EXPRESSWALA ACADEMY

Maths 1A: Compound Angles

Question Paper, Set 06

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. If $\cos A = \frac{7}{25}$ and $\sin B = \frac{9}{41}$, where A and B are acute, find $\cos(A - B)$. [2 marks]

2. If $\sin A = \frac{56}{65}$, $90^\circ < A < 180^\circ$ and $\sin B = -\frac{28}{53}$, $180^\circ < B < 270^\circ$, find $\sin(A + B)$, $\cos(A - B)$ and $\tan(A + B)$. [5 marks]

3. Find the exact value of $\cot 285^\circ$. [2 marks]

4. Evaluate $\cos 159^\circ \cos 39^\circ + \sin 159^\circ \sin 39^\circ$. [1 mark]

5. Write $2 \sin 75^\circ \sin 45^\circ$ as a sum or difference and simplify. [1 mark]

6. Write $\sin 4\theta + \sin 2\theta$ as a product of two trigonometric ratios. [2 marks]

7. Show that $\cos 76^\circ + \cos 44^\circ = \cos 16^\circ$. [3 marks]

8. If $\cot A = 6$ and $\cot B = \frac{7}{4}$, find $\cot(A + B)$. [2 marks]

9. Find the value of $\frac{\cot 108^\circ \cot 12^\circ - 1}{\cot 12^\circ + \cot 108^\circ}$. [1 mark]

10. Prove that $\cos(A + B)\cos(A - B) = \cos^2 A - \sin^2 B$ and hence find $\cos^2 97\frac{1}{2}^\circ - \sin^2 37\frac{1}{2}^\circ$. [5 marks]

11. Find the value of $\tan 72^\circ \tan 18^\circ$. [1 mark]

12. If $\tan A = \frac{1}{4}$, $\tan B = \frac{1}{4}$ and $\tan C = 4$, find $\tan(A + B + C)$. [3 marks]

13. Evaluate $\tan 15^\circ + \cot 15^\circ$ using the values at 15° and 75°. [2 marks]

14. If $\tan A = 7$ and $\tan B = \frac{3}{4}$, where A and B are acute, show that $A - B = \frac{\pi}{4}$. [3 marks]

15. Without tables, find $\tan 37^\circ + \tan 98^\circ - \tan 37^\circ \tan 98^\circ$. [1 mark]

16. If $A + B = 315^\circ$, prove that $(1 - \tan A)(1 - \tan B) = 2$. Hence find $\tan 157\frac{1}{2}^\circ$. [5 marks]

17. If $A + B + C = \pi$, $\tan A = \frac{9}{8}$ and $\tan B = \frac{1}{5}$, find $\tan C$. [2 marks]

18. If $A + B + C = \pi$, $\cot A = \frac{6}{7}$ and $\cot B = 4$, find $\cot C$ and $\tan C$. [3 marks]

19. Find the value of $\sin 110^\circ - \sin 130^\circ$. [1 mark]

20. Prove that $\tan 9A - \tan 7A - \tan 2A = \tan 9A \tan 7A \tan 2A$. [5 marks]

Key sheet

Q	Answer
1	$\frac{496}{1025}$
2	$\sin(A+B) = -\frac{1596}{3445}$, $\cos(A-B) = -\frac{83}{3445}$, $\tan(A+B) = -\frac{1596}{3053}$
3	$-(2-\sqrt{3})$
4	$-\frac{1}{2}$
5	$\frac{1}{2} + \frac{\sqrt{3}}{2}$
6	$2\sin 3\theta \cos\theta$
7	Proved
8	$\frac{38}{31}$
9	$-\frac{1}{\sqrt{3}}$
10	$-\frac{\sqrt{2}}{4}$
11	1
12	-4
13	4
14	$A - B = \frac{\pi}{4}$
15	-1
16	Proved; $\tan 157\frac{1}{2}^\circ = -(\sqrt{2}-1)$
17	$\tan C = -\frac{53}{31}$
18	$\cot C = -\frac{1}{2}$, $\tan C = -2$
19	$\sin 10^\circ$
20	Proved

Solutions

1. If $\cos A = \frac{7}{25}$ and $\sin B = \frac{9}{41}$, where A and B are acute, find $\cos(A - B)$. [2]

$\sin A = \sqrt{1 - \cos^2 A} = \frac{24}{25}$

$\cos B = \sqrt{1 - \sin^2 B} = \frac{40}{41}$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

$= (\frac{7}{25})(\frac{40}{41}) + (\frac{24}{25})(\frac{9}{41}) = \frac{496}{1025}$

Ans: $\frac{496}{1025}$

---

2. If $\sin A = \frac{56}{65}$, $90° < A < 180°$ and $\sin B = -\frac{28}{53}$, $180° < B < 270°$, find $\sin(A + B)$, $\cos(A - B)$ and $\tan(A + B)$. [5]

A is in quadrant 2 and B in quadrant 3; use the signs there.

