EXPRESSWALA ACADEMY

Maths 1A: Determinants

Question Paper, Set 01

Time: 1 hour 30 minutes Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Evaluate $\begin{vmatrix} 5 & 8 \\ 5 & 5 \end{vmatrix}$. [1 mark]

2. Solve $\begin{vmatrix} x+5 & 2 \\ 4 & x-14 \end{vmatrix} = -86$. [2 marks]

3. Find the minor and the cofactor of the element 9 in $\begin{bmatrix} -2 & 6 & 6 \\ 2 & 8 & -1 \\ 8 & 9 & 0 \end{bmatrix}$. [2 marks]

4. Evaluate $\begin{vmatrix} -5 & 0 & -4 \\ 7 & 1 & -3 \\ 5 & -5 & -3 \end{vmatrix}$. [3 marks]

5. If $\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix} = 3$, find $\begin{vmatrix} x & y & z \\ p & q & r \\ a & b & c \end{vmatrix}$. [1 mark]

6. Without expanding, find the value of $\begin{vmatrix} 1 & -1 & 1 \\ 1 & 2 & 6 \\ -3 & 3 & -3 \end{vmatrix}$. [1 mark]

7. Using the properties of determinants, evaluate $\begin{vmatrix} 4 & 2 & 27 \\ -4 & -5 & -57 \\ -2 & -2 & -21 \end{vmatrix}$. [3 marks]

8. For $A = \begin{bmatrix} 2 & -3 & 3 \\ -2 & 7 & -1 \\ 7 & 0 & -5 \end{bmatrix}$, with A_{ij} the cofactor of a_{ij}, find $a_{11}A_{12} + a_{21}A_{22} + a_{31}A_{32}$. [1 mark]

9. Solve $\begin{vmatrix} x & 3 & 4 \\ 3 & x & 4 \\ 3 & 4 & x \end{vmatrix} = 0$. [5 marks]

10. A is a square matrix of order 3 with $|A| = 2$. Find $|4A|$ and the determinant of the cofactor matrix of A. [2 marks]

11. Find the determinant of $\begin{bmatrix} -3 & 12 & 12 \\ 0 & -3 & 3 \\ 0 & 0 & 2 \end{bmatrix}$. [1 mark]

12. Using a standard result, find $\begin{vmatrix} 1 & 5 & 25 \\ 1 & -1 & 1 \\ 1 & 4 & 16 \end{vmatrix}$. [2 marks]

13. Find the value of $\begin{vmatrix} 5 & 1 & 1 \\ 1 & 9 & 1 \\ 1 & 1 & 7 \end{vmatrix}$ using a standard result. [3 marks]

14. Show that $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -(a^3 + b^3 + c^3 - 3abc)$. [5 marks]

15. By writing it as a product of two determinants, evaluate $\begin{vmatrix} (1+5)^2 & (1+1)^2 & (1+4)^2 \\ (4+5)^2 & (4+1)^2 & (4+4)^2 \\ (3+5)^2 & (3+1)^2 & (3+4)^2 \end{vmatrix}$. [5 marks]

16. If $\Delta(x) = \begin{vmatrix} e^x & e^{2x} \\ x^3 & \sin x \end{vmatrix}$, find $\Delta'(x)$. [2 marks]

17. If $\Delta(x) = \begin{vmatrix} x^3 + x & x & x^2 + 2x \\ -3x - 1 & 1 - 3x & -2x - 3 \\ -1 & 3 & -1 \end{vmatrix}$, find $\Delta'(x)$ and $\Delta'(-1)$. [5 marks]

