EXPRESSWALA ACADEMY

Maths 1B: Change of Axes

Question Paper, Set 02

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. The origin is shifted to $(6, 1)$. Find the new coordinates of the point $(7, 4)$. [1 mark]

2. Find the transformed equation of $5x^2 - 4xy + 5y^2 - 3x + 5y + 3 = 0$ when the origin is shifted to $(-2, 5)$. [2 marks]

3. When the origin is shifted to $(1, 5)$, the equation of a curve becomes $Y^2 - 4X = 0$. Find its original equation. [2 marks]

4. Find the point to which the origin must be shifted to remove the first degree terms from $5x^2 + 2xy + 2y^2 + 26x + 16y - 4 = 0$. [3 marks]

5. Shift the origin to a suitable point so that the equation $-3x^2 + 4xy + 3y^2 - 20x - 4y - 7 = 0$ has no first degree terms. Find the point and the transformed equation. [5 marks]

6. When the origin is shifted to $(3, -2)$, the equation $x^2 + xy - 4y^2 + 7y - 3 = 0$ becomes $AX^2 + 2HXY + BY^2 + 2GX + 2FY + C = 0$. Find $2G$, $2F$ and C. [2 marks]

7. Show that $-3x^2 + 6xy + 6y^2 - 12x + 30y - 3 = 0$ can be reduced to the form $aX^2 + 2hXY + bY^2 = 0$ by a translation of axes. Find the new origin and the reduced equation. [5 marks]

8. The axes are rotated through $45°$. Find the new coordinates of the point $(-4, -3)$. [1 mark]

9. The axes are rotated through $180°$. A point has new coordinates $(3, -5)$. Find its original coordinates. [2 marks]

10. Find the transformed equation of $xy = 10$ when the axes are rotated through $45°$. [2 marks]

11. After the axes are rotated through $60°$, the equation of a curve becomes $X^2 - 3Y^2 = 4$. Find the original equation. [3 marks]

12. Find the angle through which the axes must be rotated to remove the xy term from $8x^2 - 2xy + 8y^2 = -16$. [1 mark]

13. Find the angle of rotation of the axes that removes the xy term from $2x^2 - 2xy + 2y^2 = 1$, and the transformed equation. [5 marks]

14. Find the angle through which the axes must be rotated so that the line $4x + 3y + 13 = 0$ takes the form $Y = \text{constant}$. [1 mark]

15. Find the angle of rotation that reduces the line $\sqrt{3}x + y - 14 = 0$ to the form $Y = \text{constant}$, and find the new equation. [3 marks]

16. The axes are rotated through $60°$ and $2x^2 - y^2 - x - 8y = 0$ becomes $AX^2 + 2HXY + BY^2 + 2GX + 2FY + C = 0$. Find C. [1 mark]

17. A rotation of the axes removes the xy term from $-4x^2 - 6xy + 4y^2 = 1$ and gives $AX^2 + BY^2 = 1$. Use the invariants to find A and B. [3 marks]

18. The origin is shifted to $(-1, 3)$ and the axes are then rotated through an angle θ with $\cos\theta = \frac{3}{5}$, $\sin\theta = \frac{4}{5}$. Find the new coordinates of $(-4, 1)$. [1 mark]

19. The origin is shifted to $(2, 1)$ and the axes are then rotated through $30°$. A point has new coordinates $(-2, 3)$. Find its original coordinates. [2 marks]

20. Shift the origin to $(1, 1)$ and then rotate the axes through $45°$. Find the transformed equation of $-4x^2 - 2xy - 4y^2 + 10x + 10y - 12 = 0$. [5 marks]

Key sheet

Q	Answer
1	$(1, 3)$
2	$5X^2 - 4XY + 5Y^2 - 43X + 63Y + 219 = 0$
3	$y^2 - 4x - 10y + 29 = 0$
4	$(-2, -3)$
5	$(-2, 2)$; $-3X^2 + 4XY + 3Y^2 + 9 = 0$
6	$2G = 4,\ 2F = 26,\ C = -30$
7	New origin $(-3, -1)$; $-3X^2 + 6XY + 6Y^2 = 0$
8	$(-\frac{7\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$
9	$(-3, 5)$
10	$X^2 - Y^2 = 20$
11	$2x^2 - 2\sqrt{3}xy = -4$
12	$\theta = \frac{\pi}{4}$
13	$\theta = \frac{\pi}{4}$; $X^2 + 3Y^2 = 1$
14	$\tan^{-1}\left(-\frac{4}{3}\right)$
15	$-60°$; $Y = 7$
16	$C = 0$
17	-5 and 5 (in some order)
18	$(-\frac{17}{5}, \frac{6}{5})$
19	$(\frac{1}{2} - \sqrt{3}, \frac{3\sqrt{3}}{2})$
20	$-5X^2 - 3Y^2 = 2$