$\cos A = -\sqrt{1 - \sin^2 A} = -\frac{33}{65}$

$\cos B = -\sqrt{1 - \sin^2 B} = -\frac{45}{53}$

$\tan A = -\frac{56}{33},\ \tan B = \frac{28}{45}$

$\sin(A + B) = (\frac{56}{65})(-\frac{45}{53}) + (-\frac{33}{65})(-\frac{28}{53}) = -\frac{1596}{3445}$

$\cos(A - B) = (-\frac{33}{65})(-\frac{45}{53}) + (\frac{56}{65})(-\frac{28}{53}) = -\frac{83}{3445}$

$\tan(A + B) = \frac{-\frac{56}{33} + \frac{28}{45}}{1 - (-\frac{56}{33})(\frac{28}{45})} = -\frac{1596}{3053}$

Ans: $\sin(A + B) = -\frac{1596}{3445}$, $\cos(A - B) = -\frac{83}{3445}$, $\tan(A + B) = -\frac{1596}{3053}$

---

3. Find the exact value of $\cot 285°$. [2]

$\tan 285° = \tan(240° + 45°)$

$= \frac{\tan 240° + \tan 45°}{1 - \tan 240° \tan 45°}$

$= \frac{\sqrt{3} + 1}{1 - (\sqrt{3})(1)}$

$= -(2 + \sqrt{3})$

$\cot 285° = \frac{1}{\tan 285°} = -(2 - \sqrt{3})$

Ans: $-(2 - \sqrt{3})$

---

4. Evaluate $\cos 159° \cos 39° + \sin 159° \sin 39°$. [1]

$\cos 159° \cos 39° + \sin 159° \sin 39° = \cos(159° - 39°)$

$= \cos 120° = -\frac{1}{2}$

Ans: $-\frac{1}{2}$

---

5. Write $2 \sin 75° \sin 45°$ as a sum or difference and simplify. [1]

$2 \sin A \sin B = \cos(A - B) - \cos(A + B)$

$2 \sin 75° \sin 45° = \cos 30° - \cos 120°$

$= \frac{1}{2} + \frac{\sqrt{3}}{2}$

Ans: $\frac{1}{2} + \frac{\sqrt{3}}{2}$

---

6. Write $\sin 4\theta + \sin 2\theta$ as a product of two trigonometric ratios. [2]

$\sin C + \sin D = 2 \sin \frac{C+D}{2} \cos \frac{C-D}{2}$

$\sin 4\theta + \sin 2\theta = 2 \sin 3\theta \cos \theta$

Ans: $2 \sin 3\theta \cos \theta$

---

7. Show that $\cos 76^\circ + \cos 44^\circ = \cos 16^\circ$. [3]

$\cos C + \cos D = 2\cos\frac{C+D}{2}\cos\frac{C-D}{2}$

$\cos 76^\circ + \cos 44^\circ = 2\cos 60^\circ \cos 16^\circ = \cos 16^\circ$

Ans: Proved

---

8. If $\cot A = 6$ and $\cot B = \frac{7}{4}$, find $\cot(A+B)$. [2]

$\cot(A+B) = \frac{\cot A \cot B - 1}{\cot B + \cot A}$

$= \frac{6\cdot\frac{7}{4}-1}{\frac{7}{4}+6} = \frac{38}{31}$

Ans: $\frac{38}{31}$

---

9. Find the value of $\frac{\cot 108^\circ \cot 12^\circ - 1}{\cot 12^\circ + \cot 108^\circ}$. [1]

$\frac{\cot 108^\circ \cot 12^\circ - 1}{\cot 12^\circ + \cot 108^\circ} = \cot(108^\circ + 12^\circ) = \cot 120^\circ$

$= -\frac{1}{\sqrt{3}}$

Ans: $-\frac{1}{\sqrt{3}}$

---

10. Prove that $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B$ and hence find $\cos^2 97\frac{1}{2}^\circ - \sin^2 37\frac{1}{2}^\circ$. [5]

$\cos(A+B)\cos(A-B) = (\cos A\cos B - \sin A\sin B)(\cos A\cos B + \sin A\sin B)$

$= \cos^2 A\cos^2 B - \sin^2 A\sin^2 B$

$= \cos^2 A(1-\sin^2 B) - (1-\cos^2 A)\sin^2 B$

$= \cos^2 A - \sin^2 B$

Put $A = 97\frac{1}{2}^\circ$, $B = 37\frac{1}{2}^\circ$:

$\cos^2 97\frac{1}{2}^\circ - \sin^2 37\frac{1}{2}^\circ = \cos 135^\circ \cos 60^\circ = \left(-\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) = -\frac{\sqrt{2}}{4}$

