EXPRESSWALA ACADEMY

Maths 1A: Compound Angles

Question Paper, Set 02

Time: 1 hour 30 minutes Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. If $\cos A = \frac{8}{17}$ and $\sin B = \frac{56}{65}$, where A and B are acute, find $\cos(A - B)$. [2 marks]

2. If $\sin A = -\frac{60}{61}$, $180^\circ < A < 270^\circ$ and $\sin B = -\frac{28}{53}$, $270^\circ < B < 360^\circ$, find $\sin(A + B)$, $\cos(A - B)$ and $\tan(A + B)$. [5 marks]

3. Using compound angles, find $\cos 345^\circ$. [2 marks]

4. Evaluate $\sin 96^\circ \cos 6^\circ - \cos 96^\circ \sin 6^\circ$. [1 mark]

5. Write $2\cos 135^\circ \sin 15^\circ$ as a sum or difference and simplify. [1 mark]

6. Write $\sin 6\theta - \sin 2\theta$ as a product of two trigonometric ratios. [2 marks]

7. Show that $\sin 57^\circ - \sin 63^\circ + \sin 3^\circ = 0$. [3 marks]

8. If $\tan A = \frac{1}{8}$ and $\tan B = \frac{2}{7}$, find $\tan(A - B)$. [2 marks]

9. Find the value of $\frac{\cot 28^\circ \cot 17^\circ - 1}{\cot 17^\circ + \cot 28^\circ}$. [1 mark]

10. Prove that $\cos(A + B)\cos(A - B) = \cos^2 A - \sin^2 B$ and hence find $\cos^2 127\frac{1}{2}^\circ - \sin^2 7\frac{1}{2}^\circ$. [5 marks]

11. Find the value of $\frac{\cos 60^\circ - \sin 60^\circ}{\cos 60^\circ + \sin 60^\circ}$. [1 mark]

12. If $\tan A = 5$, $\tan B = 3$ and $\tan C = 2$, find $\tan(A + B + C)$. [3 marks]

13. Evaluate $\sin 75^\circ$ using the values at 15° and 75°. [2 marks]

14. If $\tan A = \frac{7}{3}$ and $\tan B = \frac{2}{5}$, where A and B are acute, show that $A - B = \frac{\pi}{4}$. [3 marks]

15. Without tables, find $(1 + \tan 3^\circ)(1 + \tan 42^\circ)$. [1 mark]

16. If $A + B = 45^\circ$, prove that $(1 - \cot A)(1 - \cot B) = 2$. Hence find $\cot 22\frac{1}{2}^\circ$. [5 marks]

17. If $A + B + C = \pi$, $\tan A = \frac{1}{6}$ and $\tan B = \frac{5}{9}$, find $\tan C$. [2 marks]

18. If $A + B + C = \pi$, $\cot A = \frac{2}{5}$ and $\cot B = \frac{9}{4}$, find $\cot C$ and $\tan C$. [3 marks]

19. Find the value of $\cos 140^\circ + \cos 100^\circ$. [1 mark]

20. Prove that $\tan 7A - \tan 4A - \tan 3A = \tan 7A \tan 4A \tan 3A$. [5 marks]

Key sheet

Q	Answer
1	$\frac{1104}{1105}$
2	$\sin(A+B) = -\frac{2392}{3233}$, $\cos(A-B) = \frac{1185}{3233}$, $\tan(A+B) = \frac{2392}{2175}$
3	$\frac{\sqrt{3}+1}{2\sqrt{2}}$
4	1
5	$\frac{1}{2} - \frac{\sqrt{3}}{2}$
6	$2\cos 4\theta \sin 2\theta$
7	Proved
8	$-\frac{9}{58}$
9	1
10	$\frac{\sqrt{2}}{4}$
11	$-(2-\sqrt{3})$
12	$\frac{2}{3}$
13	$\frac{\sqrt{3}+1}{2\sqrt{2}}$
14	$A - B = \frac{\pi}{4}$
15	2
16	Proved; $\cot 22\frac{1}{2}^\circ = \sqrt{2}+1$
17	$\tan C = -\frac{39}{49}$
18	$\cot C = \frac{2}{53}$, $\tan C = \frac{53}{2}$
19	$-\cos 20^\circ$
20	Proved

Solutions

1. If $\cos A = \frac{8}{17}$ and $\sin B = \frac{56}{65}$, where A and B are acute, find $\cos(A - B)$. [2]

$\sin A = \sqrt{1 - \cos^2 A} = \frac{15}{17}$

$\cos B = \sqrt{1 - \sin^2 B} = \frac{33}{65}$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

$= (\frac{8}{17})(\frac{33}{65}) + (\frac{15}{17})(\frac{56}{65}) = \frac{1104}{1105}$

Ans: $\frac{1104}{1105}$

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2. If $\sin A = -\frac{60}{61}$, $180° < A < 270°$ and $\sin B = -\frac{28}{53}$, $270° < B < 360°$, find $\sin(A + B)$, $\cos(A - B)$ and $\tan(A + B)$. [5]

A is in quadrant 3 and B in quadrant 4; use the signs there.

