EXPRESSWALA ACADEMY

Maths 1A: Compound Angles

Question Paper, Set 04

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. If $\cos A = \frac{3}{5}$ and $\tan B = \frac{60}{11}$, where A and B are acute, find $\cos(A + B)$. [2 marks]

2. If $\cos A = -\frac{28}{53}$, $90^\circ < A < 180^\circ$ and $\sin B = \frac{35}{37}$, $0^\circ < B < 90^\circ$, find $\sin(A + B)$, $\cos(A - B)$ and $\tan(A + B)$. [5 marks]

3. Find the value of $\operatorname{cosec} 285^\circ$. [2 marks]

4. Evaluate $\sin 133^\circ \cos 13^\circ - \cos 133^\circ \sin 13^\circ$. [1 mark]

5. Write $2 \sin 67\frac{1}{2}^\circ \sin 22\frac{1}{2}^\circ$ as a sum or difference and simplify. [1 mark]

6. Write $\cos 12\theta + \cos 6\theta$ as a product of two trigonometric ratios. [2 marks]

7. Show that $\cos 9^\circ - \cos 51^\circ = \sin 21^\circ$. [3 marks]

8. If $\tan A = \frac{7}{3}$ and $\tan B = \frac{7}{5}$, find $\tan(A - B)$. [2 marks]

9. Find the value of $\frac{\tan 118^\circ + \tan 17^\circ}{1 - \tan 118^\circ \tan 17^\circ}$. [1 mark]

10. Prove that $\cos(A + B)\cos(A - B) = \cos^2 A - \sin^2 B$ and hence find $\cos^2 105^\circ - \sin^2 45^\circ$. [5 marks]

11. Find the value of $\frac{\cos 165^\circ - \sin 165^\circ}{\cos 165^\circ + \sin 165^\circ}$. [1 mark]

12. If $\tan A = \frac{1}{4}$, $\tan B = 3$ and $\tan C = 4$, find $\tan(A + B + C)$. [3 marks]

13. Evaluate $\sin 15^\circ \cos 15^\circ$ using the values at 15° and 75°. [2 marks]

14. If $\tan A = \frac{7}{6}$ and $\tan B = \frac{1}{13}$, where A and B are acute, show that $A - B = \frac{\pi}{4}$. [3 marks]

15. Without tables, find $(1 - \cot 9^\circ)(1 - \cot 36^\circ)$. [1 mark]

16. If $A + B = 135^\circ$, prove that $(1 + \cot A)(1 + \cot B) = 2$. Hence find $\cot 67\frac{1}{2}^\circ$. [5 marks]

17. If $A + B + C = \pi$, $\tan A = \frac{9}{5}$ and $\tan B = \frac{8}{5}$, find $\tan C$. [2 marks]

18. If $A + B + C = \pi$, $\cot A = \frac{1}{4}$ and $\cot B = 2$, find $\cot C$ and $\tan C$. [3 marks]

19. Find the value of $\cos 10^\circ + \cos 250^\circ + \cos 230^\circ$. [1 mark]

20. Prove that $\tan 5A - \tan 4A - \tan A = \tan 5A \tan 4A \tan A$. [5 marks]

Key sheet

Q	Answer
1	$-\frac{207}{305}$
2	$\sin(A+B) = -\frac{440}{1961}$, $\cos(A-B) = \frac{1239}{1961}$, $\tan(A+B) = \frac{440}{1911}$
3	$-(\sqrt{6}-\sqrt{2})$
4	$\frac{\sqrt{3}}{2}$
5	$\frac{\sqrt{2}}{2}$
6	$2\cos 9\theta \cos 3\theta$
7	Proved
8	$\frac{7}{32}$
9	-1
10	$-\frac{\sqrt{3}}{4}$
11	$\sqrt{3}$
12	$-\frac{1}{3}$
13	$\frac{1}{4}$
14	$A - B = \frac{\pi}{4}$
15	2
16	Proved; $\cot 67\frac{1}{2}^\circ = \sqrt{2}-1$
17	$\tan C = \frac{85}{47}$
18	$\cot C = \frac{2}{9}$, $\tan C = \frac{9}{2}$
19	0
20	Proved

Solutions

1. If $\cos A = \frac{3}{5}$ and $\tan B = \frac{60}{11}$, where A and B are acute, find $\cos(A+B)$. [2]

$\sin A = \sqrt{1-\cos^2 A} = \frac{4}{5}$

$\tan B = \frac{60}{11} \Rightarrow \sin B = \frac{60}{61},\ \cos B = \frac{11}{61}$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$= (\frac{3}{5})(\frac{11}{61}) - (\frac{4}{5})(\frac{60}{61}) = -\frac{207}{305}$

Ans: $-\frac{207}{305}$

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2. If $\cos A = -\frac{28}{53}$, $90^\circ < A < 180^\circ$ and $\sin B = \frac{35}{37}$, $0^\circ < B < 90^\circ$, find $\sin(A+B)$, $\cos(A-B)$ and $\tan(A+B)$. [5]

A is in quadrant 2 and B in quadrant 1; use the signs there.

