EXPRESSWALA ACADEMY

Maths 1A: Determinants

Question Paper, Set 10

Time: 1 hour 30 minutes Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Evaluate $\begin{vmatrix} 11 & 6 \\ -9 & 12 \end{vmatrix}$. [1 mark]

2. Find x if $\begin{vmatrix} x+3 & 2 \\ -4 & x+5 \end{vmatrix} = 11$. [2 marks]

3. Find the minor and the cofactor of the element 5 in $\begin{bmatrix} 5 & -4 & 0 \\ -7 & 7 & -3 \\ -5 & -4 & 2 \end{bmatrix}$. [2 marks]

4. Evaluate $\begin{vmatrix} -5 & 0 & 4 \\ 0 & -4 & 5 \\ 1 & -1 & -1 \end{vmatrix}$. [3 marks]

5. If $\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix} = -6$, find $\begin{vmatrix} 2a & 2b & 2c \\ p & q & r \\ x & y & z \end{vmatrix}$. [1 mark]

6. Write the value of $\begin{vmatrix} 9 & -5 & 4 \\ 27 & -15 & 12 \\ 8 & 3 & 8 \end{vmatrix}$, giving a reason. [1 mark]

7. Simplify $\begin{vmatrix} -18 & -62 & -7 \\ 7 & 51 & 6 \\ -3 & -28 & -4 \end{vmatrix}$ by row and column operations and find its value. [3 marks]

8. For $A = \begin{bmatrix} 2 & -3 & 0 \\ 4 & -1 & -4 \\ 8 & 4 & 5 \end{bmatrix}$, with A_{ij} the cofactor of a_{ij}, find $a_{11}A_{13} + a_{21}A_{23} + a_{31}A_{33}$. [1 mark]

9. Solve $\begin{vmatrix} x & 6 & 3 \\ 6 & x & 3 \\ 6 & 3 & x \end{vmatrix} = 0$. [5 marks]

10. A and B are square matrices of order 2 with $|A| = 3$ and $|B| = -2$. Find $|AB|$ and $|2A^T|$. [2 marks]

11. Write the value of $\begin{vmatrix} -2 & 0 & 0 \\ 12 & 2 & 0 \\ 2 & -8 & 4 \end{vmatrix}$. [1 mark]

12. Evaluate $\begin{vmatrix} 1 & -1 & 1 \\ 1 & 4 & 16 \\ 1 & -3 & 9 \end{vmatrix}$ by a standard result. [2 marks]

13. Evaluate $\begin{vmatrix} 1 & 16 & 64 \\ 1 & 1 & 1 \\ 1 & 4 & 8 \end{vmatrix}$. [3 marks]

14. Show that $\begin{vmatrix} n^2 & (n+1)^2 & (n+2)^2 \\ (n+3)^2 & (n+4)^2 & (n+5)^2 \\ (n+6)^2 & (n+7)^2 & (n+8)^2 \end{vmatrix} = -216.$ [5 marks]

15. Express $\begin{vmatrix} (2-3)^2 & (2-4)^2 & (2-1)^2 \\ (3-3)^2 & (3-4)^2 & (3-1)^2 \\ (5-3)^2 & (5-4)^2 & (5-1)^2 \end{vmatrix}$ as the product of two determinants and hence find its value. [5 marks]

16. Differentiate $\Delta(x) = \begin{vmatrix} x^2+1 & e^x \\ 6 & -3 \end{vmatrix}$ by differentiating one row at a time. [2 marks]

17. If $\Delta(x) = \begin{vmatrix} 2x & x^3 & 4x-1 \\ 4x+1 & 3x-3 & 4-2x \\ -3 & -3 & 5 \end{vmatrix}$, find $\Delta'(x)$ and $\Delta'(-1)$. [5 marks]

18. Evaluate $\int_0^1 \begin{vmatrix} 4x^3 & e^x \\ -1 & 4 \end{vmatrix} \, dx$. [3 marks]

19. Is $\begin{bmatrix} -2 & 3 & 3 \\ -5 & -5 & -5 \\ 3 & 1 & 4 \end{bmatrix}$ singular or non-singular? [1 mark]

20. Find p if the matrix $\begin{bmatrix} -3 & 0 & 3 \\ 3 & 2 & p \\ 3 & -1 & 2 \end{bmatrix}$ is singular. [2 marks]

Key sheet

Q	Answer
1	186
2	$x = -6$ or -2
3	Minor 2, cofactor 2
4	-29
5	-12
6	0
7	329
8	0
9	$x = -9,\ 3,\ 6$
10	-6 and 12
11	-16
12	70
13	84
14	-216
15	72
16	$\Delta'(x) = -6x - 6e^x$
17	$\Delta'(x) = -56x^3 - 51x^2 + 12x - 51$, $\Delta'(-1) = -58$
18	$e + 3$
19	Non-singular ($\lvert A\rvert = 75$)
20	$p = -13$