$\cos A = -\sqrt{1 - \sin^2 A} = -\frac{11}{61}$

$\cos B = \sqrt{1 - \sin^2 B} = \frac{45}{53}$

$\tan A = \frac{60}{11}$, $\tan B = -\frac{28}{45}$

$\sin(A + B) = (-\frac{60}{61})(\frac{45}{53}) + (-\frac{11}{61})(-\frac{28}{53}) = -\frac{2392}{3233}$

$\cos(A - B) = (-\frac{11}{61})(\frac{45}{53}) + (-\frac{60}{61})(-\frac{28}{53}) = \frac{1185}{3233}$

$\tan(A + B) = \frac{\frac{60}{11} + (-\frac{28}{45})}{1 - (\frac{60}{11})(-\frac{28}{45})} = \frac{2392}{2175}$

Ans: $\sin(A + B) = -\frac{2392}{3233}$, $\cos(A - B) = \frac{1185}{3233}$, $\tan(A + B) = \frac{2392}{2175}$

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3. Using compound angles, find $\cos 345°$. [2]

$\cos 345° = \cos(315° + 30°)$

$= \cos 315° \cos 30° - \sin 315° \sin 30°$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(-\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

Ans: $\frac{\sqrt{3}+1}{2\sqrt{2}}$

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4. Evaluate $\sin 96° \cos 6° - \cos 96° \sin 6°$. [1]

$\sin 96° \cos 6° - \cos 96° \sin 6° = \sin(96° - 6°)$

$= \sin 90° = 1$

Ans: 1

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5. Write $2 \cos 135° \sin 15°$ as a sum or difference and simplify. [1]

$2 \cos A \sin B = \sin(A + B) - \sin(A - B)$

$2 \cos 135° \sin 15° = \sin 150° - \sin 120°$

$= \frac{1}{2} - \frac{\sqrt{3}}{2}$

Ans: $\frac{1}{2} - \frac{\sqrt{3}}{2}$

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6. Write $\sin 6\theta - \sin 2\theta$ as a product of two trigonometric ratios. [2]

$\sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2}$

$\sin 6\theta - \sin 2\theta = 2 \cos 4\theta \sin 2\theta$

Ans: $2 \cos 4\theta \sin 2\theta$

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7. Show that $\sin 57^\circ - \sin 63^\circ + \sin 3^\circ = 0$. [3]

$\sin 57^\circ - \sin 63^\circ = 2\cos 60^\circ \sin(-3^\circ) = -\sin 3^\circ$

$\therefore \text{LHS} = -\sin 3^\circ + \sin 3^\circ = 0$

Ans: Proved

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8. If $\tan A = \frac{1}{8}$ and $\tan B = \frac{2}{7}$, find $\tan(A - B)$. [2]

$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$= \frac{\frac{1}{8} - \frac{2}{7}}{1 + \frac{1}{8}\cdot\frac{2}{7}} = -\frac{9}{58}$

Ans: $-\frac{9}{58}$

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9. Find the value of $\frac{\cot 28^\circ \cot 17^\circ - 1}{\cot 17^\circ + \cot 28^\circ}$. [1]

$\frac{\cot 28^\circ \cot 17^\circ - 1}{\cot 17^\circ + \cot 28^\circ} = \cot(28^\circ + 17^\circ) = \cot 45^\circ$

$= 1$

Ans: 1

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10. Prove that $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B$ and hence find $\cos^2 127\frac{1}{2}^\circ - \sin^2 7\frac{1}{2}^\circ$. [5]

$\cos(A+B)\cos(A-B) = (\cos A\cos B - \sin A\sin B)(\cos A\cos B + \sin A\sin B)$

$= \cos^2 A\cos^2 B - \sin^2 A\sin^2 B$

$= \cos^2 A(1 - \sin^2 B) - (1 - \cos^2 A)\sin^2 B$

$= \cos^2 A - \sin^2 B$

Put $A = 127\frac{1}{2}^\circ$, $B = 7\frac{1}{2}^\circ$:

$\cos^2 127\frac{1}{2}^\circ - \sin^2 7\frac{1}{2}^\circ = \cos 135^\circ \cos 120^\circ = \left(-\frac{1}{\sqrt{2}}\right)\left(-\frac{1}{2}\right) = \frac{\sqrt{2}}{4}$

Ans: $\frac{\sqrt{2}}{4}$

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11. Find the value of $\frac{\cos 60^\circ - \sin 60^\circ}{\cos 60^\circ + \sin 60^\circ}$. [1]

