EXPRESSWALA ACADEMY

Maths 1A: Compound Angles

Question Paper, Set 07

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. If $\cos A = \frac{45}{53}$ and $\sin B = \frac{11}{61}$, where A and B are acute, find $\sin(A - B)$. [2 marks]

2. If $\cos A = -\frac{56}{65}$, $180^\circ < A < 270^\circ$ and $\tan B = \frac{60}{11}$, $0^\circ < B < 90^\circ$, find $\sin(A + B)$, $\cos(A - B)$ and $\tan(A + B)$. [5 marks]

3. Find the value of $\sin 165^\circ$. [2 marks]

4. Find the value of $\sin 133^\circ \cos 43^\circ - \cos 133^\circ \sin 43^\circ$. [1 mark]

5. Express $2\cos 6\theta \sin 3\theta$ as a sum or difference. [1 mark]

6. Express $\cos 3\theta + \cos \theta$ as a product. [2 marks]

7. Show that $\cos 27^\circ + \cos 63^\circ = \sqrt{2}\,\cos 18^\circ$. [3 marks]

8. If $\tan A = \frac{3}{2}$ and $\tan B = \frac{9}{7}$, find $\tan(A + B)$. [2 marks]

9. Find the value of $\frac{\cot 136^\circ \cot 14^\circ - 1}{\cot 14^\circ + \cot 136^\circ}$. [1 mark]

10. Prove that $\sin(A + B)\sin(A - B) = \sin^2 A - \sin^2 B$ and hence find $\sin^2 90^\circ - \sin^2 60^\circ$. [5 marks]

11. Find the value of $\frac{\cos 165^\circ + \sin 165^\circ}{\cos 165^\circ - \sin 165^\circ}$. [1 mark]

12. If $\tan A = 1$, $\tan B = 6$ and $\tan C = 4$, find $\tan(A + B + C)$. [3 marks]

13. Find the value of $\cot 75^\circ$. [2 marks]

14. If $\tan A = \frac{1}{9}$ and $\tan B = \frac{4}{5}$, where A and B are acute, show that $A + B = \frac{\pi}{4}$. [3 marks]

15. Find the value of $(1 + \cot 19^\circ)(1 + \cot 116^\circ)$. [1 mark]

16. If $A + B = 225^\circ$, prove that $(1 - \cot A)(1 - \cot B) = 2$. Hence find $\cot 112\frac{1}{2}^\circ$. [5 marks]

17. If $A + B + C = \frac{\pi}{2}$, $\tan A = \frac{9}{8}$ and $\tan B = 5$, find $\tan C$. [2 marks]

18. If $A + B + C = \pi$, $\cot A = 2$ and $\cot B = 3$, find $\cot C$ and $\tan C$. [3 marks]

19. Find the value of $\sin 10^\circ + \sin 250^\circ - \sin 230^\circ$. [1 mark]

20. Show that $\tan 54^\circ = \tan 36^\circ + 2\tan 18^\circ$. [5 marks]

Key sheet

Q	Answer
1	$\frac{1185}{3233}$
2	$\sin(A+B) = -\frac{3723}{3965}$, $\cos(A-B) = -\frac{2596}{3965}$, $\tan(A+B) = -\frac{3723}{1364}$
3	$\frac{\sqrt{3}-1}{2\sqrt{2}}$
4	1
5	$-\sin 3\theta + \sin 9\theta$
6	$2\cos 2\theta \cos\theta$
7	Proved
8	-3
9	$-\sqrt{3}$
10	$\frac{1}{4}$
11	$\frac{1}{\sqrt{3}}$
12	$\frac{13}{33}$
13	$2-\sqrt{3}$
14	$A+B = \frac{\pi}{4}$
15	2
16	Proved; $\cot 112\frac{1}{2}^\circ = -(\sqrt{2}-1)$
17	$\tan C = -\frac{37}{49}$
18	$\cot C = -1$, $\tan C = -1$
19	0
20	Proved

Solutions

1. If $\cos A = \frac{45}{53}$ and $\sin B = \frac{11}{61}$, where A and B are acute, find $\sin(A - B)$. [2]

$\sin A = \sqrt{1 - \cos^2 A} = \frac{28}{53}$

$\cos B = \sqrt{1 - \sin^2 B} = \frac{60}{61}$

$\sin(A - B) = \sin A \cos B - \cos A \sin B$

$= (\frac{28}{53})(\frac{60}{61}) - (\frac{45}{53})(\frac{11}{61}) = \frac{1185}{3233}$

Ans: $\frac{1185}{3233}$

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2. If $\cos A = -\frac{56}{65}$, $180° < A < 270°$ and $\tan B = \frac{60}{11}$, $0° < B < 90°$, find $\sin(A + B)$, $\cos(A - B)$ and $\tan(A + B)$. [5]

A is in quadrant 3 and B in quadrant 1; use the signs there.

