EXPRESSWALA ACADEMY

Maths 1A: Compound Angles

Question Paper, Set 10

Time: 1 hour 30 minutes Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. If $\tan A = \frac{12}{5}$ and $\sin B = \frac{3}{5}$, where A and B are acute, find $\cos(A - B)$. [2 marks]

2. If $\cos A = -\frac{12}{37}$, $90^\circ < A < 180^\circ$ and $\sin B = \frac{4}{5}$, $0^\circ < B < 90^\circ$, find $\sin(A + B)$, $\cos(A - B)$ and $\tan(A + B)$. [5 marks]

3. Find the value of $\tan 105^\circ$. [2 marks]

4. Evaluate $\sin 168^\circ \cos 18^\circ - \cos 168^\circ \sin 18^\circ$. [1 mark]

5. Write $2 \sin 165^\circ \sin 15^\circ$ as a sum or difference and simplify. [1 mark]

6. Write $\sin 3\theta - \sin \theta$ as a product of two trigonometric ratios. [2 marks]

7. Show that $\sin 38^\circ + \sin 22^\circ = \cos 8^\circ$. [3 marks]

8. If $\tan A = 4$ and $\tan B = \frac{7}{4}$, find $\tan(A + B)$. [2 marks]

9. Find the value of $\frac{\tan 93^\circ - \tan 33^\circ}{1 + \tan 93^\circ \tan 33^\circ}$. [1 mark]

10. Prove that $\cos(A + B)\cos(A - B) = \cos^2 A - \sin^2 B$ and hence find $\cos^2 67\frac{1}{2}^\circ - \sin^2 22\frac{1}{2}^\circ$. [5 marks]

11. Find the value of $\tan 84^\circ \tan 6^\circ$. [1 mark]

12. If $\tan A = 2$, $\tan B = 6$ and $\tan C = \frac{1}{5}$, find $\tan(A + B + C)$. [3 marks]

13. Evaluate $\tan 75^\circ - \tan 15^\circ$ using the values at 15° and 75°. [2 marks]

14. If $\tan A = \frac{12}{7}$ and $\tan B = \frac{5}{19}$, where A and B are acute, show that $A - B = \frac{\pi}{4}$. [3 marks]

15. Without tables, find $\tan 7^\circ + \tan 128^\circ - \tan 7^\circ \tan 128^\circ$. [1 mark]

16. If $A + B = 135^\circ$, prove that $(1 + \tan A)(1 + \tan B) = 2 \tan A \tan B$. Hence find $\tan 67\frac{1}{2}^\circ$. [5 marks]

17. If $A + B + C = \frac{\pi}{2}$, $\tan A = 1$ and $\tan B = \frac{1}{4}$, find $\tan C$. [2 marks]

18. If $A + B + C = \frac{\pi}{2}$, $\cot A = \frac{2}{3}$ and $\cot B = 3$, find $\cot C$ and $\tan C$. [3 marks]

19. Find the value of $\sin 85^\circ - \sin 155^\circ$. [1 mark]

20. Prove that $\tan 7A - \tan 6A - \tan A = \tan 7A \tan 6A \tan A$. [5 marks]

Key sheet

Q	Answer
1	$\frac{56}{65}$
2	$\sin(A+B)=\frac{57}{185}$, $\cos(A-B)=\frac{104}{185}$, $\tan(A+B)=-\frac{57}{176}$
3	$-(2+\sqrt{3})$
4	$\frac{1}{2}$
5	$1-\frac{\sqrt{3}}{2}$
6	$2\cos 2\theta \sin\theta$
7	Proved
8	$-\frac{23}{24}$
9	$\sqrt{3}$
10	0
11	1
12	$-\frac{29}{63}$
13	$2\sqrt{3}$
14	$A-B=\frac{\pi}{4}$
15	-1
16	Proved; $\tan 67\frac{1}{2}^\circ=\sqrt{2}+1$
17	$\tan C=\frac{3}{5}$
18	$\cot C=\frac{11}{3}$, $\tan C=\frac{3}{11}$
19	$\sin 35^\circ$
20	Proved

Solutions

1. If $\tan A = \frac{12}{5}$ and $\sin B = \frac{3}{5}$, where A and B are acute, find $\cos(A - B)$. [2]

$\tan A = \frac{12}{5} \Rightarrow \sin A = \frac{12}{13},\ \cos A = \frac{5}{13}$

$\cos B = \sqrt{1 - \sin^2 B} = \frac{4}{5}$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

$= (\frac{5}{13})(\frac{4}{5}) + (\frac{12}{13})(\frac{3}{5}) = \frac{56}{65}$

Ans: $\frac{56}{65}$

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2. If $\cos A = -\frac{12}{37}$, $90° < A < 180°$ and $\sin B = \frac{4}{5}$, $0° < B < 90°$, find $\sin(A + B)$, $\cos(A - B)$ and $\tan(A + B)$. [5]

A is in quadrant 2 and B in quadrant 1; use the signs there.

