EXPRESSWALA ACADEMY

Maths 1B: Change of Axes

Question Paper, Set 03

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. The origin is shifted to $(-5, 1)$. Find the new coordinates of the point $(-8, 4)$. [1 mark]

2. Find the transformed equation of $y^2 + 8x - 2y - 28 = 0$ when the origin is shifted to $(4, 1)$. [2 marks]

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3. When the origin is shifted to $(5, -4)$, the equation of a curve becomes $Y^2 - 12X = 0$. Find its original equation. [2 marks]

4. Find the point to which the origin must be shifted to remove the first degree terms from $-3x^2 + 6xy + y^2 + 6x + 10y - 3 = 0$. [3 marks]

5. Shift the origin to a suitable point so that the equation $-x^2 - 4xy - 3y^2 - 12x - 22y + 12 = 0$ has no first degree terms. Find the point and the transformed equation. [5 marks]

6. When the origin is shifted to $(-2, 2)$, the equation $-x^2 + 4xy + 6y^2 - 6x + 5y + 1 = 0$ becomes $AX^2 + 2HXY + BY^2 + 2GX + 2FY + C = 0$. Find $2G$, $2F$ and C. [2 marks]

7. Show that $-3x^2 - 4xy - 3y^2 + 16x + 4y - 28 = 0$ can be reduced to the form $aX^2 + 2hXY + bY^2 = 0$ by a translation of axes. Find the new origin and the reduced equation. [5 marks]

8. The axes are rotated through $90°$. Find the new coordinates of the point $(4, -6)$. [1 mark]

9. The axes are rotated through $90°$ in the clockwise direction. Find the new coordinates of the point $(-4, 5)$. [2 marks]

10. Find the transformed equation of $xy = 6$ when the axes are rotated through $135°$. [2 marks]

11. After the axes are rotated through an angle θ with $\cos\theta = \frac{3}{5}$, $\sin\theta = \frac{4}{5}$, the equation of a curve becomes $X^2 - Y^2 = 4$. Find the original equation. [3 marks]

12. Find the angle through which the axes must be rotated to remove the xy term from $x^2 + 12xy + y^2 = 12$. [1 mark]

13. Transform $3x^2 - 4xy + 6y^2 = 4$ by rotating the axes so that the xy term disappears. [5 marks]

14. Find the angle through which the axes must be rotated so that the line $\sqrt{3}x - y - 8 = 0$ takes the form $Y = \text{constant}$. [1 mark]

15. Find the angle of rotation that reduces the line $12x + 5y - 8 = 0$ to the form $Y = \text{constant}$, and find the new equation. [3 marks]

16. The axes are rotated through $30°$ and $6x^2 - 2xy + 5y^2 + x - 8y - 2 = 0$ becomes $AX^2 + 2HXY + BY^2 + 2GX + 2FY + C = 0$. Find C. [1 mark]

17. A rotation of the axes removes the xy term from $4x^2 + 6xy - 4y^2 = 6$ and gives $AX^2 + BY^2 = 6$. Use the invariants to find A and B. [3 marks]

18. The origin is shifted to $(3, -1)$ and the axes are then rotated through $90°$. Find the new coordinates of $(5, -5)$. [1 mark]

19. The origin is shifted to $(-2, 5)$ and the axes are then rotated through $45°$. A point has new coordinates $(-5, -2)$. Find its original coordinates. [2 marks]

20. Shift the origin to $(2, -3)$ and then rotate the axes through $\tan^{-1}\frac{1}{3}$. Find the transformed equation of $-4x^2 - 6xy + 4y^2 - 2x + 36y + 53 = 0$. [5 marks]

Key sheet

Q	Answer
1	$(-3, 3)$
2	$Y^2 + 8X + 3 = 0$
3	$y^2 - 12x + 8y + 76 = 0$
4	$(-1, -2)$
5	$(-4, -1)$; $-X^2 - 4XY - 3Y^2 + 47 = 0$
6	$2G = 6,\ 2F = 21,\ C = 27$
7	New origin $(4, -2)$; $-3X^2 - 4XY - 3Y^2 = 0$
8	$(-6, -4)$
9	$(-5, -4)$
10	$X^2 - Y^2 = -12$
11	$7x^2 - 48xy - 7y^2 = -100$
12	$\theta = \frac{\pi}{4}$
13	$\theta = \tan^{-1}\frac{1}{2}$; $2X^2 + 7Y^2 = 4$
14	60°
15	$\tan^{-1}\left(-\frac{12}{5}\right)$; $Y = \frac{8}{13}$
16	$C = -2$
17	5 and -5 (in some order)
18	$(-4, -2)$
19	$(-\frac{3\sqrt{2}}{2} - 2, 5 - \frac{7\sqrt{2}}{2})$
20	$-5X^2 + 5Y^2 = 3$

Solutions

1. The origin is shifted to $(-5, 1)$. Find the new coordinates of the point $(-8, 4)$. [1]

$X = x - h,\ Y = y - k$

$X = -8 - (-5) = -3,\ Y = 4 - (1) = 3$

Ans: $(-3, 3)$

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2. Find the transformed equation of $y^2 + 8x - 2y - 28 = 0$ when the origin is shifted to $(4, 1)$. [2]

Put $x = X + 4,\ y = Y + 1$ in the equation.

