EXPRESSWALA ACADEMY

Maths 1A: Compound Angles

Question Paper, Set 01

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. If $\cos A = \frac{11}{61}$ and $\sin B = \frac{45}{53}$, where A and B are acute, find $\sin(A+B)$. [2 marks]

2. If $\cos A = -\frac{20}{29}$, $180^\circ < A < 270^\circ$ and $\cos B = \frac{15}{17}$, $0^\circ < B < 90^\circ$, find $\sin(A+B)$, $\cos(A-B)$ and $\tan(A+B)$. [5 marks]

3. Find the value of $\cot 165^\circ$. [2 marks]

4. Find the value of $\cos 41^\circ \cos 11^\circ + \sin 41^\circ \sin 11^\circ$. [1 mark]

5. Express $2\sin 6\theta \sin 4\theta$ as a sum or difference. [1 mark]

6. Express $\sin 11\theta - \sin 9\theta$ as a product. [2 marks]

7. Show that $\cos 48^\circ + \cos 72^\circ + \cos 168^\circ = 0$. [3 marks]

8. If $\tan A = \frac{9}{4}$ and $\tan B = 4$, find $\tan(A-B)$. [2 marks]

9. Find the value of $\frac{\cot 131^\circ \cot 4^\circ - 1}{\cot 4^\circ + \cot 131^\circ}$. [1 mark]

10. Prove that $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$ and hence find $\sin^2 97\frac{1}{2}^\circ - \sin^2 52\frac{1}{2}^\circ$. [5 marks]

11. Find the value of $\tan 59^\circ \tan 31^\circ$. [1 mark]

12. If $\tan A = 2$, $\tan B = \frac{1}{4}$ and $\tan C = 6$, find $\tan(A+B+C)$. [3 marks]

13. Find the value of $\cos 75^\circ$. [2 marks]

14. If $\tan A = \frac{1}{11}$ and $\tan B = \frac{5}{6}$, where A and B are acute, show that $A + B = \frac{\pi}{4}$. [3 marks]

15. Find the value of $(1 - \cot 42^\circ)(1 - \cot 3^\circ)$. [1 mark]

16. If $A + B = 45^\circ$, prove that $(1+\tan A)(1+\tan B) = 2$. Hence find $\tan 22\frac{1}{2}^\circ$. [5 marks]

17. If $A + B + C = \pi$, $\tan A = \frac{8}{3}$ and $\tan B = 8$, find $\tan C$. [2 marks]

18. If $A + B + C = \pi$, $\cot A = \frac{8}{3}$ and $\cot B = 1$, find $\cot C$ and $\tan C$. [3 marks]

19. Find the value of $\cos 160^\circ + \cos 80^\circ$. [1 mark]

20. Show that $\tan 51^\circ = \tan 39^\circ + 2\tan 12^\circ$. [5 marks]

Key sheet

Q	Answer
1	$\frac{2175}{3233}$
2	$\sin(A+B) = -\frac{475}{493}$, $\cos(A-B) = -\frac{468}{493}$, $\tan(A+B) = \frac{475}{132}$
3	$-(2+\sqrt{3})$
4	$\frac{\sqrt{3}}{2}$
5	$\cos 2\theta - \cos 10\theta$
6	$2\cos 10\theta \sin\theta$
7	Proved
8	$-\frac{7}{40}$
9	-1
10	$\frac{\sqrt{2}}{4}$
11	1
12	$-\frac{21}{52}$
13	$\frac{\sqrt{3}-1}{2\sqrt{2}}$
14	$A+B = \frac{\pi}{4}$
15	2
16	Proved; $\tan 22\frac{1}{2}^\circ = \sqrt{2}-1$
17	$\tan C = \frac{32}{61}$
18	$\cot C = -\frac{5}{11}$, $\tan C = -\frac{11}{5}$
19	$-\cos 40^\circ$
20	Proved

Solutions

1. If $\cos A = \frac{11}{61}$ and $\sin B = \frac{45}{53}$, where A and B are acute, find $\sin(A+B)$. [2]

$\sin A = \sqrt{1-\cos^2 A} = \frac{60}{61}$

$\cos B = \sqrt{1-\sin^2 B} = \frac{28}{53}$

$\sin(A+B) = \sin A\cos B + \cos A \sin B$

$= (\frac{60}{61})(\frac{28}{53}) + (\frac{11}{61})(\frac{45}{53}) = \frac{2175}{3233}$

Ans: $\frac{2175}{3233}$

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2. If $\cos A = -\frac{20}{29}$, $180° < A < 270°$ and $\cos B = \frac{15}{17}$, $0° < B < 90°$, find $\sin(A+B)$, $\cos(A-B)$ and $\tan(A+B)$. [5]

A is in quadrant 3 and B in quadrant 1; use the signs there.