$\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1 - \tan\theta}{1 + \tan\theta} = \tan(45^\circ - \theta)$

$\frac{\cos 60^\circ - \sin 60^\circ}{\cos 60^\circ + \sin 60^\circ} = \tan(-15^\circ) = -(2 - \sqrt{3})$

Ans: $-(2 - \sqrt{3})$

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12. If $\tan A = 5$, $\tan B = 3$ and $\tan C = 2$, find $\tan(A + B + C)$. [3]

$\tan(A+B+C) = \frac{S_1 - S_3}{1 - S_2}$

$S_1 = 10,\ S_2 = 31,\ S_3 = 30$

$\tan(A+B+C) = \frac{10 - 30}{1 - 31} = \frac{2}{3}$

Ans: $\frac{2}{3}$

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13. Evaluate $\sin 75^\circ$ using the values at 15° and 75°. [2]

$\sin 75^\circ = \sin(45^\circ + 30^\circ)$

$= \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

Ans: $\frac{\sqrt{3}+1}{2\sqrt{2}}$

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14. If $\tan A = \frac{7}{3}$ and $\tan B = \frac{2}{5}$, where A and B are acute, show that $A - B = \frac{\pi}{4}$. [3]

$\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B} = \frac{\frac{7}{3}-\frac{2}{5}}{1+\frac{7}{3}\cdot\frac{2}{5}}$

$= \frac{\frac{29}{15}}{\frac{29}{15}} = 1$

$\therefore A - B = \frac{\pi}{4}$

Ans: $A - B = \frac{\pi}{4}$

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15. Without tables, find $(1+\tan 3^\circ)(1+\tan 42^\circ)$. [1]

$3^\circ + 42^\circ = 45^\circ,\ \tan(3^\circ+42^\circ) = 1$

$\Rightarrow \tan A + \tan B = 1 - \tan A \tan B$

$\therefore (1+\tan 3^\circ)(1+\tan 42^\circ) = 2$

Ans: 2

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16. If $A + B = 45^\circ$, prove that $(1-\cot A)(1-\cot B) = 2$. Hence find $\cot 22\frac{1}{2}^\circ$. [5]

$\cot(A+B) = \cot 45^\circ = 1$

$\frac{\cot A \cot B - 1}{\cot A + \cot B} = 1 \Rightarrow \cot A \cot B - 1 = (\cot A + \cot B)$

Rearranging, $(1-\cot A)(1-\cot B) = 2$.

$A = B = 22\frac{1}{2}^\circ :\ (1-t)^2 = 2,\ t = \cot 22\frac{1}{2}^\circ$

$\Rightarrow 1 - t = \pm\sqrt{2};\ \cot 22\frac{1}{2}^\circ > 0,\ \text{so } t = \sqrt{2}+1$

Ans: Proved; $\cot 22\frac{1}{2}^\circ = \sqrt{2}+1$

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17. If $A + B + C = \pi$, $\tan A = \frac{1}{6}$ and $\tan B = \frac{5}{9}$, find $\tan C$. [2]

$A + B + C = \pi \Rightarrow \sum \tan A = \prod \tan A$

$\frac{1}{6} + \frac{5}{9} + \tan C = \frac{5}{54}\tan C$

$\tan C = -\frac{39}{49}$

Ans: $\tan C = -\frac{39}{49}$

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18. If $A + B + C = \pi$, $\cot A = \frac{2}{5}$ and $\cot B = \frac{9}{4}$, find $\cot C$ and $\tan C$. [3]

$A + B + C = \pi \Rightarrow \tan(A+B+C) = 0$

$\Rightarrow \sum \cot A \cot B = 1$

$\frac{9}{10} + (\frac{2}{5} + \frac{9}{4})\cot C = 1$

$\cot C = \frac{2}{53},\ \tan C = \frac{53}{2}$

Ans: $\cot C = \frac{2}{53}$, $\tan C = \frac{53}{2}$

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19. Find the value of $\cos 140^\circ + \cos 100^\circ$. [1]

$\cos 140^\circ + \cos 100^\circ = 2\cos 120^\circ \cos 20^\circ = -\cos 20^\circ$

Ans: $-\cos 20^\circ$

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20. Prove that $\tan 7A - \tan 4A - \tan 3A = \tan 7A \tan 4A \tan 3A$. [5]

$7A = 4A + 3A$

$\tan 7A = \frac{\tan 4A + \tan 3A}{1 - \tan 4A \tan 3A}$

$\tan 7A - \tan 7A \tan 4A \tan 3A = \tan 4A + \tan 3A$

$\therefore \tan 7A - \tan 4A - \tan 3A = \tan 7A \tan 4A \tan 3A$

Ans: Proved