Solutions

1. Evaluate $\begin{vmatrix} 11 & 6 \\ -9 & 12 \end{vmatrix}$. [1]

$\begin{vmatrix} 11 & 6 \\ -9 & 12 \end{vmatrix} = 11 \cdot 12 - 6 \cdot (-9)$

$= 132 - (-54) = 186$

Ans: 186

---

2. Find x if $\begin{vmatrix} x+3 & 2 \\ -4 & x+5 \end{vmatrix} = 11$. [2]

$(x+3)(x+5) - 2 \cdot (-4) = 11$

$\Rightarrow x^2 + 8x + 12 = 0$

$\Rightarrow (x+6)(x+2) = 0$

$\therefore x = -6$ or $x = -2$

Ans: $x = -6$ or -2

---

3. Find the minor and the cofactor of the element 5 in $\begin{bmatrix} 5 & -4 & 0 \\ -7 & 7 & -3 \\ -5 & -4 & 2 \end{bmatrix}$. [2]

Delete row 1 and column 1.

$M_{11} = \begin{vmatrix} 7 & -3 \\ -4 & 2 \end{vmatrix} = 14 - 12 = 2$

$A_{11} = (-1)^{1+1} M_{11} = 2$

Ans: Minor 2**, cofactor** 2

---

4. Evaluate $\begin{vmatrix} -5 & 0 & 4 \\ 0 & -4 & 5 \\ 1 & -1 & -1 \end{vmatrix}$. [3]

Expand along the first row.

$\Delta = -5 \begin{vmatrix} -4 & 5 \\ -1 & -1 \end{vmatrix} + 4 \begin{vmatrix} 0 & -4 \\ 1 & -1 \end{vmatrix}$

$= -5(4 - (-5)) + 4(0 - (-4))$

$= -5(9) + 4(4) = -29$

Ans: -29

---

5. If $\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix} = -6$, find $\begin{vmatrix} 2a & 2b & 2c \\ p & q & r \\ x & y & z \end{vmatrix}$. [1]

One row is multiplied by 2, so the value is multiplied by 2.

$\therefore$ value $= 2 \times (-6) = -12$

Ans: -12

---

6. Write the value of $\begin{vmatrix} 9 & -5 & 4 \\ 27 & -15 & 12 \\ 8 & 3 & 8 \end{vmatrix}$, giving a reason. [1]

$R_2 = 3R_1$: two rows are proportional.

$\therefore \Delta = 0$

Ans: 0

---

7. Simplify $\begin{vmatrix} -18 & -62 & -7 \\ 7 & 51 & 6 \\ -3 & -28 & -4 \end{vmatrix}$ by row and column operations and find its value. [3]

Apply $R_1 \to R_1 + 2R_2$ (the value does not change):

$$\Delta = \begin{vmatrix} -4 & 40 & 5 \\ 7 & 51 & 6 \\ -3 & -28 & -4 \end{vmatrix}$$

Apply $C_2 \to C_2 - 8C_3$:

$$\Delta = \begin{vmatrix} -4 & 0 & 5 \\ 7 & 3 & 6 \\ -3 & 4 & -4 \end{vmatrix}$$

$$\Delta = -4\begin{vmatrix} 3 & 6 \\ 4 & -4 \end{vmatrix} + 5\begin{vmatrix} 7 & 3 \\ -3 & 4 \end{vmatrix}$$

$$= -4(-12 - 24) + 5(28 - (-9))$$

$$= -4(-36) + 5(37) = 329$$

Ans: 329

---

8. For $A = \begin{bmatrix} 2 & -3 & 0 \\ 4 & -1 & -4 \\ 8 & 4 & 5 \end{bmatrix}$, with A_{ij} the cofactor of a_{ij}, find $a_{11}A_{13} + a_{21}A_{23} + a_{31}A_{33}$. [1]

This is the elements of column 1 with the cofactors of column 3.