Expanding and collecting terms: $Y^2 + 8X + 3 = 0$

Ans: $Y^2 + 8X + 3 = 0$

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3. When the origin is shifted to $(5, -4)$, the equation of a curve becomes $Y^2 - 12X = 0$. Find its original equation. [2]

Replace X by $x - 5$ and Y by $y + 4$.

$y^2 - 12x + 8y + 76 = 0$

Ans: $y^2 - 12x + 8y + 76 = 0$

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4. Find the point to which the origin must be shifted to remove the first degree terms from $-3x^2 + 6xy + y^2 + 6x + 10y - 3 = 0$. [3]

$a = -3,\ h = 3,\ b = 1,\ g = 3,\ f = 5,\ c = -3$

Solve $ax + hy + g = 0$ and $hx + by + f = 0$: $-3x + 3y + 3 = 0$, $3x + y + 5 = 0$

So $(x_1, y_1) = (-1, -2)$.

Ans: $(-1, -2)$

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5. Shift the origin to a suitable point so that the equation $-x^2 - 4xy - 3y^2 - 12x - 22y + 12 = 0$ has no first degree terms. Find the point and the transformed equation. [5]

$a = -1,\ h = -2,\ b = -3,\ g = -6,\ f = -11,\ c = 12$

Solve $ax + hy + g = 0$ and $hx + by + f = 0$: $-x - 2y - 6 = 0$, $-2x - 3y - 11 = 0$

So $(x_1, y_1) = (-4, -1)$.

Constant $= gx_1 + fy_1 + c = (-6)(-4) + (-11)(-1) + (12) = 47$

Transformed equation: $-X^2 - 4XY - 3Y^2 + 47 = 0$

Ans: $(-4, -1)$; $-X^2 - 4XY - 3Y^2 + 47 = 0$

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6. When the origin is shifted to $(-2, 2)$, the equation $-x^2 + 4xy + 6y^2 - 6x + 5y + 1 = 0$ becomes $AX^2 + 2HXY + BY^2 + 2GX + 2FY + C = 0$. Find $2G$, $2F$ and C. [2]

$2G = \left(\frac{\partial S}{\partial x}\right)_{(-2,2)} = 2(-1)(-2) + (4)(2) + (-6) = 6$

$2F = \left(\frac{\partial S}{\partial y}\right)_{(-2,2)} = (4)(-2) + 2(6)(2) + (5) = 21$

$C = S(-2, 2) = 27$

Ans: $2G = 6,\ 2F = 21,\ C = 27$

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7. Show that $-3x^2 - 4xy - 3y^2 + 16x + 4y - 28 = 0$ can be reduced to the form $aX^2 + 2hXY + bY^2 = 0$ by a translation of axes. Find the new origin and the reduced equation. [5]

$a = -3,\ h = -2,\ b = -3,\ g = 8,\ f = 2,\ c = -28$

$abc + 2fgh - af^2 - bg^2 - ch^2 = -252 - 64 + 12 + 192 + 112 = 0$

So the reduction is possible.

Solve $-3x - 2y + 8 = 0$ and $-2x - 3y + 2 = 0$: new origin $(4, -2)$.

Constant $gx_1 + fy_1 + c = 0$, so the reduced equation is $-3X^2 - 4XY - 3Y^2 = 0$

Ans: New origin $(4, -2)$; $-3X^2 - 4XY - 3Y^2 = 0$

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8. The axes are rotated through $90°$. Find the new coordinates of the point $(4, -6)$. [1]

$X = x\cos\theta + y\sin\theta = -6,\ Y = -x\sin\theta + y\cos\theta = -4$

Ans: $(-6, -4)$

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9. The axes are rotated through $90°$ in the clockwise direction. Find the new coordinates of the point $(-4, 5)$. [2]

Clockwise: use $\theta = -90°$.

$X = x\cos\theta + y\sin\theta = -5,\ Y = -x\sin\theta + y\cos\theta = -4$

Ans: $(-5, -4)$

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10. Find the transformed equation of $xy = 6$ when the axes are rotated through $135°$. [2]

Put $x = X\cos\theta - Y\sin\theta = -\frac{\sqrt{2}X}{2} - \frac{\sqrt{2}Y}{2},\ y = X\sin\theta + Y\cos\theta = \frac{\sqrt{2}X}{2} - \frac{\sqrt{2}Y}{2}$.

Simplifying: $X^2 - Y^2 = -12$

Ans: $X^2 - Y^2 = -12$

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11. After the axes are rotated through an angle θ with $\cos\theta = \frac{3}{5}$, $\sin\theta = \frac{4}{5}$, the equation of a curve becomes $X^2 - Y^2 = 4$. Find the original equation. [3]

Replace X by $x\cos\theta + y\sin\theta = \frac{3x}{5} + \frac{4y}{5}$ and Y by $-x\sin\theta + y\cos\theta = -\frac{4x}{5} + \frac{3y}{5}$.

