EXPRESSWALA ACADEMY

Maths 1A: Compound Angles

Question Paper, Set 05

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. If $\tan A = \frac{15}{8}$ and $\sin B = \frac{5}{13}$, where A and B are acute, find $\sin(A+B)$. [2 marks]

2. If $\cos A = -\frac{45}{53}$, $90^\circ < A < 180^\circ$ and $\cos B = \frac{60}{61}$, $270^\circ < B < 360^\circ$, find $\sin(A+B)$, $\cos(A-B)$ and $\tan(A+B)$. [5 marks]

3. Using compound angles, find $\sin 105^\circ$. [2 marks]

4. Find the value of $\sin 18^\circ \cos 12^\circ + \cos 18^\circ \sin 12^\circ$. [1 mark]

5. Express $2\sin 4\theta \cos\theta$ as a sum or difference. [1 mark]

6. Express $\cos 7\theta - \cos 3\theta$ as a product. [2 marks]

7. Show that $\sin 65^\circ - \sin 25^\circ = \sqrt{2}\,\sin 20^\circ$. [3 marks]

8. If $\tan A = 4$ and $\tan B = 1$, find $\tan(A+B)$. [2 marks]

9. Find the value of $\frac{\tan 111^\circ + \tan 39^\circ}{1 - \tan 111^\circ \tan 39^\circ}$. [1 mark]

10. Prove that $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$ and hence find $\sin^2 105^\circ - \sin^2 45^\circ$. [5 marks]

11. Find the value of $\frac{\cos 105^\circ + \sin 105^\circ}{\cos 105^\circ - \sin 105^\circ}$. [1 mark]

12. If $\tan A = 1$, $\tan B = 3$ and $\tan C = 6$, find $\tan(A+B+C)$. [3 marks]

13. Find the value of $\sec 15^\circ$. [2 marks]

14. If $\tan A = \frac{3}{11}$ and $\tan B = \frac{4}{7}$, where A and B are acute, show that $A + B = \frac{\pi}{4}$. [3 marks]

15. Find the value of $\tan 126^\circ + \tan 189^\circ - \tan 126^\circ \tan 189^\circ$. [1 mark]

16. If $A + B = 225^\circ$, prove that $(1+\tan A)(1+\tan B) = 2$. Hence find $\tan 112\frac{1}{2}^\circ$. [5 marks]

17. If $A + B + C = \frac{\pi}{2}$, $\tan A = \frac{9}{4}$ and $\tan B = 2$, find $\tan C$. [2 marks]

18. If $A + B + C = \frac{\pi}{2}$, $\cot A = 2$ and $\cot B = 5$, find $\cot C$ and $\tan C$. [3 marks]

19. Find the value of $\sin 45^\circ + \sin 285^\circ - \sin 195^\circ$. [1 mark]

20. Show that $\tan 72^\circ = \tan 18^\circ + 2\tan 54^\circ$. [5 marks]

Key sheet

Q	Answer
1	$\frac{220}{221}$
2	$\sin(A+B)=\frac{2175}{3233}$, $\cos(A-B)=-\frac{3008}{3233}$, $\tan(A+B)=-\frac{2175}{2392}$
3	$\frac{\sqrt{3}+1}{2\sqrt{2}}$
4	$\frac{1}{2}$
5	$\sin 3\theta+\sin 5\theta$
6	$-2\sin 5\theta\sin 2\theta$
7	Proved
8	$-\frac{5}{3}$
9	$-\frac{1}{\sqrt{3}}$
10	$\frac{\sqrt{3}}{4}$
11	$-\frac{1}{\sqrt{3}}$
12	$\frac{4}{13}$
13	$\sqrt{6}-\sqrt{2}$
14	$A+B=\frac{\pi}{4}$
15	-1
16	Proved; $\tan 112\frac{1}{2}^\circ=-(\sqrt{2}+1)$
17	$\tan C=-\frac{14}{17}$
18	$\cot C=\frac{7}{9}$, $\tan C=\frac{9}{7}$
19	0
20	Proved

Solutions

1. If $\tan A = \frac{15}{8}$ and $\sin B = \frac{5}{13}$, where A and B are acute, find $\sin(A+B)$. [2]

$\tan A = \frac{15}{8} \Rightarrow \sin A = \frac{15}{17},\ \cos A = \frac{8}{17}$

$\cos B = \sqrt{1-\sin^2 B} = \frac{12}{13}$

$\sin(A+B) = \sin A\cos B + \cos A \sin B$

$= (\frac{15}{17})(\frac{12}{13}) + (\frac{8}{17})(\frac{5}{13}) = \frac{220}{221}$

Ans: $\frac{220}{221}$

---

2. If $\cos A = -\frac{45}{53}$, $90° < A < 180°$ and $\cos B = \frac{60}{61}$, $270° < B < 360°$, find $\sin(A+B)$, $\cos(A-B)$ and $\tan(A+B)$. [5]

A is in quadrant 2 and B in quadrant 4; use the signs there.