$\sin A = \sqrt{1-\cos^2 A} = \frac{45}{53}$

$\cos B = \sqrt{1-\sin^2 B} = \frac{12}{37}$

$\tan A = -\frac{45}{28},\ \tan B = \frac{35}{12}$

$\sin(A+B) = (\frac{45}{53})(\frac{12}{37}) + (-\frac{28}{53})(\frac{35}{37}) = -\frac{440}{1961}$

$\cos(A-B) = (-\frac{28}{53})(\frac{12}{37}) + (\frac{45}{53})(\frac{35}{37}) = \frac{1239}{1961}$

$\tan(A+B) = \frac{-\frac{45}{28}+\frac{35}{12}}{1-(-\frac{45}{28})(\frac{35}{12})} = \frac{440}{1911}$

Ans: $\sin(A+B) = -\frac{440}{1961}$, $\cos(A-B) = \frac{1239}{1961}$, $\tan(A+B) = \frac{440}{1911}$

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3. Find the value of $\operatorname{cosec} 285^\circ$. [2]

$\sin 285^\circ = \sin(240^\circ + 45^\circ)$

$= \sin 240^\circ \cos 45^\circ + \cos 240^\circ \sin 45^\circ$

$= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(-\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$

$= -\frac{\sqrt{3}+1}{2\sqrt{2}}$

$\operatorname{cosec} 285^\circ = \frac{1}{\sin 285^\circ} = -(\sqrt{6}-\sqrt{2})$

Ans: $-(\sqrt{6}-\sqrt{2})$

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4. Evaluate $\sin 133^\circ \cos 13^\circ - \cos 133^\circ \sin 13^\circ$. [1]

$\sin 133^\circ \cos 13^\circ - \cos 133^\circ \sin 13^\circ = \sin(133^\circ - 13^\circ)$

$= \sin 120^\circ = \frac{\sqrt{3}}{2}$

Ans: $\frac{\sqrt{3}}{2}$

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5. Write $2 \sin 67\frac{1}{2}^\circ \sin 22\frac{1}{2}^\circ$ as a sum or difference and simplify. [1]

$2 \sin A \sin B = \cos(A-B) - \cos(A+B)$

$2 \sin 67\frac{1}{2}^\circ \sin 22\frac{1}{2}^\circ = \cos 45^\circ - \cos 90^\circ$

$= \frac{\sqrt{2}}{2}$

Ans: $\frac{\sqrt{2}}{2}$

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6. Write $\cos 12\theta + \cos 6\theta$ as a product of two trigonometric ratios. [2]

$\cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2}$

$\cos 12\theta + \cos 6\theta = 2 \cos 9\theta \cos 3\theta$

Ans: $2 \cos 9\theta \cos 3\theta$

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7. Show that $\cos 9^\circ - \cos 51^\circ = \sin 21^\circ$. [3]

$\cos C - \cos D = 2\sin\frac{C+D}{2}\sin\frac{D-C}{2}$

$\cos 9^\circ - \cos 51^\circ = 2\sin 30^\circ \sin 21^\circ = \sin 21^\circ$

Ans: Proved

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8. If $\tan A = \frac{7}{3}$ and $\tan B = \frac{7}{5}$, find $\tan(A-B)$. [2]

$\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B}$

$= \frac{\frac{7}{3}-\frac{7}{5}}{1+\frac{7}{3}\cdot\frac{7}{5}} = \frac{7}{32}$

Ans: $\frac{7}{32}$

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9. Find the value of $\frac{\tan 118^\circ + \tan 17^\circ}{1-\tan 118^\circ \tan 17^\circ}$. [1]

$\frac{\tan 118^\circ + \tan 17^\circ}{1-\tan 118^\circ \tan 17^\circ} = \tan(118^\circ + 17^\circ) = \tan 135^\circ$

$= -1$

Ans: -1

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10. Prove that $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B$ and hence find $\cos^2 105^\circ - \sin^2 45^\circ$. [5]

$\cos(A+B)\cos(A-B) = (\cos A\cos B - \sin A\sin B)(\cos A\cos B + \sin A\sin B)$

$= \cos^2 A\cos^2 B - \sin^2 A\sin^2 B$

$= \cos^2 A(1-\sin^2 B) - (1-\cos^2 A)\sin^2 B$

$= \cos^2 A - \sin^2 B$

Put $A = 105^\circ$, $B = 45^\circ$:

$\cos^2 105^\circ - \sin^2 45^\circ = \cos 150^\circ \cos 60^\circ = \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) = -\frac{\sqrt{3}}{4}$

Ans: $-\frac{\sqrt{3}}{4}$

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11. Find the value of $\frac{\cos 165^\circ - \sin 165^\circ}{\cos 165^\circ + \sin 165^\circ}$. [1]