The sum of products of the elements of one line with the cofactors of a different line is 0.

$\therefore a_{11}A_{13} + a_{21}A_{23} + a_{31}A_{33} = 0$

Ans: 0

---

9. Solve $\begin{vmatrix} x & 6 & 3 \\ 6 & x & 3 \\ 6 & 3 & x \end{vmatrix} = 0$. [5]

$C_1 \to C_1 + C_2 + C_3$: every entry of C_1 becomes $x + 9$.

$$\Delta = (x+9)\begin{vmatrix} 1 & 6 & 3 \\ 1 & x & 3 \\ 1 & 3 & x \end{vmatrix}$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$:

$$\Delta = (x+9)\begin{vmatrix} 1 & 6 & 3 \\ 0 & x-6 & 0 \\ 0 & -3 & x-3 \end{vmatrix}$$

$$= (x+9)(x-6)(x-3)$$

$\Delta = 0 \Rightarrow x = -9,\ 6,\ 3$

Ans: $x = -9,\ 3,\ 6$

---

10. A and B are square matrices of order 2 with $|A| = 3$ and $|B| = -2$. Find $|AB|$ and $|2A^T|$. [2]

$|AB| = |A|\,|B| = 3 \times (-2) = -6$

$|2A^T| = 2^2|A^T| = 4 \times 3 = 12$

Ans: -6 **and** 12

---

11. Write the value of $\begin{vmatrix} -2 & 0 & 0 \\ 12 & 2 & 0 \\ 2 & -8 & 4 \end{vmatrix}$. [1]

The matrix is triangular: the determinant is the product of the diagonal elements.

$(-2) \times 2 \times 4 = -16$

Ans: -16

---

12. Evaluate $\begin{vmatrix} 1 & -1 & 1 \\ 1 & 4 & 16 \\ 1 & -3 & 9 \end{vmatrix}$ by a standard result. [2]

Use result 3, $(a-b)(b-c)(c-a)$ with $a, b, c = -1, 4, -3$.

$(-1-4)(4-(-3))(-3-(-1)) = 70$

Ans: 70

---

13. Evaluate $\begin{vmatrix} 1 & 16 & 64 \\ 1 & 1 & 1 \\ 1 & 4 & 8 \end{vmatrix}$. [3]

Result 5 with $a, b, c = 4, 1, 2$:

$(ab+bc+ca)(a-b)(b-c)(c-a) = (14)(6) = 84$

Ans: 84

---

14. Show that $\begin{vmatrix} n^2 & (n+1)^2 & (n+2)^2 \\ (n+3)^2 & (n+4)^2 & (n+5)^2 \\ (n+6)^2 & (n+7)^2 & (n+8)^2 \end{vmatrix} = -216$. [5]

$C_3 \to C_3 - C_2$, $C_2 \to C_2 - C_1$:

$$\Delta = \begin{vmatrix} n^2 & 2n+1 & 2n+3 \\ (n+3)^2 & 2n+7 & 2n+9 \\ (n+6)^2 & 2n+13 & 2n+15 \end{vmatrix}$$

$C_3 \to C_3 - C_2$:

$$\Delta = \begin{vmatrix} n^2 & 2n+1 & 2 \\ (n+3)^2 & 2n+7 & 2 \\ (n+6)^2 & 2n+13 & 2 \end{vmatrix}$$

$R_3 \to R_3 - R_2$, $R_2 \to R_2 - R_1$:

$$\Delta = \begin{vmatrix} n^2 & 2n+1 & 2 \\ 6n+9 & 6 & 0 \\ 6n+27 & 6 & 0 \end{vmatrix}$$

Expand along C_3:

$= 2\,[6(6n+9) - 6(6n+27)] = 2(-108) = -216$

Ans: -216

---

15. Express $\begin{vmatrix}(2-3)^2 & (2-4)^2 & (2-1)^2\\(3-3)^2 & (3-4)^2 & (3-1)^2\\(5-3)^2 & (5-4)^2 & (5-1)^2\end{vmatrix}$ as the product of two determinants and hence find its value. [5]

