \end{vmatrix} - 2\begin{vmatrix} -4 & -2 \\ 7 & 5 \end{vmatrix} - 5\begin{vmatrix} -4 & -4 \\ 7 & -4 \end{vmatrix}$

$= -(-20-8) - 2(-20-(-14)) - 5(16-(-28))$

$= -(-28) - 2(-6) - 5(44) = -180$

$|A| \neq 0$, so the matrix is non-singular.

Ans: Non-singular ($|A| = -180$)

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20. Find p if the matrix $\begin{bmatrix} p & 0 & -1 \\ 4 & 6 & 2 \\ -4 & -3 & -3 \end{bmatrix}$ is singular. [2]

Expand along any row; $|A|$ is linear in p:

$|A| = -12p - 12$

$|A| = 0 \Rightarrow p = -1$

Ans: $p = -1$
