EXPRESSWALA ACADEMY

Maths 1A: Compound Angles

Question Paper, Set 03

Time: 1 hour 30 minutes Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. If $\sin A = \frac{11}{61}$ and $\tan B = \frac{45}{28}$, where A and B are acute, find $\sin(A - B)$. [2 marks]

2. If $\sin A = -\frac{8}{17}$, $180^\circ < A < 270^\circ$ and $\sin B = -\frac{4}{5}$, $270^\circ < B < 360^\circ$, find $\sin(A + B)$, $\cos(A - B)$ and $\tan(A + B)$. [5 marks]

3. Find the exact value of $\sec 165^\circ$. [2 marks]

4. Find the value of $\sin 20^\circ \cos 40^\circ + \cos 20^\circ \sin 40^\circ$. [1 mark]

5. Express $2\cos 9\theta \cos 5\theta$ as a sum or difference. [1 mark]

6. Express $\cos 9\theta + \cos 7\theta$ as a product. [2 marks]

7. Show that $\sin 58^\circ + \sin 2^\circ = \cos 28^\circ$. [3 marks]

8. If $\cot A = \frac{1}{2}$ and $\cot B = \frac{9}{2}$, find $\cot(A + B)$. [2 marks]

9. Find the value of $\frac{\cot 13^\circ \cot 17^\circ - 1}{\cot 17^\circ + \cot 13^\circ}$. [1 mark]

10. Prove that $\sin(A + B)\sin(A - B) = \sin^2 A - \sin^2 B$ and hence find $\sin^2 82\frac{1}{2}^\circ - \sin^2 52\frac{1}{2}^\circ$. [5 marks]

11. Find the value of $\tan 78^\circ \tan 12^\circ$. [1 mark]

12. If $\tan A = 4$, $\tan B = 2$ and $\tan C = 4$, find $\tan(A + B + C)$. [3 marks]

13. Find the value of $\cos 15^\circ$. [2 marks]

14. If $\tan A = \frac{1}{8}$ and $\tan B = \frac{7}{9}$, where A and B are acute, show that $A + B = \frac{\pi}{4}$. [3 marks]

15. Find the value of $(1 + \cot 11^\circ)(1 + \cot 124^\circ)$. [1 mark]

16. If $A + B = 135^\circ$, prove that $(1 - \tan A)(1 - \tan B) = 2$. Hence find $\tan 67\frac{1}{2}^\circ$. [5 marks]

17. If $A + B + C = \frac{\pi}{2}$, $\tan A = 6$ and $\tan B = 1$, find $\tan C$. [2 marks]

18. If $A + B + C = \pi$, $\cot A = \frac{1}{8}$ and $\cot B = \frac{1}{3}$, find $\cot C$ and $\tan C$. [3 marks]

19. Find the value of $\cos 25^\circ + \cos 265^\circ + \cos 215^\circ$. [1 mark]

20. Show that $\tan 49^\circ = \tan 41^\circ + 2\tan 8^\circ$. [5 marks]

Key sheet

Q	Answer
1	$-\frac{2392}{3233}$
2	$\sin(A+B)=\frac{36}{85}$, $\cos(A-B)=-\frac{13}{85}$, $\tan(A+B)=-\frac{36}{77}$
3	$-(\sqrt{6}-\sqrt{2})$
4	$\frac{\sqrt{3}}{2}$
5	$\cos 4\theta+\cos 14\theta$
6	$2\cos 8\theta\cos\theta$
7	Proved
8	$\frac{1}{4}$
9	$\sqrt{3}$
10	$\frac{\sqrt{2}}{4}$
11	1
12	$\frac{22}{31}$
13	$\frac{\sqrt{3}+1}{2\sqrt{2}}$
14	$A+B=\frac{\pi}{4}$
15	2
16	Proved; $\tan 67\frac{1}{2}^\circ=\sqrt{2}+1$
17	$\tan C=-\frac{5}{7}$
18	$\cot C=\frac{23}{11}$, $\tan C=\frac{11}{23}$
19	0
20	Proved

Solutions

1. If $\sin A = \frac{11}{61}$ and $\tan B = \frac{45}{28}$, where A and B are acute, find $\sin(A - B)$. [2]

$\cos A = \sqrt{1 - \sin^2 A} = \frac{60}{61}$

$\tan B = \frac{45}{28} \Rightarrow \sin B = \frac{45}{53},\ \cos B = \frac{28}{53}$

$\sin(A - B) = \sin A \cos B - \cos A \sin B$

$= (\frac{11}{61})(\frac{28}{53}) - (\frac{60}{61})(\frac{45}{53}) = -\frac{2392}{3233}$

Ans: $-\frac{2392}{3233}$

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2. If $\sin A = -\frac{8}{17}$, $180° < A < 270°$ and $\sin B = -\frac{4}{5}$, $270° < B < 360°$, find $\sin(A + B)$, $\cos(A - B)$ and $\tan(A + B)$. [5]

A is in quadrant 3 and B in quadrant 4; use the signs there.