Simplifying: $7x^2 - 48xy - 7y^2 = -100$

Ans: $7x^2 - 48xy - 7y^2 = -100$

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12. Find the angle through which the axes must be rotated to remove the xy term from $x^2 + 12xy + y^2 = 12$. [1]

Here $a = b = 1$, so $\theta = \frac{\pi}{4}$.

Ans: $\theta = \frac{\pi}{4}$

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13. Transform $3x^2 - 4xy + 6y^2 = 4$ by rotating the axes so that the xy term disappears. [5]

$\tan 2\theta = \frac{2h}{a-b} = \frac{4}{3}$, so $\cos\theta = \frac{2\sqrt{5}}{5},\ \sin\theta = \frac{\sqrt{5}}{5}$.

Put $x = \frac{2\sqrt{5}X}{5} - \frac{\sqrt{5}Y}{5},\ y = \frac{\sqrt{5}X}{5} + \frac{2\sqrt{5}Y}{5}$.

The XY terms cancel and the equation becomes $2X^2 + 7Y^2 = 4$

Check: $A + B = 9 = a + b$.

Ans: $\theta = \tan^{-1}\frac{1}{2}$; $2X^2 + 7Y^2 = 4$

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14. Find the angle through which the axes must be rotated so that the line $\sqrt{3}x - y - 8 = 0$ takes the form $Y = \text{constant}$. [1]

Angle $= \tan^{-1}\left(-\frac{a}{b}\right) = 60^\circ$.

Ans: 60°

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15. Find the angle of rotation that reduces the line $12x + 5y - 8 = 0$ to the form $Y = \text{constant}$, and find the new equation. [3]

Angle $= \tan^{-1}\left(-\frac{a}{b}\right) = \tan^{-1}\left(-\frac{12}{5}\right)$.

Here $\cos\theta = \frac{5}{13},\ \sin\theta = -\frac{12}{13}$; put $x = \frac{5X}{13} + \frac{12Y}{13},\ y = -\frac{12X}{13} + \frac{5Y}{13}$.

The line becomes $13Y - 8 = 0$, that is $Y = \frac{8}{13}$

Ans: $\tan^{-1}\left(-\frac{12}{5}\right)$; $Y = \frac{8}{13}$

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16. The axes are rotated through 30° and $6x^2 - 2xy + 5y^2 + x - 8y - 2 = 0$ becomes $AX^2 + 2HXY + BY^2 + 2GX + 2FY + C = 0$. Find C. [1]

$C = c$ (an invariant under rotation).

$= -2$

Ans: $C = -2$

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17. A rotation of the axes removes the xy term from $4x^2 + 6xy - 4y^2 = 6$ and gives $AX^2 + BY^2 = 6$. Use the invariants to find A and B. [3]

$A + B = a + b = 0$

$H^2 - AB = h^2 - ab$ with $H = 0$: $AB = ab - h^2 = -25$

So A and B are the roots of $t^2 - (0)t + (-25) = 0$: 5 and -5 (in some order).

Ans: 5 **and** -5 **(in some order)**

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18. The origin is shifted to $(3, -1)$ and the axes are then rotated through 90°. Find the new coordinates of $(5, -5)$. [1]

$x - h = 2,\ y - k = -4$

$X = (x - h)\cos\theta + (y - k)\sin\theta = -4$

$Y = -(x - h)\sin\theta + (y - k)\cos\theta = -2$

Ans: $(-4, -2)$

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19. The origin is shifted to $(-2, 5)$ and the axes are then rotated through 45°. A point has new coordinates $(-5, -2)$. Find its original coordinates. [2]

$x - h = X\cos\theta - Y\sin\theta = -\frac{3\sqrt{2}}{2}$

$y - k = X\sin\theta + Y\cos\theta = -\frac{7\sqrt{2}}{2}$

So $(x, y) = (-\frac{3\sqrt{2}}{2} - 2, 5 - \frac{7\sqrt{2}}{2})$.

Ans: $(-\frac{3\sqrt{2}}{2} - 2, 5 - \frac{7\sqrt{2}}{2})$

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20. Shift the origin to $(2, -3)$ and then rotate the axes through $\tan^{-1}\frac{1}{3}$. Find the transformed equation of $-4x^2 - 6xy + 4y^2 - 2x + 36y + 53 = 0$. [5]

Shift the origin: $x - h = x - 2,\ y - k = y + 3$. The equation is $-4(x-2)^2 - 6(x-2)(y+3) + 4(y+3)^2 = 3$.

Rotate: $\cos\theta = \frac{3\sqrt{10}}{10},\ \sin\theta = \frac{\sqrt{10}}{10}$; put $x - h = \frac{3\sqrt{10}X}{10} - \frac{\sqrt{10}Y}{10},\ y - k = \frac{\sqrt{10}X}{10} + \frac{3\sqrt{10}Y}{10}$.

The XY terms cancel: $-5X^2 + 5Y^2 = 3$

Ans: $-5X^2 + 5Y^2 = 3$