18. Evaluate $\int_0^1 \begin{vmatrix} 2x + 1 & x^2 + x \\ 6 & 3 \end{vmatrix} \, dx$. [3 marks]

19. Is $\begin{bmatrix} -5 & 1 & -5 \\ 6 & 5 & -2 \\ -16 & -3 & -8 \end{bmatrix}$ singular or non-singular? [1 mark]

20. For what value of p is $\begin{bmatrix} p & 6 \\ -12 & 4 \end{bmatrix}$ singular? [2 marks]

Key sheet

Q	Answer
1	-15
2	$x = 1$ or 8
3	Minor -10, cofactor 10
4	250
5	-3
6	0
7	-30
8	0
9	$x = -7,\ 3,\ 4$
10	128 and 4
11	18
12	30
13	296
14	$-(a^3 + b^3 + c^3 - 3abc)$
15	144
16	$\Delta'(x) = -2x^3e^{2x} - 3x^2e^{2x} + e^x \sin x + e^x \cos x$
17	$\Delta'(x) = 36x^3 - 12x^2 - 36x + 6$, $\Delta'(-1) = -6$
18	1
19	Singular ($\lvert A\rvert = 0$)
20	$p = -18$

Solutions

1. Evaluate $\begin{vmatrix} 5 & 8 \\ 5 & 5 \end{vmatrix}$. [1]

$\begin{vmatrix} 5 & 8 \\ 5 & 5 \end{vmatrix} = 5 \cdot 5 - 8 \cdot 5$

$= 25 - 40 = -15$

Ans: -15

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2. Solve $\begin{vmatrix} x+5 & 2 \\ 4 & x-14 \end{vmatrix} = -86$. [2]

$(x+5)(x-14) - 2 \cdot 4 = -86$

$\Rightarrow x^2 - 9x + 8 = 0$

$\Rightarrow (x-1)(x-8) = 0$

$\therefore x = 1 \text{ or } x = 8$

Ans: $x = 1$ or 8

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3. Find the minor and the cofactor of the element 9 in $\begin{bmatrix} -2 & 6 & 6 \\ 2 & 8 & -1 \\ 8 & 9 & 0 \end{bmatrix}$. [2]

Delete row 3 and column 2.

$M_{32} = \begin{vmatrix} -2 & 6 \\ 2 & -1 \end{vmatrix} = 2 - 12 = -10$

$A_{32} = (-1)^{3+2} M_{32} = 10$

Ans: Minor -10**, cofactor** 10

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4. Evaluate $\begin{vmatrix} -5 & 0 & -4 \\ 7 & 1 & -3 \\ 5 & -5 & -3 \end{vmatrix}$. [3]

Expand along the first row.

$\Delta = -5 \begin{vmatrix} 1 & -3 \\ -5 & -3 \end{vmatrix} - 4 \begin{vmatrix} 7 & 1 \\ 5 & -5 \end{vmatrix}$

$= -5(-3 - 15) - 4(-35 - 5)$

$= -5(-18) - 4(-40) = 250$

Ans: 250

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5. If $\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix} = 3$, find $\begin{vmatrix} x & y & z \\ p & q & r \\ a & b & c \end{vmatrix}$. [1]

Interchanging two rows changes the sign.

$\therefore \text{value} = (-1) \times 3 = -3$

Ans: -3

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6. Without expanding, find the value of $\begin{vmatrix} 1 & -1 & 1 \\ 1 & 2 & 6 \\ -3 & 3 & -3 \end{vmatrix}$. [1]

$R_3 = -3R_1$: two rows are proportional.

$\therefore \Delta = 0$

Ans: 0

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7. Using the properties of determinants, evaluate $\begin{vmatrix} 4 & 2 & 27 \\ -4 & -5 & -57 \\ -2 & -2 & -21 \end{vmatrix}$. [3]

Apply $R_2 \to R_2 - 2R_3$ (the value does not change):

$$\Delta = \begin{vmatrix} 4 & 2 & 27 \\ 0 & -1 & -15 \\ -2 & -2 & -21 \end{vmatrix}$$

Apply $C_3 \to C_3 - 12C_2$:

$$\Delta = \begin{vmatrix} 4 & 2 & 3 \\ 0 & -1 & -3 \\ -2 & -2 & 3 \end{vmatrix}$$

$$\Delta = -\begin{vmatrix} 4 & 3 \\ -2 & 3 \end{vmatrix} + 3\begin{vmatrix} 4 & 2 \\ -2 & -2 \end{vmatrix}$$

$$= -(12 - (-6)) + 3(-8 - (-4))$$

$$= -(18) + 3(-4) = -30$$

Ans: -30

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8. For $A = \begin{bmatrix} 2 & -3 & 3 \\ -2 & 7 & -1 \\ 7 & 0 & -5 \end{bmatrix}$, with A_{ij} the cofactor of a_{ij}, find $a_{11}A_{12} + a_{21}A_{22} + a_{31}A_{32}$. [1]

This is the elements of column 1 with the cofactors of column 2.

