EXPRESSWALA ACADEMY

Maths 1A: Compound Angles

Question Paper, Set 09

Time: 1 hour 30 minutes Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. If $\cos A = \frac{33}{65}$ and $\cos B = \frac{5}{13}$, where A and B are acute, find $\sin(A+B)$. [2 marks]

2. If $\cos A = \frac{12}{13}$, $270^\circ < A < 360^\circ$ and $\sin B = -\frac{9}{41}$, $180^\circ < B < 270^\circ$, find $\sin(A+B)$, $\cos(A-B)$ and $\tan(A+B)$. [5 marks]

3. Find the exact value of $\cot 345^\circ$. [2 marks]

4. Find the value of $\cos 89^\circ \cos 31^\circ - \sin 89^\circ \sin 31^\circ$. [1 mark]

5. Express $2\sin 2\theta \cos\theta$ as a sum or difference. [1 mark]

6. Express $\cos 8\theta + \cos 4\theta$ as a product. [2 marks]

7. Show that $\sin 53^\circ - \sin 67^\circ + \sin 7^\circ = 0$. [3 marks]

8. If $\cot A = 5$ and $\cot B = 1$, find $\cot(A+B)$. [2 marks]

9. Find the value of $\frac{1+\tan 195^\circ}{1-\tan 195^\circ}$. [1 mark]

10. Prove that $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$ and hence find $\sin^2 75^\circ - \sin^2 15^\circ$. [5 marks]

11. Find the value of $\frac{\cos 30^\circ + \sin 30^\circ}{\cos 30^\circ - \sin 30^\circ}$. [1 mark]

12. If $\tan A = 5$, $\tan B = \frac{1}{2}$ and $\tan C = 3$, find $\tan(A+B+C)$. [3 marks]

13. Find the value of $\tan 75^\circ$. [2 marks]

14. If $\tan A = \frac{7}{10}$ and $\tan B = \frac{3}{17}$, where A and B are acute, show that $A + B = \frac{\pi}{4}$. [3 marks]

15. Find the value of $(1+\cot 18^\circ)(1+\cot 117^\circ)$. [1 mark]

16. If $A + B = 45^\circ$, prove that $(1+\cot A)(1+\cot B) = 2\cot A \cot B$. Hence find $\cot 22\frac{1}{2}^\circ$. [5 marks]

17. If $A + B + C = \frac{\pi}{2}$, $\tan A = \frac{1}{5}$ and $\tan B = \frac{1}{6}$, find $\tan C$. [2 marks]

18. If $A + B + C = \pi$, $\cot A = 1$ and $\cot B = \frac{5}{3}$, find $\cot C$ and $\tan C$. [3 marks]

19. Find the value of $\cos 135^\circ + \cos 105^\circ$. [1 mark]

20. Show that $\tan 53^\circ = \tan 37^\circ + 2\tan 16^\circ$. [5 marks]

Key sheet

Q	Answer
1	$\frac{4}{5}$
2	$\sin(A+B)=\frac{92}{533}$, $\cos(A-B)=-\frac{435}{533}$, $\tan(A+B)=-\frac{92}{525}$
3	$-(2+\sqrt{3})$
4	$-\frac{1}{2}$
5	$\sin\theta+\sin 3\theta$
6	$2\cos 6\theta\cos 2\theta$
7	Proved
8	$\frac{2}{3}$
9	$\sqrt{3}$
10	$\frac{\sqrt{3}}{2}$
11	$2+\sqrt{3}$
12	$-\frac{1}{18}$
13	$2+\sqrt{3}$
14	$A+B=\frac{\pi}{4}$
15	2
16	Proved; $\cot 22\frac{1}{2}^\circ=\sqrt{2}+1$
17	$\tan C=\frac{29}{11}$
18	$\cot C=-\frac{1}{4}$, $\tan C=-4$
19	$-\cos 15^\circ$
20	Proved

Solutions

1. If $\cos A = \frac{33}{65}$ and $\cos B = \frac{5}{13}$, where A and B are acute, find $\sin(A+B)$. [2]

$\sin A = \sqrt{1-\cos^2 A} = \frac{56}{65}$

$\sin B = \sqrt{1-\cos^2 B} = \frac{12}{13}$

$\sin(A+B) = \sin A\cos B + \cos A \sin B$

$= (\frac{56}{65})(\frac{5}{13}) + (\frac{33}{65})(\frac{12}{13}) = \frac{4}{5}$

Ans: $\frac{4}{5}$

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2. If $\cos A = \frac{12}{13}$, $270° < A < 360°$ and $\sin B = -\frac{9}{41}$, $180° < B < 270°$, find $\sin(A+B)$, $\cos(A-B)$ and $\tan(A+B)$. [5]

A is in quadrant 4 and B in quadrant 3; use the signs there.