Solutions

1. The origin is shifted to $(6, 1)$. Find the new coordinates of the point $(7, 4)$. [1]

$X = x - h,\ Y = y - k$

$X = 7 - (6) = 1,\ Y = 4 - (1) = 3$

Ans: $(1, 3)$

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2. Find the transformed equation of $5x^2 - 4xy + 5y^2 - 3x + 5y + 3 = 0$ when the origin is shifted to $(-2, 5)$. [2]

Put $x = X - 2,\ y = Y + 5$ in the equation.

Expanding and collecting terms: $5X^2 - 4XY + 5Y^2 - 43X + 63Y + 219 = 0$

Ans: $5X^2 - 4XY + 5Y^2 - 43X + 63Y + 219 = 0$

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3. When the origin is shifted to $(1, 5)$, the equation of a curve becomes $Y^2 - 4X = 0$. Find its original equation. [2]

Replace X by $x - 1$ and Y by $y - 5$.

$y^2 - 4x - 10y + 29 = 0$

Ans: $y^2 - 4x - 10y + 29 = 0$

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4. Find the point to which the origin must be shifted to remove the first degree terms from $5x^2 + 2xy + 2y^2 + 26x + 16y - 4 = 0$. [3]

$a = 5,\ h = 1,\ b = 2,\ g = 13,\ f = 8,\ c = -4$

Solve $ax + hy + g = 0$ and $hx + by + f = 0$: $5x + y + 13 = 0$, $x + 2y + 8 = 0$

So $(x_1, y_1) = (-2, -3)$.

Ans: $(-2, -3)$

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5. Shift the origin to a suitable point so that the equation $-3x^2 + 4xy + 3y^2 - 20x - 4y - 7 = 0$ has no first degree terms. Find the point and the transformed equation. [5]

$a = -3,\ h = 2,\ b = 3,\ g = -10,\ f = -2,\ c = -7$

Solve $ax + hy + g = 0$ and $hx + by + f = 0$: $-3x + 2y - 10 = 0$, $2x + 3y - 2 = 0$

So $(x_1, y_1) = (-2, 2)$.

Constant $= gx_1 + fy_1 + c = (-10)(-2) + (-2)(2) + (-7) = 9$

Transformed equation: $-3X^2 + 4XY + 3Y^2 + 9 = 0$

Ans: $(-2, 2)$; $-3X^2 + 4XY + 3Y^2 + 9 = 0$

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6. When the origin is shifted to $(3, -2)$, the equation $x^2 + xy - 4y^2 + 7y - 3 = 0$ becomes $AX^2 + 2HXY + BY^2 + 2GX + 2FY + C = 0$. Find $2G$, $2F$ and C. [2]

$2G = \left(\frac{\partial S}{\partial x}\right)_{(3,-2)} = 2(1)(3) + (1)(-2) + (0) = 4$

$2F = \left(\frac{\partial S}{\partial y}\right)_{(3,-2)} = (1)(3) + 2(-4)(-2) + (7) = 26$

$C = S(3, -2) = -30$

Ans: $2G = 4,\ 2F = 26,\ C = -30$

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7. Show that $-3x^2 + 6xy + 6y^2 - 12x + 30y - 3 = 0$ can be reduced to the form $aX^2 + 2hXY + bY^2 = 0$ by a translation of axes. Find the new origin and the reduced equation. [5]

$a = -3,\ h = 3,\ b = 6,\ g = -6,\ f = 15,\ c = -3$

$abc + 2fgh - af^2 - bg^2 - ch^2 = 54 - 540 + 675 - 216 + 27 = 0$

So the reduction is possible.

Solve $-3x + 3y - 6 = 0$ and $3x + 6y + 15 = 0$: new origin $(-3, -1)$.