$\sin A = -\sqrt{1-\cos^2 A} = -\frac{21}{29}$

$\sin B = \sqrt{1-\cos^2 B} = \frac{8}{17}$

$\tan A = \frac{21}{20},\ \tan B = \frac{8}{15}$

$\sin(A+B) = (-\frac{21}{29})(\frac{15}{17}) + (-\frac{20}{29})(\frac{8}{17}) = -\frac{475}{493}$

$\cos(A-B) = (-\frac{20}{29})(\frac{15}{17}) + (-\frac{21}{29})(\frac{8}{17}) = -\frac{468}{493}$

$\tan(A+B) = \frac{\frac{21}{20}+\frac{8}{15}}{1-(\frac{21}{20})(\frac{8}{15})} = \frac{475}{132}$

Ans: $\sin(A+B) = -\frac{475}{493}$, $\cos(A-B) = -\frac{468}{493}$, $\tan(A+B) = \frac{475}{132}$

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3. Find the value of $\cot 165°$. [2]

$\tan 165° = \tan(135° + 30°)$

$= \frac{\tan 135° + \tan 30°}{1 - \tan 135° \tan 30°}$

$= \frac{-1 + \frac{1}{\sqrt{3}}}{1-(-1)\left(\frac{1}{\sqrt{3}}\right)}$

$= -(2-\sqrt{3})$

$\cot 165° = \frac{1}{\tan 165°} = -(2+\sqrt{3})$

Ans: $-(2+\sqrt{3})$

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4. Find the value of $\cos 41°\cos 11° + \sin 41°\sin 11°$. [1]

$\cos 41°\cos 11° + \sin 41°\sin 11° = \cos(41° - 11°)$

$= \cos 30° = \frac{\sqrt{3}}{2}$

Ans: $\frac{\sqrt{3}}{2}$

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5. Express $2\sin 6\theta \sin 4\theta$ as a sum or difference. [1]

$2\sin A \sin B = \cos(A-B) - \cos(A+B)$

$2\sin 6\theta \sin 4\theta = \cos 2\theta - \cos 10\theta$

Ans: $\cos 2\theta - \cos 10\theta$

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6. Express $\sin 11\theta - \sin 9\theta$ as a product. [2]

$\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}$

$\sin 11\theta - \sin 9\theta = 2\cos 10\theta \sin\theta$

Ans: $2\cos 10\theta \sin\theta$

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7. Show that $\cos 48^\circ + \cos 72^\circ + \cos 168^\circ = 0$. [3]

$\cos 72^\circ + \cos 48^\circ = 2\cos 60^\circ \cos 12^\circ = \cos 12^\circ$

$\cos 168^\circ = \cos(180^\circ - 12^\circ) = -\cos 12^\circ$

$\therefore \text{LHS} = \cos 12^\circ - \cos 12^\circ = 0$

Ans: Proved

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8. If $\tan A = \frac{9}{4}$ and $\tan B = 4$, find $\tan(A - B)$. [2]

$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$= \frac{\frac{9}{4} - 4}{1 + \frac{9}{4} \cdot 4} = -\frac{7}{40}$

Ans: $-\frac{7}{40}$

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9. Find the value of $\frac{\cot 131^\circ \cot 4^\circ - 1}{\cot 4^\circ + \cot 131^\circ}$. [1]

$\frac{\cot 131^\circ \cot 4^\circ - 1}{\cot 4^\circ + \cot 131^\circ} = \cot(131^\circ + 4^\circ) = \cot 135^\circ$

$= -1$

Ans: -1

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10. Prove that $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$ and hence find $\sin^2 97\frac{1}{2}^\circ - \sin^2 52\frac{1}{2}^\circ$. [5]

$\sin(A+B)\sin(A-B) = (\sin A \cos B + \cos A \sin B)(\sin A \cos B - \cos A \sin B)$

$= \sin^2 A \cos^2 B - \cos^2 A \sin^2 B$

$= \sin^2 A(1 - \sin^2 B) - (1 - \sin^2 A)\sin^2 B$

$= \sin^2 A - \sin^2 B$

Put $A = 97\frac{1}{2}^\circ$, $B = 52\frac{1}{2}^\circ$:

$\sin^2 97\frac{1}{2}^\circ - \sin^2 52\frac{1}{2}^\circ = \sin 150^\circ \sin 45^\circ = \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) = \frac{\sqrt{2}}{4}$

Ans: $\frac{\sqrt{2}}{4}$

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11. Find the value of $\tan 59^\circ \tan 31^\circ$. [1]