$\sin A = \sqrt{1 - \cos^2 A} = \frac{35}{37}$

$\cos B = \sqrt{1 - \sin^2 B} = \frac{3}{5}$

$\tan A = -\frac{35}{12},\ \tan B = \frac{4}{3}$

$\sin(A + B) = (\frac{35}{37})(\frac{3}{5}) + (-\frac{12}{37})(\frac{4}{5}) = \frac{57}{185}$

$\cos(A - B) = (-\frac{12}{37})(\frac{3}{5}) + (\frac{35}{37})(\frac{4}{5}) = \frac{104}{185}$

$\tan(A + B) = \frac{-\frac{35}{12} + \frac{4}{3}}{1 - (-\frac{35}{12})(\frac{4}{3})} = -\frac{57}{176}$

Ans: $\sin(A + B) = \frac{57}{185}$, $\cos(A - B) = \frac{104}{185}$, $\tan(A + B) = -\frac{57}{176}$

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3. Find the value of $\tan 105°$. [2]

$\tan 105° = \tan(135° - 30°)$

$= \frac{\tan 135° - \tan 30°}{1 + \tan 135° \tan 30°}$

$= \frac{-1 - \frac{1}{\sqrt{3}}}{1 + (-1)\left(\frac{1}{\sqrt{3}}\right)}$

$= -(2 + \sqrt{3})$

Ans: $-(2 + \sqrt{3})$

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4. Evaluate $\sin 168° \cos 18° - \cos 168° \sin 18°$. [1]

$\sin 168° \cos 18° - \cos 168° \sin 18° = \sin(168° - 18°)$

$= \sin 150° = \frac{1}{2}$

Ans: $\frac{1}{2}$

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5. Write $2 \sin 165° \sin 15°$ as a sum or difference and simplify. [1]

$2 \sin A \sin B = \cos(A - B) - \cos(A + B)$

$2 \sin 165° \sin 15° = \cos 150° - \cos 180°$

$= 1 - \frac{\sqrt{3}}{2}$

Ans: $1 - \frac{\sqrt{3}}{2}$

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6. Write $\sin 3\theta - \sin \theta$ as a product of two trigonometric ratios. [2]

$\sin C - \sin D = 2 \cos \frac{C+D}{2} \sin \frac{C-D}{2}$

$\sin 3\theta - \sin \theta = 2 \cos 2\theta \sin \theta$

Ans: $2 \cos 2\theta \sin \theta$

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7. Show that $\sin 38^\circ + \sin 22^\circ = \cos 8^\circ$. [3]

$\sin 38^\circ + \sin 22^\circ = 2\sin\frac{38^\circ+22^\circ}{2}\cos\frac{38^\circ-22^\circ}{2}$

$= 2\sin 30^\circ \cos 8^\circ = 2 \cdot \frac{1}{2}\cos 8^\circ = \cos 8^\circ$

Ans: Proved

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8. If $\tan A = 4$ and $\tan B = \frac{7}{4}$, find $\tan(A+B)$. [2]

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$= \frac{4+\frac{7}{4}}{1-4\cdot\frac{7}{4}} = -\frac{23}{24}$

Ans: $-\frac{23}{24}$

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9. Find the value of $\frac{\tan 93^\circ - \tan 33^\circ}{1+\tan 93^\circ \tan 33^\circ}$. [1]

$\frac{\tan 93^\circ - \tan 33^\circ}{1+\tan 93^\circ \tan 33^\circ} = \tan(93^\circ - 33^\circ) = \tan 60^\circ$

$= \sqrt{3}$

Ans: $\sqrt{3}$

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10. Prove that $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B$ and hence find $\cos^2 67\frac{1}{2}^\circ - \sin^2 22\frac{1}{2}^\circ$. [5]

$\cos(A+B)\cos(A-B) = (\cos A\cos B - \sin A\sin B)(\cos A\cos B + \sin A\sin B)$

$= \cos^2 A\cos^2 B - \sin^2 A\sin^2 B$

$= \cos^2 A(1-\sin^2 B) - (1-\cos^2 A)\sin^2 B$

$= \cos^2 A - \sin^2 B$

Put $A = 67\frac{1}{2}^\circ$, $B = 22\frac{1}{2}^\circ$:

$\cos^2 67\frac{1}{2}^\circ - \sin^2 22\frac{1}{2}^\circ = \cos 90^\circ \cos 45^\circ = (0)\left(\frac{1}{\sqrt{2}}\right) = 0$

Ans: 0

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11. Find the value of $\tan 84^\circ \tan 6^\circ$. [1]