$\sin A = \sqrt{1-\cos^2 A} = \frac{28}{53}$

$\sin B = -\sqrt{1-\cos^2 B} = -\frac{11}{61}$

$\tan A = -\frac{28}{45},\ \tan B = -\frac{11}{60}$

$\sin(A+B) = (\frac{28}{53})(\frac{60}{61}) + (-\frac{45}{53})(-\frac{11}{61}) = \frac{2175}{3233}$

$\cos(A-B) = (-\frac{45}{53})(\frac{60}{61}) + (\frac{28}{53})(-\frac{11}{61}) = -\frac{3008}{3233}$

$\tan(A+B) = \frac{-\frac{28}{45}+(-\frac{11}{60})}{1-(-\frac{28}{45})(-\frac{11}{60})} = -\frac{2175}{2392}$

Ans: $\sin(A+B) = \frac{2175}{3233}$, $\cos(A-B) = -\frac{3008}{3233}$, $\tan(A+B) = -\frac{2175}{2392}$

---

3. Using compound angles, find $\sin 105°$. [2]

$\sin 105° = \sin(135° - 30°)$

$= \sin 135°\cos 30° - \cos 135°\sin 30°$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(-\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

Ans: $\frac{\sqrt{3}+1}{2\sqrt{2}}$

---

4. Find the value of $\sin 18°\cos 12° + \cos 18°\sin 12°$. [1]

$\sin 18°\cos 12° + \cos 18°\sin 12° = \sin(18° + 12°)$

$= \sin 30° = \frac{1}{2}$

Ans: $\frac{1}{2}$

---

5. Express $2\sin 4\theta\cos\theta$ as a sum or difference. [1]

$2\sin A\cos B = \sin(A+B) + \sin(A-B)$

$2\sin 4\theta\cos\theta = \sin 3\theta + \sin 5\theta$

Ans: $\sin 3\theta + \sin 5\theta$

---

6. Express $\cos 7\theta - \cos 3\theta$ as a product. [2]

$\cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}$

$\cos 7\theta - \cos 3\theta = -2\sin 5\theta\sin 2\theta$

Ans: $-2\sin 5\theta\sin 2\theta$

---

7. Show that $\sin 65^\circ - \sin 25^\circ = \sqrt{2}\,\sin 20^\circ$. [3]

$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$

$\sin 65^\circ - \sin 25^\circ = 2\cos 45^\circ \sin 20^\circ = 2\cdot\frac{1}{\sqrt{2}}\sin 20^\circ = \sqrt{2}\,\sin 20^\circ$

Ans: Proved

---

8. If $\tan A = 4$ and $\tan B = 1$, find $\tan(A+B)$. [2]

$\tan(A+B) = \frac{\tan A+\tan B}{1-\tan A\tan B}$

$= \frac{4+1}{1-4\cdot 1} = -\frac{5}{3}$

Ans: $-\frac{5}{3}$

---

9. Find the value of $\frac{\tan 111^\circ+\tan 39^\circ}{1-\tan 111^\circ\tan 39^\circ}$. [1]

$\frac{\tan 111^\circ+\tan 39^\circ}{1-\tan 111^\circ\tan 39^\circ} = \tan(111^\circ+39^\circ) = \tan 150^\circ$

$= -\frac{1}{\sqrt{3}}$

Ans: $-\frac{1}{\sqrt{3}}$

---

10. Prove that $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$ and hence find $\sin^2 105^\circ - \sin^2 45^\circ$. [5]

$\sin(A+B)\sin(A-B) = (\sin A\cos B + \cos A\sin B)(\sin A\cos B - \cos A\sin B)$

$= \sin^2 A\cos^2 B - \cos^2 A\sin^2 B$

$= \sin^2 A(1-\sin^2 B) - (1-\sin^2 A)\sin^2 B$

$= \sin^2 A - \sin^2 B$

Put $A = 105^\circ$, $B = 45^\circ$:

$\sin^2 105^\circ - \sin^2 45^\circ = \sin 150^\circ \sin 60^\circ = \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{4}$

Ans: $\frac{\sqrt{3}}{4}$

---

11. Find the value of $\frac{\cos 105^\circ+\sin 105^\circ}{\cos 105^\circ-\sin 105^\circ}$. [1]

$\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta} = \frac{1+\tan\theta}{1-\tan\theta} = \tan(45^\circ+\theta)$

$\frac{\cos 105^\circ+\sin 105^\circ}{\cos 105^\circ-\sin 105^\circ} = \tan 150^\circ = -\frac{1}{\sqrt{3}}$