$\tan(45^\circ + \theta) = \cot(45^\circ - \theta)$

$\tan 59^\circ \tan 31^\circ = \cot 31^\circ \tan 31^\circ = 1$

Ans: 1

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12. If $\tan A = 2$, $\tan B = \frac{1}{4}$ and $\tan C = 6$, find $\tan(A+B+C)$. [3]

$\tan(A+B+C) = \frac{S_1 - S_3}{1 - S_2}$

$S_1 = \frac{33}{4},\ S_2 = 14,\ S_3 = 3$

$\tan(A+B+C) = \frac{\frac{33}{4} - 3}{1 - 14} = -\frac{21}{52}$

Ans: $-\frac{21}{52}$

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13. Find the value of $\cos 75^\circ$. [2]

$\cos 75^\circ = \cos(45^\circ + 30^\circ)$

$= \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$

Ans: $\frac{\sqrt{3}-1}{2\sqrt{2}}$

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14. If $\tan A = \frac{1}{11}$ and $\tan B = \frac{5}{6}$, where A and B are acute, show that $A + B = \frac{\pi}{4}$. [3]

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{\frac{1}{11} + \frac{5}{6}}{1 - \frac{1}{11} \cdot \frac{5}{6}}$

$= \frac{\frac{61}{66}}{\frac{61}{66}} = 1$

$\therefore A + B = \frac{\pi}{4}$

Ans: $A + B = \frac{\pi}{4}$

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15. Find the value of $(1 - \cot 42^\circ)(1 - \cot 3^\circ)$. [1]

$42^\circ + 3^\circ = 45^\circ,\ \cot(42^\circ + 3^\circ) = 1$

$\Rightarrow \cot A \cot B - 1 = (\cot A + \cot B)$

$\therefore (1 - \cot 42^\circ)(1 - \cot 3^\circ) = 2$

Ans: 2

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16. If $A + B = 45^\circ$, prove that $(1 + \tan A)(1 + \tan B) = 2$. Hence find $\tan 22\frac{1}{2}^\circ$. [5]

$\tan(A+B) = \tan 45^\circ = 1$

$\frac{\tan A + \tan B}{1 - \tan A \tan B} = 1 \Rightarrow \tan A + \tan B = 1 - \tan A \tan B$

Rearranging, $(1 + \tan A)(1 + \tan B) = 2$.

$A = B = 22\frac{1}{2}^\circ :\ (1+t)^2 = 2,\ t = \tan 22\frac{1}{2}^\circ$

$\Rightarrow 1 + t = \pm\sqrt{2};\ \tan 22\frac{1}{2}^\circ > 0,\ \text{so } t = \sqrt{2} - 1$

Ans: Proved; $\tan 22\frac{1}{2}^\circ = \sqrt{2} - 1$

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17. If $A + B + C = \pi$, $\tan A = \frac{8}{3}$ and $\tan B = 8$, find $\tan C$. [2]

$A + B + C = \pi \Rightarrow \sum \tan A = \prod \tan A$

$\frac{8}{3} + 8 + \tan C = \frac{64}{3} \tan C$

$\tan C = \frac{32}{61}$

Ans: $\tan C = \frac{32}{61}$

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18. If $A + B + C = \pi$, $\cot A = \frac{8}{3}$ and $\cot B = 1$, find $\cot C$ and $\tan C$. [3]

$A + B + C = \pi \Rightarrow \tan(A + B + C) = 0$

$\Rightarrow \sum \cot A \cot B = 1$

$\frac{8}{3} + (\frac{8}{3} + 1)\cot C = 1$

$\cot C = -\frac{5}{11},\ \tan C = -\frac{11}{5}$

Ans: $\cot C = -\frac{5}{11}$, $\tan C = -\frac{11}{5}$

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19. Find the value of $\cos 160^\circ + \cos 80^\circ$. [1]

$\cos 160^\circ + \cos 80^\circ = 2\cos 120^\circ \cos 40^\circ = -\cos 40^\circ$

Ans: $-\cos 40^\circ$

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20. Show that $\tan 51^\circ = \tan 39^\circ + 2\tan 12^\circ$. [5]

$39^\circ + 12^\circ = 51^\circ$, so with $A + C = B$:

$\tan 51^\circ - \tan 39^\circ - \tan 12^\circ = \tan 39^\circ \tan 51^\circ \tan 12^\circ$

$\tan 39^\circ \tan 51^\circ = \tan 39^\circ \cot 39^\circ = 1$

$\Rightarrow \tan 51^\circ - \tan 39^\circ - \tan 12^\circ = \tan 12^\circ$

$\therefore \tan 51^\circ = \tan 39^\circ + 2\tan 12^\circ$

Ans: Proved