$\sin A = -\sqrt{1-\cos^2 A} = -\frac{5}{13}$

$\cos B = -\sqrt{1-\sin^2 B} = -\frac{40}{41}$

$\tan A = -\frac{5}{12},\ \tan B = \frac{9}{40}$

$\sin(A+B) = (-\frac{5}{13})(-\frac{40}{41}) + (\frac{12}{13})(-\frac{9}{41}) = \frac{92}{533}$

$\cos(A-B) = (\frac{12}{13})(-\frac{40}{41}) + (-\frac{5}{13})(-\frac{9}{41}) = -\frac{435}{533}$

$\tan(A+B) = \frac{-\frac{5}{12}+\frac{9}{40}}{1-(-\frac{5}{12})(\frac{9}{40})} = -\frac{92}{525}$

Ans: $\sin(A+B) = \frac{92}{533}$, $\cos(A-B) = -\frac{435}{533}$, $\tan(A+B) = -\frac{92}{525}$

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3. Find the exact value of $\cot 345°$. [2]

$\tan 345° = \tan(315° + 30°)$

$= \frac{\tan 315° + \tan 30°}{1 - \tan 315° \tan 30°}$

$= \frac{-1+\frac{1}{\sqrt{3}}}{1-(-1)\left(\frac{1}{\sqrt{3}}\right)}$

$= -(2-\sqrt{3})$

$\cot 345° = \frac{1}{\tan 345°} = -(2+\sqrt{3})$

Ans: $-(2+\sqrt{3})$

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4. Find the value of $\cos 89°\cos 31° - \sin 89°\sin 31°$. [1]

$\cos 89°\cos 31° - \sin 89°\sin 31° = \cos(89° + 31°)$

$= \cos 120° = -\frac{1}{2}$

Ans: $-\frac{1}{2}$

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5. Express $2\sin 2\theta \cos\theta$ as a sum or difference. [1]

$2\sin A\cos B = \sin(A+B) + \sin(A-B)$

$2\sin 2\theta\cos\theta = \sin\theta + \sin 3\theta$

Ans: $\sin\theta + \sin 3\theta$

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6. Express $\cos 8\theta + \cos 4\theta$ as a product. [2]

$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

$\cos 8\theta + \cos 4\theta = 2\cos 6\theta\cos 2\theta$

Ans: $2\cos 6\theta\cos 2\theta$

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7. Show that $\sin 53^\circ - \sin 67^\circ + \sin 7^\circ = 0$. [3]

$\sin 53^\circ - \sin 67^\circ = 2\cos 60^\circ \sin(-7^\circ) = -\sin 7^\circ$

$\therefore \text{LHS} = -\sin 7^\circ + \sin 7^\circ = 0$

Ans: Proved

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8. If $\cot A = 5$ and $\cot B = 1$, find $\cot(A+B)$. [2]

$\cot(A+B) = \frac{\cot A \cot B - 1}{\cot B + \cot A}$

$= \frac{5 \cdot 1 - 1}{1+5} = \frac{2}{3}$

Ans: $\frac{2}{3}$

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9. Find the value of $\frac{1+\tan 195^\circ}{1-\tan 195^\circ}$. [1]

$\frac{1+\tan 195^\circ}{1-\tan 195^\circ} = \tan(45^\circ + 195^\circ) = \tan 240^\circ$

$= \sqrt{3}$

Ans: $\sqrt{3}$

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10. Prove that $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$ and hence find $\sin^2 75^\circ - \sin^2 15^\circ$. [5]

$\sin(A+B)\sin(A-B) = (\sin A \cos B + \cos A \sin B)(\sin A \cos B - \cos A \sin B)$

$= \sin^2 A \cos^2 B - \cos^2 A \sin^2 B$

$= \sin^2 A(1 - \sin^2 B) - (1 - \sin^2 A)\sin^2 B$

$= \sin^2 A - \sin^2 B$

Put $A = 75^\circ$, $B = 15^\circ$:

$\sin^2 75^\circ - \sin^2 15^\circ = \sin 90^\circ \sin 60^\circ = (1)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{2}$

Ans: $\frac{\sqrt{3}}{2}$

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11. Find the value of $\frac{\cos 30^\circ + \sin 30^\circ}{\cos 30^\circ - \sin 30^\circ}$. [1]

$\frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \frac{1+\tan\theta}{1-\tan\theta} = \tan(45^\circ + \theta)$

$\frac{\cos 30^\circ + \sin 30^\circ}{\cos 30^\circ - \sin 30^\circ} = \tan 75^\circ = 2 + \sqrt{3}$