Ans: $-\frac{\sqrt{2}}{4}$

---

11. Find the value of $\tan 72^\circ \tan 18^\circ$. [1]

$\tan(45^\circ + \theta) = \cot(45^\circ - \theta)$

$\tan 72^\circ \tan 18^\circ = \cot 18^\circ \tan 18^\circ = 1$

Ans: 1

---

12. If $\tan A = \frac{1}{4}$, $\tan B = \frac{1}{4}$ and $\tan C = 4$, find $\tan(A+B+C)$. [3]

$\tan(A+B+C) = \frac{S_1 - S_3}{1 - S_2}$

$S_1 = \frac{9}{2},\ S_2 = \frac{33}{16},\ S_3 = \frac{1}{4}$

$\tan(A+B+C) = \frac{\frac{9}{2} - \frac{1}{4}}{1 - \frac{33}{16}} = -4$

Ans: -4

---

13. Evaluate $\tan 15^\circ + \cot 15^\circ$ using the values at 15° and 75°. [2]

$\tan 15^\circ = 2 - \sqrt{3}$

$\cot 15^\circ = 2 + \sqrt{3}$

$\therefore \tan 15^\circ + \cot 15^\circ = 4$

Ans: 4

---

14. If $\tan A = 7$ and $\tan B = \frac{3}{4}$, where A and B are acute, show that $A - B = \frac{\pi}{4}$. [3]

$\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B} = \frac{7-\frac{3}{4}}{1+7\cdot\frac{3}{4}}$

$= \frac{\frac{25}{4}}{\frac{25}{4}} = 1$

$\therefore A - B = \frac{\pi}{4}$

Ans: $A - B = \frac{\pi}{4}$

---

15. Without tables, find $\tan 37^\circ + \tan 98^\circ - \tan 37^\circ \tan 98^\circ$. [1]

$37^\circ + 98^\circ = 135^\circ,\ \tan(37^\circ + 98^\circ) = -1$

$\Rightarrow \tan A + \tan B = -1 + \tan A \tan B$

$\therefore \tan 37^\circ + \tan 98^\circ - \tan 37^\circ \tan 98^\circ = -1$

Ans: -1

---

16. If $A + B = 315^\circ$, prove that $(1 - \tan A)(1 - \tan B) = 2$. Hence find $\tan 157\frac{1}{2}^\circ$. [5]

$\tan(A+B) = \tan 315^\circ = -1$

$\frac{\tan A + \tan B}{1 - \tan A \tan B} = -1 \Rightarrow \tan A + \tan B = \tan A \tan B - 1$

Rearranging, $(1 - \tan A)(1 - \tan B) = 2$.

$A = B = 157\frac{1}{2}^\circ :\ (1-t)^2 = 2,\ t = \tan 157\frac{1}{2}^\circ$

$\Rightarrow 1 - t = \pm\sqrt{2};\ \tan 157\frac{1}{2}^\circ < 0,\ \text{so } t = -(\sqrt{2}-1)$

Ans: **Proved;** $\tan 157\frac{1}{2}^\circ = -(\sqrt{2}-1)$

---

17. If $A + B + C = \pi$, $\tan A = \frac{9}{8}$ and $\tan B = \frac{1}{5}$, find $\tan C$. [2]

$A + B + C = \pi \Rightarrow \sum \tan A = \prod \tan A$

$\frac{9}{8} + \frac{1}{5} + \tan C = \frac{9}{40}\tan C$

$\tan C = -\frac{53}{31}$

Ans: $\tan C = -\frac{53}{31}$

---

18. If $A + B + C = \pi$, $\cot A = \frac{6}{7}$ and $\cot B = 4$, find $\cot C$ and $\tan C$. [3]

$A + B + C = \pi \Rightarrow \tan(A+B+C) = 0$

$\Rightarrow \sum \cot A \cot B = 1$

$\frac{24}{7} + (\frac{6}{7} + 4)\cot C = 1$

$\cot C = -\frac{1}{2},\ \tan C = -2$

Ans: $\cot C = -\frac{1}{2}$, $\tan C = -2$

---

19. Find the value of $\sin 110^\circ - \sin 130^\circ$. [1]

$\sin 110^\circ - \sin 130^\circ = 2\cos 120^\circ \sin(-10^\circ) = \sin 10^\circ$

Ans: $\sin 10^\circ$

---

20. Prove that $\tan 9A - \tan 7A - \tan 2A = \tan 9A \tan 7A \tan 2A$. [5]

$9A = 7A + 2A$

$\tan 9A = \frac{\tan 7A + \tan 2A}{1 - \tan 7A \tan 2A}$

$\tan 9A - \tan 9A \tan 7A \tan 2A = \tan 7A + \tan 2A$

$\therefore \tan 9A - \tan 7A - \tan 2A = \tan 9A \tan 7A \tan 2A$

Ans: **Proved**