$(u-v)^2 = u^2\cdot 1 + 1\cdot v^2 - 2u\cdot v$

So, multiplying row by column,

$$\Delta = \begin{vmatrix}4 & 1 & -4\\9 & 1 & -6\\25 & 1 & -10\end{vmatrix}\begin{vmatrix}1 & 1 & 1\\9 & 16 & 1\\3 & 4 & 1\end{vmatrix}$$

$$\begin{vmatrix}4 & 1 & -4\\9 & 1 & -6\\25 & 1 & -10\end{vmatrix} = -12$$

$$\begin{vmatrix}1 & 1 & 1\\9 & 16 & 1\\3 & 4 & 1\end{vmatrix} = -6$$

$\therefore \Delta = (-12)\times(-6) = 72$

Ans: 72

---

16. Differentiate $\Delta(x) = \begin{vmatrix}x^2+1 & e^x\\6 & -3\end{vmatrix}$ by differentiating one row at a time. [2]

$$\Delta'(x) = \begin{vmatrix}2x & e^x\\6 & -3\end{vmatrix} + \begin{vmatrix}x^2+1 & e^x\\0 & 0\end{vmatrix}$$

$= (-6x - 6e^x) + (0)$

$= -6x - 6e^x$

Ans: $\Delta'(x) = -6x - 6e^x$

---

17. If $\Delta(x) = \begin{vmatrix}2x & x^3 & 4x-1\\4x+1 & 3x-3 & 4-2x\\-3 & -3 & 5\end{vmatrix}$, find $\Delta'(x)$ and $\Delta'(-1)$. [5]

Differentiate one row at a time and add:

$$\Delta'(x) = \begin{vmatrix}2 & 3x^2 & 4\\4x+1 & 3x-3 & 4-2x\\-3 & -3 & 5\end{vmatrix}$$

$$+\begin{vmatrix}2x & x^3 & 4x-1\\4 & 3 & -2\\-3 & -3 & 5\end{vmatrix}$$

$$+\begin{vmatrix}2x & x^3 & 4x-1\\4x+1 & 3x-3 & 4-2x\\0 & 0 & 0\end{vmatrix}$$

The third determinant has a row of zeros, so it is 0.

$$\begin{vmatrix}2 & 3x^2 & 4\\4x+1 & 3x-3 & 4-2x\\-3 & -3 & 5\end{vmatrix} = -42x^3 - 51x^2 + 6x - 54$$

$$\begin{vmatrix}2x & x^3 & 4x-1\\4 & 3 & -2\\-3 & -3 & 5\end{vmatrix} = -14x^3 + 6x + 3$$

$\therefore \Delta'(x) = -56x^3 - 51x^2 + 12x - 51$

$\Delta'(-1) = -58$

Ans: $\Delta'(x) = -56x^3 - 51x^2 + 12x - 51$, $\Delta'(-1) = -58$

---

18. Evaluate $\int_0^1 \begin{vmatrix} 4x^3 & e^x \\ -1 & 4 \end{vmatrix} dx$. [3]

Only one row contains x, so integrate that row:

$\begin{vmatrix} \int_0^1 4x^3\,dx & \int_0^1 e^x\,dx \\ -1 & 4 \end{vmatrix} = \begin{vmatrix} 1 & -1+e \\ -1 & 4 \end{vmatrix}$

$= e + 3$

Ans: $e + 3$

---

19. Is $\begin{bmatrix} -2 & 3 & 3 \\ -5 & -5 & -5 \\ 3 & 1 & 4 \end{bmatrix}$ singular or non-singular? [1]

Expand along the first row.

$|A| = -2\begin{vmatrix} -5 & -5 \\ 1 & 4 \end{vmatrix} - 3\begin{vmatrix} -5 & -5 \\ 3 & 4 \end{vmatrix} + 3\begin{vmatrix} -5 & -5 \\ 3 & 1 \end{vmatrix}$

$= -2(-20-(-5)) - 3(-20-(-15)) + 3(-5-(-15))$

$= -2(-15) - 3(-5) + 3(10) = 75$

$|A| \neq 0$, so the matrix is non-singular.

Ans: Non-singular ($|A| = 75$)

---

20. Find p if the matrix $\begin{bmatrix} -3 & 0 & 3 \\ 3 & 2 & p \\ 3 & -1 & 2 \end{bmatrix}$ is singular. [2]

Expand along any row; $|A|$ is linear in p:

$|A| = -3p - 39$

$|A| = 0 \Rightarrow p = -13$

Ans: $p = -13$