Ans: $2 + \sqrt{3}$

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12. If $\tan A = 5$, $\tan B = \frac{1}{2}$ and $\tan C = 3$, find $\tan(A+B+C)$. [3]

$\tan(A+B+C) = \frac{S_1 - S_3}{1 - S_2}$

$S_1 = \frac{17}{2},\ S_2 = 19,\ S_3 = \frac{15}{2}$

$\tan(A+B+C) = \frac{\frac{17}{2} - \frac{15}{2}}{1-19} = -\frac{1}{18}$

Ans: $-\frac{1}{18}$

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13. Find the value of $\tan 75^\circ$. [2]

$\tan 75^\circ = \tan(45^\circ + 30^\circ)$

$= \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ}$

$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - (1)\left(\frac{1}{\sqrt{3}}\right)}$

$= 2 + \sqrt{3}$

Ans: $2 + \sqrt{3}$

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14. If $\tan A = \frac{7}{10}$ and $\tan B = \frac{3}{17}$, where A and B are acute, show that $A + B = \frac{\pi}{4}$. [3]

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{7}{10} + \frac{3}{17}}{1 - \frac{7}{10} \cdot \frac{3}{17}}$

$= \frac{\frac{149}{170}}{\frac{149}{170}} = 1$

$\therefore A + B = \frac{\pi}{4}$

Ans: $A + B = \frac{\pi}{4}$

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15. Find the value of $(1 + \cot 18^\circ)(1 + \cot 117^\circ)$. [1]

$18^\circ + 117^\circ = 135^\circ,\ \cot(18^\circ + 117^\circ) = -1$

$\Rightarrow \cot A \cot B - 1 = -(\cot A + \cot B)$

$\therefore (1 + \cot 18^\circ)(1 + \cot 117^\circ) = 2$

Ans: 2

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16. If $A + B = 45^\circ$, prove that $(1 + \cot A)(1 + \cot B) = 2 \cot A \cot B$. Hence find $\cot 22\frac{1}{2}^\circ$. [5]

$\cot(A + B) = \cot 45^\circ = 1$

$\frac{\cot A \cot B - 1}{\cot A + \cot B} = 1 \Rightarrow \cot A \cot B - 1 = (\cot A + \cot B)$

Rearranging, $(1 + \cot A)(1 + \cot B) = 2 \cot A \cot B$.

$A = B = 22\frac{1}{2}^\circ :\ (1 + t)^2 = 2t^2,\ t = \cot 22\frac{1}{2}^\circ$

$\Rightarrow 1 + t = \sqrt{2}\, t \Rightarrow t = \frac{1}{\sqrt{2} - 1} = \sqrt{2} + 1$

Ans: Proved; $\cot 22\frac{1}{2}^\circ = \sqrt{2} + 1$

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17. If $A + B + C = \frac{\pi}{2}$, $\tan A = \frac{1}{5}$ and $\tan B = \frac{1}{6}$, find $\tan C$. [2]

$A + B + C = \frac{\pi}{2} \Rightarrow \sum \tan A \tan B = 1$

$\frac{1}{30} + (\frac{1}{5} + \frac{1}{6}) \tan C = 1$

$\tan C = \frac{29}{11}$

Ans: $\tan C = \frac{29}{11}$

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18. If $A + B + C = \pi$, $\cot A = 1$ and $\cot B = \frac{5}{3}$, find $\cot C$ and $\tan C$. [3]

$A + B + C = \pi \Rightarrow \tan(A + B + C) = 0$

$\Rightarrow \sum \cot A \cot B = 1$

$\frac{5}{3} + (1 + \frac{5}{3}) \cot C = 1$

$\cot C = -\frac{1}{4},\ \tan C = -4$

Ans: $\cot C = -\frac{1}{4}$, $\tan C = -4$

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19. Find the value of $\cos 135^\circ + \cos 105^\circ$. [1]

$\cos 135^\circ + \cos 105^\circ = 2 \cos 120^\circ \cos 15^\circ = -\cos 15^\circ$

Ans: $-\cos 15^\circ$

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20. Show that $\tan 53^\circ = \tan 37^\circ + 2\tan 16^\circ$. [5]

$37^\circ + 16^\circ = 53^\circ$, so with $A + C = B$:

$\tan 53^\circ - \tan 37^\circ - \tan 16^\circ = \tan 37^\circ \tan 53^\circ \tan 16^\circ$

$\tan 37^\circ \tan 53^\circ = \tan 37^\circ \cot 37^\circ = 1$

$\Rightarrow \tan 53^\circ - \tan 37^\circ - \tan 16^\circ = \tan 16^\circ$

$\therefore \tan 53^\circ = \tan 37^\circ + 2\tan 16^\circ$

Ans: Proved