$\cos A = -\sqrt{1 - \sin^2 A} = -\frac{15}{17}$

$\cos B = \sqrt{1 - \sin^2 B} = \frac{3}{5}$

$\tan A = \frac{8}{15},\ \tan B = -\frac{4}{3}$

$\sin(A + B) = (-\frac{8}{17})(\frac{3}{5}) + (-\frac{15}{17})(-\frac{4}{5}) = \frac{36}{85}$

$\cos(A - B) = (-\frac{15}{17})(\frac{3}{5}) + (-\frac{8}{17})(-\frac{4}{5}) = -\frac{13}{85}$

$\tan(A + B) = \frac{\frac{8}{15} + (-\frac{4}{3})}{1 - (\frac{8}{15})(-\frac{4}{3})} = -\frac{36}{77}$

Ans: $\sin(A + B) = \frac{36}{85}$, $\cos(A - B) = -\frac{13}{85}$, $\tan(A + B) = -\frac{36}{77}$

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3. Find the exact value of $\sec 165°$. [2]

$\cos 165° = \cos(120° + 45°)$

$= \cos 120° \cos 45° - \sin 120° \sin 45°$

$= \left(-\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$

$= -\frac{\sqrt{3}+1}{2\sqrt{2}}$

$\sec 165° = \frac{1}{\cos 165°} = -(\sqrt{6} - \sqrt{2})$

Ans: $-(\sqrt{6} - \sqrt{2})$

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4. Find the value of $\sin 20° \cos 40° + \cos 20° \sin 40°$. [1]

$\sin 20° \cos 40° + \cos 20° \sin 40° = \sin(20° + 40°)$

$= \sin 60° = \frac{\sqrt{3}}{2}$

Ans: $\frac{\sqrt{3}}{2}$

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5. Express $2 \cos 9\theta \cos 5\theta$ as a sum or difference. [1]

$2 \cos A \cos B = \cos(A + B) + \cos(A - B)$

$2 \cos 9\theta \cos 5\theta = \cos 4\theta + \cos 14\theta$

Ans: $\cos 4\theta + \cos 14\theta$

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6. Express $\cos 9\theta + \cos 7\theta$ as a product. [2]

$\cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2}$

$\cos 9\theta + \cos 7\theta = 2 \cos 8\theta \cos \theta$

Ans: $2 \cos 8\theta \cos \theta$

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7. Show that $\sin 58^\circ + \sin 2^\circ = \cos 28^\circ$. [3]

$\sin 58^\circ + \sin 2^\circ = 2\sin\frac{58^\circ+2^\circ}{2}\cos\frac{58^\circ-2^\circ}{2}$

$= 2\sin 30^\circ \cos 28^\circ = 2 \cdot \frac{1}{2}\cos 28^\circ = \cos 28^\circ$

Ans: Proved

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8. If $\cot A = \frac{1}{2}$ and $\cot B = \frac{9}{2}$, find $\cot(A+B)$. [2]

$\cot(A+B) = \frac{\cot A \cot B - 1}{\cot B + \cot A}$

$= \frac{\frac{1}{2}\cdot\frac{9}{2}-1}{\frac{9}{2}+\frac{1}{2}} = \frac{1}{4}$

Ans: $\frac{1}{4}$

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9. Find the value of $\frac{\cot 13^\circ \cot 17^\circ - 1}{\cot 17^\circ + \cot 13^\circ}$. [1]

$\frac{\cot 13^\circ \cot 17^\circ - 1}{\cot 17^\circ + \cot 13^\circ} = \cot(13^\circ + 17^\circ) = \cot 30^\circ$

$= \sqrt{3}$

Ans: $\sqrt{3}$

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10. Prove that $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$ and hence find $\sin^2 82\frac{1}{2}^\circ - \sin^2 52\frac{1}{2}^\circ$. [5]

$\sin(A+B)\sin(A-B) = (\sin A\cos B + \cos A\sin B)(\sin A\cos B - \cos A\sin B)$

$= \sin^2 A\cos^2 B - \cos^2 A\sin^2 B$

$= \sin^2 A(1-\sin^2 B) - (1-\sin^2 A)\sin^2 B$

$= \sin^2 A - \sin^2 B$

Put $A = 82\frac{1}{2}^\circ$, $B = 52\frac{1}{2}^\circ$:

$\sin^2 82\frac{1}{2}^\circ - \sin^2 52\frac{1}{2}^\circ = \sin 135^\circ \sin 30^\circ = \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) = \frac{\sqrt{2}}{4}$