$\sin A = -\sqrt{1 - \cos^2 A} = -\frac{33}{65}$

$\tan B = \frac{60}{11} \Rightarrow \sin B = \frac{60}{61},\ \cos B = \frac{11}{61}$

$\tan A = \frac{33}{56},\ \tan B = \frac{60}{11}$

$\sin(A + B) = (-\frac{33}{65})(\frac{11}{61}) + (-\frac{56}{65})(\frac{60}{61}) = -\frac{3723}{3965}$

$\cos(A - B) = (-\frac{56}{65})(\frac{11}{61}) + (-\frac{33}{65})(\frac{60}{61}) = -\frac{2596}{3965}$

$\tan(A + B) = \frac{\frac{33}{56} + \frac{60}{11}}{1 - (\frac{33}{56})(\frac{60}{11})} = -\frac{3723}{1364}$

Ans: $\sin(A + B) = -\frac{3723}{3965}$, $\cos(A - B) = -\frac{2596}{3965}$, $\tan(A + B) = -\frac{3723}{1364}$

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3. Find the value of $\sin 165°$. [2]

$\sin 165° = \sin(120° + 45°)$

$= \sin 120° \cos 45° + \cos 120° \sin 45°$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(-\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$

Ans: $\frac{\sqrt{3}-1}{2\sqrt{2}}$

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4. Find the value of $\sin 133° \cos 43° - \cos 133° \sin 43°$. [1]

$\sin 133° \cos 43° - \cos 133° \sin 43° = \sin(133° - 43°)$

$= \sin 90° = 1$

Ans: 1

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5. Express $2 \cos 6\theta \sin 3\theta$ as a sum or difference. [1]

$2 \cos A \sin B = \sin(A + B) - \sin(A - B)$

$2 \cos 6\theta \sin 3\theta = -\sin 3\theta + \sin 9\theta$

Ans: $-\sin 3\theta + \sin 9\theta$

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6. Express $\cos 3\theta + \cos \theta$ as a product. [2]

$\cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2}$

$\cos 3\theta + \cos \theta = 2 \cos 2\theta \cos \theta$

Ans: $2 \cos 2\theta \cos \theta$

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7. Show that $\cos 27^\circ + \cos 63^\circ = \sqrt{2}\cos 18^\circ$. [3]

$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

$\cos 27^\circ + \cos 63^\circ = 2\cos 45^\circ \cos 18^\circ = \sqrt{2}\cos 18^\circ$

Ans: Proved

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8. If $\tan A = \frac{3}{2}$ and $\tan B = \frac{9}{7}$, find $\tan(A+B)$. [2]

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$= \frac{\frac{3}{2}+\frac{9}{7}}{1-\frac{3}{2}\cdot\frac{9}{7}} = -3$

Ans: -3

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9. Find the value of $\frac{\cot 136^\circ \cot 14^\circ - 1}{\cot 14^\circ + \cot 136^\circ}$. [1]

$\frac{\cot 136^\circ \cot 14^\circ - 1}{\cot 14^\circ + \cot 136^\circ} = \cot(136^\circ + 14^\circ) = \cot 150^\circ$

$= -\sqrt{3}$

Ans: $-\sqrt{3}$

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10. Prove that $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$ and hence find $\sin^2 90^\circ - \sin^2 60^\circ$. [5]

$\sin(A+B)\sin(A-B) = (\sin A\cos B + \cos A\sin B)(\sin A\cos B - \cos A\sin B)$

$= \sin^2 A\cos^2 B - \cos^2 A\sin^2 B$

$= \sin^2 A(1-\sin^2 B) - (1-\sin^2 A)\sin^2 B$

$= \sin^2 A - \sin^2 B$

Put $A = 90^\circ$, $B = 60^\circ$:

$\sin^2 90^\circ - \sin^2 60^\circ = \sin 150^\circ \sin 30^\circ = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$

Ans: $\frac{1}{4}$

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11. Find the value of $\frac{\cos 165^\circ + \sin 165^\circ}{\cos 165^\circ - \sin 165^\circ}$. [1]

$\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta} = \frac{1+\tan\theta}{1-\tan\theta} = \tan(45^\circ+\theta)$

$\frac{\cos 165^\circ+\sin 165^\circ}{\cos 165^\circ-\sin 165^\circ} = \tan 210^\circ = \frac{1}{\sqrt{3}}$