The sum of products of the elements of one line with the cofactors of a different line is 0.

$\therefore a_{11}A_{12} + a_{21}A_{22} + a_{31}A_{32} = 0$

Ans: 0

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9. Solve $\begin{vmatrix} x & 3 & 4 \\ 3 & x & 4 \\ 3 & 4 & x \end{vmatrix} = 0$. [5]

$C_1 \to C_1 + C_2 + C_3$: every entry of C_1 becomes $x + 7$.

$$\Delta = (x+7)\begin{vmatrix} 1 & 3 & 4 \\ 1 & x & 4 \\ 1 & 4 & x \end{vmatrix}$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$:

$$\Delta = (x+7)\begin{vmatrix} 1 & 3 & 4 \\ 0 & x-3 & 0 \\ 0 & 1 & x-4 \end{vmatrix}$$

$$= (x+7)(x-3)(x-4)$$

$\Delta = 0 \Rightarrow x = -7,\ 3,\ 4$

Ans: $x = -7,\ 3,\ 4$

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10. A is a square matrix of order 3 with $|A| = 2$. Find $|4A|$ and the determinant of the cofactor matrix of A. [2]

$|4A| = 4^3|A| = 64 \times 2 = 128$

$\det \text{ of cofactor matrix} = |A|^{3-1} = 2^2 = 4$

Ans: 128 **and** 4

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11. Find the determinant of $\begin{bmatrix} -3 & 12 & 12 \\ 0 & -3 & 3 \\ 0 & 0 & 2 \end{bmatrix}$. [1]

The matrix is triangular: the determinant is the product of the diagonal elements.

$(-3) \times (-3) \times 2 = 18$

Ans: 18

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12. Using a standard result, find $\begin{vmatrix} 1 & 5 & 25 \\ 1 & -1 & 1 \\ 1 & 4 & 16 \end{vmatrix}$. [2]

Use result 3, $(a-b)(b-c)(c-a)$ with $a, b, c = 5, -1, 4$.

$(5-(-1))(-1-4)(4-5) = 30$

Ans: 30

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13. Find the value of $\begin{vmatrix} 5 & 1 & 1 \\ 1 & 9 & 1 \\ 1 & 1 & 7 \end{vmatrix}$ using a standard result. [3]

Result 9 with $a, b, c = 4, 8, 6$:

$abc\left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) = 192 + 48 + 24 + 32 = 296$

Ans: 296

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14. Show that $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -(a^3 + b^3 + c^3 - 3abc)$. [5]

$C_1 \to C_1 + C_2 + C_3$; take $a+b+c$ out of C_1:

$$\Delta = (a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix}$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$:

$$\Delta = (a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & c-b & a-c \\ 0 & a-b & b-c \end{vmatrix}$$

$= (a+b+c)\left[(c-b)(b-c) - (a-c)(a-b)\right]$

$= -(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$

$= -(a^3+b^3+c^3-3abc)$

Ans: $-(a^3+b^3+c^3-3abc)$

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15. By writing it as a product of two determinants, evaluate $\begin{vmatrix}(1+5)^2 & (1+1)^2 & (1+4)^2\\(4+5)^2 & (4+1)^2 & (4+4)^2\\(3+5)^2 & (3+1)^2 & (3+4)^2\end{vmatrix}$. [5]