Ans: $\frac{\sqrt{2}}{4}$

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11. Find the value of $\tan 78^\circ \tan 12^\circ$. [1]

$\tan(45^\circ + \theta) = \cot(45^\circ - \theta)$

$\tan 78^\circ \tan 12^\circ = \cot 12^\circ \tan 12^\circ = 1$

Ans: 1

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12. If $\tan A = 4$, $\tan B = 2$ and $\tan C = 4$, find $\tan(A+B+C)$. [3]

$\tan(A+B+C) = \frac{S_1 - S_3}{1 - S_2}$

$S_1 = 10,\ S_2 = 32,\ S_3 = 32$

$\tan(A+B+C) = \frac{10-32}{1-32} = \frac{22}{31}$

Ans: $\frac{22}{31}$

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13. Find the value of $\cos 15^\circ$. [2]

$\cos 15^\circ = \cos(45^\circ - 30^\circ)$

$= \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

Ans: $\frac{\sqrt{3}+1}{2\sqrt{2}}$

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14. If $\tan A = \frac{1}{8}$ and $\tan B = \frac{7}{9}$, where A and B are acute, show that $A + B = \frac{\pi}{4}$. [3]

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{1}{8}+\frac{7}{9}}{1-\frac{1}{8}\cdot\frac{7}{9}}$

$= \frac{\frac{65}{72}}{\frac{65}{72}} = 1$

$\therefore A + B = \frac{\pi}{4}$

Ans: $A + B = \frac{\pi}{4}$

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15. Find the value of $(1+\cot 11^\circ)(1+\cot 124^\circ)$. [1]

$11^\circ + 124^\circ = 135^\circ,\ \cot(11^\circ + 124^\circ) = -1$

$\Rightarrow \cot A \cot B - 1 = -(\cot A + \cot B)$

$\therefore (1+\cot 11^\circ)(1+\cot 124^\circ) = 2$

Ans: 2

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16. If $A + B = 135^\circ$, prove that $(1-\tan A)(1-\tan B) = 2$. Hence find $\tan 67\frac{1}{2}^\circ$. [5]

$\tan(A+B) = \tan 135^\circ = -1$

$\frac{\tan A + \tan B}{1 - \tan A \tan B} = -1 \Rightarrow \tan A + \tan B = \tan A \tan B - 1$

Rearranging, $(1-\tan A)(1-\tan B) = 2$.

$A = B = 67\frac{1}{2}^\circ :\ (1-t)^2 = 2,\ t = \tan 67\frac{1}{2}^\circ$

$\Rightarrow 1 - t = \pm\sqrt{2};\ \tan 67\frac{1}{2}^\circ > 0,\ \text{so } t = \sqrt{2}+1$

Ans: Proved; $\tan 67\frac{1}{2}^\circ = \sqrt{2}+1$

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17. If $A + B + C = \frac{\pi}{2}$, $\tan A = 6$ and $\tan B = 1$, find $\tan C$. [2]

$A + B + C = \frac{\pi}{2} \Rightarrow \sum \tan A \tan B = 1$

$6 + (6+1)\tan C = 1$

$\tan C = -\frac{5}{7}$

Ans: $\tan C = -\frac{5}{7}$

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18. If $A + B + C = \pi$, $\cot A = \frac{1}{8}$ and $\cot B = \frac{1}{3}$, find $\cot C$ and $\tan C$. [3]

$A + B + C = \pi \Rightarrow \tan(A+B+C) = 0$

$\Rightarrow \sum \cot A \cot B = 1$

$\frac{1}{24} + (\frac{1}{8}+\frac{1}{3})\cot C = 1$

$\cot C = \frac{23}{11},\ \tan C = \frac{11}{23}$

Ans: $\cot C = \frac{23}{11}$, $\tan C = \frac{11}{23}$

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19. Find the value of $\cos 25^\circ + \cos 265^\circ + \cos 215^\circ$. [1]

$\cos 265^\circ + \cos 215^\circ = 2\cos 240^\circ \cos 25^\circ = -\cos 25^\circ$

$\therefore \text{value} = 0$

Ans: 0

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20. Show that $\tan 49^\circ = \tan 41^\circ + 2\tan 8^\circ$. [5]

$41^\circ + 8^\circ = 49^\circ$, so with $A + C = B$:

$\tan 49^\circ - \tan 41^\circ - \tan 8^\circ = \tan 41^\circ \tan 49^\circ \tan 8^\circ$

$\tan 41^\circ \tan 49^\circ = \tan 41^\circ \cot 41^\circ = 1$

$\Rightarrow \tan 49^\circ - \tan 41^\circ - \tan 8^\circ = \tan 8^\circ$

$\therefore \tan 49^\circ = \tan 41^\circ + 2\tan 8^\circ$

Ans: Proved