$\tan(45^\circ + \theta) = \cot(45^\circ - \theta)$

$\tan 84^\circ \tan 6^\circ = \cot 6^\circ \tan 6^\circ = 1$

Ans: 1

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12. If $\tan A = 2$, $\tan B = 6$ and $\tan C = \frac{1}{5}$, find $\tan(A+B+C)$. [3]

$\tan(A+B+C) = \frac{S_1 - S_3}{1 - S_2}$

$S_1 = \frac{41}{5},\ S_2 = \frac{68}{5},\ S_3 = \frac{12}{5}$

$\tan(A+B+C) = \frac{\frac{41}{5}-\frac{12}{5}}{1-\frac{68}{5}} = -\frac{29}{63}$

Ans: $-\frac{29}{63}$

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13. Evaluate $\tan 75^\circ - \tan 15^\circ$ using the values at 15° and 75°. [2]

$\tan 75^\circ = 2 + \sqrt{3}$

$\tan 15^\circ = 2 - \sqrt{3}$

$\therefore \tan 75^\circ - \tan 15^\circ = 2\sqrt{3}$

Ans: $2\sqrt{3}$

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14. If $\tan A = \frac{12}{7}$ and $\tan B = \frac{5}{19}$, where A and B are acute, show that $A - B = \frac{\pi}{4}$. [3]

$\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B} = \frac{\frac{12}{7} - \frac{5}{19}}{1+\frac{12}{7}\cdot\frac{5}{19}}$

$= \frac{\frac{193}{133}}{\frac{193}{133}} = 1$

$\therefore A - B = \frac{\pi}{4}$

Ans: $A - B = \frac{\pi}{4}$

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15. Without tables, find $\tan 7^\circ + \tan 128^\circ - \tan 7^\circ \tan 128^\circ$. [1]

$7^\circ + 128^\circ = 135^\circ,\ \tan(7^\circ + 128^\circ) = -1$

$\Rightarrow \tan A + \tan B = -1 + \tan A \tan B$

$\therefore \tan 7^\circ + \tan 128^\circ - \tan 7^\circ \tan 128^\circ = -1$

Ans: -1

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16. If $A + B = 135^\circ$, prove that $(1+\tan A)(1+\tan B) = 2\tan A \tan B$. Hence find $\tan 67\frac{1}{2}^\circ$. [5]

$\tan(A+B) = \tan 135^\circ = -1$

$\frac{\tan A + \tan B}{1 - \tan A \tan B} = -1 \Rightarrow \tan A + \tan B = \tan A \tan B - 1$

Rearranging, $(1+\tan A)(1+\tan B) = 2\tan A \tan B$.

$A = B = 67\frac{1}{2}^\circ :\ (1+t)^2 = 2t^2,\ t = \tan 67\frac{1}{2}^\circ$

$\Rightarrow 1 + t = \sqrt{2}\,t \Rightarrow t = \frac{1}{\sqrt{2}-1} = \sqrt{2}+1$

Ans: Proved; $\tan 67\frac{1}{2}^\circ = \sqrt{2}+1$

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17. If $A + B + C = \frac{\pi}{2}$, $\tan A = 1$ and $\tan B = \frac{1}{4}$, find $\tan C$. [2]

$A + B + C = \frac{\pi}{2} \Rightarrow \sum \tan A \tan B = 1$

$\frac{1}{4} + (1 + \frac{1}{4})\tan C = 1$

$\tan C = \frac{3}{5}$

Ans: $\tan C = \frac{3}{5}$

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18. If $A + B + C = \frac{\pi}{2}$, $\cot A = \frac{2}{3}$ and $\cot B = 3$, find $\cot C$ and $\tan C$. [3]

$A + B + C = \frac{\pi}{2} \Rightarrow \cot(A+B+C) = 0$

$\Rightarrow \sum \cot A = \prod \cot A$

$\frac{2}{3} + 3 + \cot C = 2\cot C$

$\cot C = \frac{11}{3},\ \tan C = \frac{3}{11}$

Ans: $\cot C = \frac{11}{3}$, $\tan C = \frac{3}{11}$

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19. Find the value of $\sin 85^\circ - \sin 155^\circ$. [1]

$\sin 85^\circ - \sin 155^\circ = 2\cos 120^\circ \sin(-35^\circ) = \sin 35^\circ$

Ans: $\sin 35^\circ$

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20. Prove that $\tan 7A - \tan 6A - \tan A = \tan 7A \tan 6A \tan A$. [5]

$7A = 6A + A$

$\tan 7A = \frac{\tan 6A + \tan A}{1 - \tan 6A \tan A}$

$\tan 7A - \tan 7A \tan 6A \tan A = \tan 6A + \tan A$

$\therefore \tan 7A - \tan 6A - \tan A = \tan 7A \tan 6A \tan A$

Ans: Proved