$(u+v)^2 = u^2\cdot 1 + 1\cdot v^2 + 2u\cdot v$

So, multiplying row by column,

$$\Delta = \begin{vmatrix}1 & 1 & 2\\16 & 1 & 8\\9 & 1 & 6\end{vmatrix}\begin{vmatrix}1 & 1 & 1\\25 & 1 & 16\\5 & 1 & 4\end{vmatrix}$$

$$\begin{vmatrix}1 & 1 & 2\\16 & 1 & 8\\9 & 1 & 6\end{vmatrix} = -12$$

$$\begin{vmatrix}1 & 1 & 1\\25 & 1 & 16\\5 & 1 & 4\end{vmatrix} = -12$$

$\therefore \Delta = (-12)\times(-12) = 144$

Ans: 144

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16. If $\Delta(x) = \begin{vmatrix}e^x & e^{2x}\\x^3 & \sin x\end{vmatrix}$, find $\Delta'(x)$. [2]

$$\Delta'(x) = \begin{vmatrix}e^x & 2e^{2x}\\x^3 & \sin x\end{vmatrix} + \begin{vmatrix}e^x & e^{2x}\\3x^2 & \cos x\end{vmatrix}$$

$$= (-2x^3e^{2x} + e^x\sin x) + (-3x^2e^{2x} + e^x\cos x)$$

$$= -2x^3e^{2x} - 3x^2e^{2x} + e^x\sin x + e^x\cos x$$

Ans: $\Delta'(x) = -2x^3e^{2x} - 3x^2e^{2x} + e^x\sin x + e^x\cos x$

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17. If $\Delta(x) = \begin{vmatrix}x^3+x & x & x^2+2x\\-3x-1 & 1-3x & -2x-3\\-1 & 3 & -1\end{vmatrix}$, find $\Delta'(x)$ and $\Delta'(-1)$. [5]

Differentiate one row at a time and add:

$$\Delta'(x) = \begin{vmatrix}3x^2+1 & 1 & 2x+2\\-3x-1 & 1-3x & -2x-3\\-1 & 3 & -1\end{vmatrix}$$

$$+\begin{vmatrix}x^3+x & x & x^2+2x\\-3 & -3 & -2\\-1 & 3 & -1\end{vmatrix}$$

$$+\begin{vmatrix}x^3+x & x & x^2+2x\\-3x-1 & 1-3x & -2x-3\\0 & 0 & 0\end{vmatrix}$$

The third determinant has a row of zeros, so it is 0.

$$\begin{vmatrix}3x^2+1 & 1 & 2x+2\\-3x-1 & 1-3x & -2x-3\\-1 & 3 & -1\end{vmatrix} = 27x^3 - 20x + 6$$

$$\begin{vmatrix}x^3+x & x & x^2+2x\\-3 & -3 & -2\\-1 & 3 & -1\end{vmatrix} = 9x^3 - 12x^2 - 16x$$

$\therefore \Delta'(x) = 36x^3 - 12x^2 - 36x + 6$

$\Delta'(-1) = -6$

Ans: $\Delta'(x) = 36x^3 - 12x^2 - 36x + 6$, $\Delta'(-1) = -6$

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18. Evaluate $\int_0^1 \begin{vmatrix} 2x+1 & x^2+x \\ 6 & 3 \end{vmatrix} dx$. [3]

Only one row contains x, so integrate that row:

$\begin{vmatrix} \int_0^1 (2x+1)\,dx & \int_0^1 (x^2+x)\,dx \\ 6 & 3 \end{vmatrix} = \begin{vmatrix} 2 & \frac{5}{6} \\ 6 & 3 \end{vmatrix}$

$= 1$

Ans: 1

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19. Is $\begin{bmatrix} -5 & 1 & -5 \\ 6 & 5 & -2 \\ -16 & -3 & -8 \end{bmatrix}$ singular or non-singular? [1]

$R_3 = 2R_1 - R_2$, so $R_3 \to R_3 - (2R_1 - R_2)$ gives a row of zeros.

$|A| = 0$

So the matrix is singular.

Ans: Singular ($|A| = 0$)

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20. For what value of p is $\begin{bmatrix} p & 6 \\ -12 & 4 \end{bmatrix}$ singular? [2]

$\begin{vmatrix} p & 6 \\ -12 & 4 \end{vmatrix} = 4p + 72 = 0$

$\Rightarrow p = -18$

Ans: $p = -18$