Ans: $-\frac{1}{\sqrt{3}}$

---

12. If $\tan A = 1$, $\tan B = 3$ and $\tan C = 6$, find $\tan(A+B+C)$. [3]

$\tan(A+B+C) = \frac{S_1-S_3}{1-S_2}$

$S_1 = 10,\ S_2 = 27,\ S_3 = 18$

$\tan(A+B+C) = \frac{10-18}{1-27} = \frac{4}{13}$

Ans: $\frac{4}{13}$

---

13. Find the value of $\sec 15^\circ$. [2]

$\cos 15^\circ = \cos(45^\circ - 30^\circ)$

$= \cos 45^\circ\cos 30^\circ + \sin 45^\circ\sin 30^\circ$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

$\sec 15^\circ = \frac{1}{\cos 15^\circ} = \sqrt{6}-\sqrt{2}$

Ans: $\sqrt{6}-\sqrt{2}$

---

14. If $\tan A = \frac{3}{11}$ and $\tan B = \frac{4}{7}$, where A and B are acute, show that $A + B = \frac{\pi}{4}$. [3]

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{3}{11} + \frac{4}{7}}{1 - \frac{3}{11} \cdot \frac{4}{7}}$

$= \frac{\frac{65}{77}}{\frac{65}{77}} = 1$

$\therefore A + B = \frac{\pi}{4}$

Ans: $A + B = \frac{\pi}{4}$

---

15. Find the value of $\tan 126^\circ + \tan 189^\circ - \tan 126^\circ \tan 189^\circ$. [1]

$126^\circ + 189^\circ = 315^\circ,\ \tan(126^\circ + 189^\circ) = -1$

$\Rightarrow \tan A + \tan B = -1 + \tan A \tan B$

$\therefore \tan 126^\circ + \tan 189^\circ - \tan 126^\circ \tan 189^\circ = -1$

Ans: -1

---

16. If $A + B = 225^\circ$, prove that $(1 + \tan A)(1 + \tan B) = 2$. Hence find $\tan 112\frac{1}{2}^\circ$. [5]

$\tan(A+B) = \tan 225^\circ = 1$

$\frac{\tan A + \tan B}{1 - \tan A \tan B} = 1 \Rightarrow \tan A + \tan B = 1 - \tan A \tan B$

Rearranging, $(1 + \tan A)(1 + \tan B) = 2$.

$A = B = 112\frac{1}{2}^\circ :\ (1+t)^2 = 2,\ t = \tan 112\frac{1}{2}^\circ$

$\Rightarrow 1 + t = \pm\sqrt{2};\ \tan 112\frac{1}{2}^\circ < 0,\ \text{so } t = -(\sqrt{2}+1)$

Ans: Proved; $\tan 112\frac{1}{2}^\circ = -(\sqrt{2}+1)$

---

17. If $A + B + C = \frac{\pi}{2}$, $\tan A = \frac{9}{4}$ and $\tan B = 2$, find $\tan C$. [2]

$A + B + C = \frac{\pi}{2} \Rightarrow \sum \tan A \tan B = 1$

$\frac{9}{2} + (\frac{9}{4} + 2)\tan C = 1$

$\tan C = -\frac{14}{17}$

Ans: $\tan C = -\frac{14}{17}$

---

18. If $A + B + C = \frac{\pi}{2}$, $\cot A = 2$ and $\cot B = 5$, find $\cot C$ and $\tan C$. [3]

$A + B + C = \frac{\pi}{2} \Rightarrow \cot(A + B + C) = 0$

$\Rightarrow \sum \cot A = \prod \cot A$

$2 + 5 + \cot C = 10 \cot C$

$\cot C = \frac{7}{9},\ \tan C = \frac{9}{7}$

Ans: $\cot C = \frac{7}{9}$, $\tan C = \frac{9}{7}$

---

19. Find the value of $\sin 45^\circ + \sin 285^\circ - \sin 195^\circ$. [1]

$\sin 285^\circ - \sin 195^\circ = 2\cos 240^\circ \sin 45^\circ = -\sin 45^\circ$

$\therefore \text{value} = 0$

Ans: 0

---

20. Show that $\tan 72^\circ = \tan 18^\circ + 2\tan 54^\circ$. [5]

$18^\circ + 54^\circ = 72^\circ$, so with $A + C = B$:

$\tan 72^\circ - \tan 18^\circ - \tan 54^\circ = \tan 18^\circ \tan 72^\circ \tan 54^\circ$

$\tan 18^\circ \tan 72^\circ = \tan 18^\circ \cot 18^\circ = 1$

$\Rightarrow \tan 72^\circ - \tan 18^\circ - \tan 54^\circ = \tan 54^\circ$

$\therefore \tan 72^\circ = \tan 18^\circ + 2\tan 54^\circ$

Ans: Proved