Ans: $\frac{1}{\sqrt{3}}$

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12. If $\tan A = 1$, $\tan B = 6$ and $\tan C = 4$, find $\tan(A+B+C)$. [3]

$\tan(A+B+C) = \frac{S_1 - S_3}{1 - S_2}$

$S_1 = 11,\ S_2 = 34,\ S_3 = 24$

$\tan(A+B+C) = \frac{11-24}{1-34} = \frac{13}{33}$

Ans: $\frac{13}{33}$

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13. Find the value of $\cot 75^\circ$. [2]

$\tan 75^\circ = \tan(45^\circ + 30^\circ)$

$= \frac{\tan 45^\circ + \tan 30^\circ}{1 - \tan 45^\circ \tan 30^\circ}$

$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - (1)\left(\frac{1}{\sqrt{3}}\right)}$

$= 2 + \sqrt{3}$

$\cot 75^\circ = \frac{1}{\tan 75^\circ} = 2 - \sqrt{3}$

Ans: $2 - \sqrt{3}$

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14. If $\tan A = \frac{1}{9}$ and $\tan B = \frac{4}{5}$, where A and B are acute, show that $A + B = \frac{\pi}{4}$. [3]

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{1}{9}+\frac{4}{5}}{1-\frac{1}{9}\cdot\frac{4}{5}}$

$= \frac{\frac{41}{45}}{\frac{41}{45}} = 1$

$\therefore A + B = \frac{\pi}{4}$

Ans: $A + B = \frac{\pi}{4}$

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15. Find the value of $(1+\cot 19^\circ)(1+\cot 116^\circ)$. [1]

$19^\circ + 116^\circ = 135^\circ,\ \cot(19^\circ + 116^\circ) = -1$

$\Rightarrow \cot A \cot B - 1 = -(\cot A + \cot B)$

$\therefore (1+\cot 19^\circ)(1+\cot 116^\circ) = 2$

Ans: 2

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16. If $A + B = 225^\circ$, prove that $(1-\cot A)(1-\cot B) = 2$. Hence find $\cot 112\frac{1}{2}^\circ$. [5]

$\cot(A+B) = \cot 225^\circ = 1$

$\frac{\cot A \cot B - 1}{\cot A + \cot B} = 1 \Rightarrow \cot A \cot B - 1 = (\cot A + \cot B)$

Rearranging, $(1-\cot A)(1-\cot B) = 2$.

$A = B = 112\frac{1}{2}^\circ :\ (1-t)^2 = 2,\ t = \cot 112\frac{1}{2}^\circ$

$\Rightarrow 1 - t = \pm\sqrt{2};\ \cot 112\frac{1}{2}^\circ < 0,\ \text{so } t = -(\sqrt{2}-1)$

Ans: **Proved;** $\cot 112\frac{1}{2}^\circ = -(\sqrt{2}-1)$

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17. If $A + B + C = \frac{\pi}{2}$, $\tan A = \frac{9}{8}$ and $\tan B = 5$, find $\tan C$. [2]

$A + B + C = \frac{\pi}{2} \Rightarrow \sum \tan A \tan B = 1$

$\frac{45}{8} + (\frac{9}{8} + 5)\tan C = 1$

$\tan C = -\frac{37}{49}$

Ans: $\tan C = -\frac{37}{49}$

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18. If $A + B + C = \pi$, $\cot A = 2$ and $\cot B = 3$, find $\cot C$ and $\tan C$. [3]

$A + B + C = \pi \Rightarrow \tan(A+B+C) = 0$

$\Rightarrow \sum \cot A \cot B = 1$

$6 + (2+3)\cot C = 1$

$\cot C = -1,\ \tan C = -1$

Ans: $\cot C = -1$, $\tan C = -1$

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19. Find the value of $\sin 10^\circ + \sin 250^\circ - \sin 230^\circ$. [1]

$\sin 250^\circ - \sin 230^\circ = 2\cos 240^\circ \sin 10^\circ = -\sin 10^\circ$

$\therefore \text{value} = 0$

Ans: 0

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20. Show that $\tan 54^\circ = \tan 36^\circ + 2\tan 18^\circ$. [5]

$36^\circ + 18^\circ = 54^\circ$, so with $A + C = B$:

$\tan 54^\circ - \tan 36^\circ - \tan 18^\circ = \tan 36^\circ \tan 54^\circ \tan 18^\circ$

$\tan 36^\circ \tan 54^\circ = \tan 36^\circ \cot 36^\circ = 1$

$\Rightarrow \tan 54^\circ - \tan 36^\circ - \tan 18^\circ = \tan 18^\circ$

$\therefore \tan 54^\circ = \tan 36^\circ + 2\tan 18^\circ$

Ans: Proved