$\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1-\tan\theta}{1+\tan\theta} = \tan(45^\circ - \theta)$

$\frac{\cos 165^\circ - \sin 165^\circ}{\cos 165^\circ + \sin 165^\circ} = \tan(-120^\circ) = \sqrt{3}$

Ans: $\sqrt{3}$

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12. If $\tan A = \frac{1}{4}$, $\tan B = 3$ and $\tan C = 4$, find $\tan(A+B+C)$. [3]

$\tan(A+B+C) = \frac{S_1 - S_3}{1-S_2}$

$S_1 = \frac{29}{4},\ S_2 = \frac{55}{4},\ S_3 = 3$

$\tan(A+B+C) = \frac{\frac{29}{4}-3}{1-\frac{55}{4}} = -\frac{1}{3}$

Ans: $-\frac{1}{3}$

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13. Evaluate $\sin 15^\circ \cos 15^\circ$ using the values at 15° and 75°. [2]

$\sin 15^\circ = \frac{\sqrt{3}-1}{2\sqrt{2}}$

$\cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$

$\therefore \sin 15^\circ \cos 15^\circ = \frac{1}{4}$

Ans: $\frac{1}{4}$

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14. If $\tan A = \frac{7}{6}$ and $\tan B = \frac{1}{13}$, where A and B are acute, show that $A - B = \frac{\pi}{4}$. [3]

$\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B} = \frac{\frac{7}{6}-\frac{1}{13}}{1+\frac{7}{6}\cdot\frac{1}{13}}$

$= \frac{\frac{85}{78}}{\frac{85}{78}} = 1$

$\therefore A - B = \frac{\pi}{4}$

Ans: $A - B = \frac{\pi}{4}$

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15. Without tables, find $(1-\cot 9^\circ)(1-\cot 36^\circ)$. [1]

$9^\circ + 36^\circ = 45^\circ,\ \cot(9^\circ+36^\circ) = 1$

$\Rightarrow \cot A \cot B - 1 = (\cot A + \cot B)$

$\therefore (1-\cot 9^\circ)(1-\cot 36^\circ) = 2$

Ans: 2

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16. If $A + B = 135^\circ$, prove that $(1+\cot A)(1+\cot B) = 2$. Hence find $\cot 67\frac{1}{2}^\circ$. [5]

$\cot(A+B) = \cot 135^\circ = -1$

$\frac{\cot A \cot B - 1}{\cot A + \cot B} = -1 \Rightarrow \cot A \cot B - 1 = -(\cot A + \cot B)$

Rearranging, $(1+\cot A)(1+\cot B) = 2$.

$A = B = 67\frac{1}{2}^\circ :\ (1+t)^2 = 2,\ t = \cot 67\frac{1}{2}^\circ$

$\Rightarrow 1 + t = \pm\sqrt{2};\ \cot 67\frac{1}{2}^\circ > 0,\ \text{so } t = \sqrt{2} - 1$

Ans: Proved; $\cot 67\frac{1}{2}^\circ = \sqrt{2} - 1$

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17. If $A + B + C = \pi$, $\tan A = \frac{9}{5}$ and $\tan B = \frac{8}{5}$, find $\tan C$. [2]

$A + B + C = \pi \Rightarrow \sum \tan A = \prod \tan A$

$\frac{9}{5} + \frac{8}{5} + \tan C = \frac{72}{25}\tan C$

$\tan C = \frac{85}{47}$

Ans: $\tan C = \frac{85}{47}$

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18. If $A + B + C = \pi$, $\cot A = \frac{1}{4}$ and $\cot B = 2$, find $\cot C$ and $\tan C$. [3]

$A + B + C = \pi \Rightarrow \tan(A+B+C) = 0$

$\Rightarrow \sum \cot A \cot B = 1$

$\frac{1}{2} + (\frac{1}{4} + 2)\cot C = 1$

$\cot C = \frac{2}{9},\ \tan C = \frac{9}{2}$

Ans: $\cot C = \frac{2}{9}$, $\tan C = \frac{9}{2}$

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19. Find the value of $\cos 10^\circ + \cos 250^\circ + \cos 230^\circ$. [1]

$\cos 250^\circ + \cos 230^\circ = 2\cos 240^\circ \cos 10^\circ = -\cos 10^\circ$

$\therefore \text{value} = 0$

Ans: 0

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20. Prove that $\tan 5A - \tan 4A - \tan A = \tan 5A \tan 4A \tan A$. [5]

$5A = 4A + A$

$\tan 5A = \frac{\tan 4A + \tan A}{1 - \tan 4A \tan A}$

$\tan 5A - \tan 5A \tan 4A \tan A = \tan 4A + \tan A$

$\therefore \tan 5A - \tan 4A - \tan A = \tan 5A \tan 4A \tan A$

Ans: Proved