Constant $gx_1 + fy_1 + c = 0$, so the reduced equation is $-3X^2 + 6XY + 6Y^2 = 0$

Ans: New origin $(-3, -1)$; $-3X^2 + 6XY + 6Y^2 = 0$

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8. The axes are rotated through $45°$. Find the new coordinates of the point $(-4, -3)$. [1]

$X = x\cos\theta + y\sin\theta = -\frac{7\sqrt{2}}{2},\ Y = -x\sin\theta + y\cos\theta = \frac{\sqrt{2}}{2}$

Ans: $(-\frac{7\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$

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9. The axes are rotated through $180°$. A point has new coordinates $(3, -5)$. Find its original coordinates. [2]

$x = X\cos\theta - Y\sin\theta = -3,\ y = X\sin\theta + Y\cos\theta = 5$

Ans: $(-3, 5)$

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10. Find the transformed equation of $xy = 10$ when the axes are rotated through $45°$. [2]

Put $x = X\cos\theta - Y\sin\theta = \frac{\sqrt{2}X}{2} - \frac{\sqrt{2}Y}{2},\ y = X\sin\theta + Y\cos\theta = \frac{\sqrt{2}X}{2} + \frac{\sqrt{2}Y}{2}$.

Simplifying: $X^2 - Y^2 = 20$

Ans: $X^2 - Y^2 = 20$

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11. After the axes are rotated through $60°$, the equation of a curve becomes $X^2 - 3Y^2 = 4$. Find the original equation. [3]

Replace X by $x\cos\theta + y\sin\theta = \frac{x}{2} + \frac{\sqrt{3}y}{2}$ and Y by $-x\sin\theta + y\cos\theta = -\frac{\sqrt{3}x}{2} + \frac{y}{2}$.

Simplifying: $2x^2 - 2\sqrt{3}xy = -4$

Ans: $2x^2 - 2\sqrt{3}xy = -4$

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12. Find the angle through which the axes must be rotated to remove the xy term from $8x^2 - 2xy + 8y^2 = -16$. [1]

Here $a = b = 8$, so $\theta = \frac{\pi}{4}$.

Ans: $\theta = \frac{\pi}{4}$

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13. Find the angle of rotation of the axes that removes the xy term from $2x^2 - 2xy + 2y^2 = 1$, and the transformed equation. [5]

Here $a = b$, so $\theta = \frac{\pi}{4}$.

Put $x = \frac{\sqrt{2}X}{2} - \frac{\sqrt{2}Y}{2},\ y = \frac{\sqrt{2}X}{2} + \frac{\sqrt{2}Y}{2}$.

The XY terms cancel and the equation becomes $X^2 + 3Y^2 = 1$

Check: $A + B = 4 = a + b$.

Ans: $\theta = \frac{\pi}{4}$; $X^2 + 3Y^2 = 1$

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14. Find the angle through which the axes must be rotated so that the line $4x + 3y + 13 = 0$ takes the form $Y = \text{constant}$. [1]

Angle $= \tan^{-1}\left(-\frac{a}{b}\right) = \tan^{-1}\left(-\frac{4}{3}\right)$.

Ans: $\tan^{-1}\left(-\frac{4}{3}\right)$

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15. Find the angle of rotation that reduces the line $\sqrt{3}x + y - 14 = 0$ to the form $Y = \text{constant}$, and find the new equation. [3]

Angle $= \tan^{-1}\left(-\frac{a}{b}\right) = -60^\circ$.

Here $\cos\theta = \frac{1}{2},\ \sin\theta = -\frac{\sqrt{3}}{2}$; put $x = \frac{X}{2} + \frac{\sqrt{3}Y}{2},\ y = -\frac{\sqrt{3}X}{2} + \frac{Y}{2}$.

The line becomes $2Y - 14 = 0$, that is $Y = 7$

Ans: -60°; $Y = 7$

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16. The axes are rotated through 60° and $2x^2 - y^2 - x - 8y = 0$ becomes $AX^2 + 2HXY + BY^2 + 2GX + 2FY + C = 0$. Find C. [1]

$C = c$ (an invariant under rotation).

$= 0$

Ans: $C = 0$

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17. A rotation of the axes removes the xy term from $-4x^2 - 6xy + 4y^2 = 1$ and gives $AX^2 + BY^2 = 1$. Use the invariants to find A and B. [3]

$A + B = a + b = 0$

$H^2 - AB = h^2 - ab$ with $H = 0$: $AB = ab - h^2 = -25$

So A and B are the roots of $t^2 - (0)t + (-25) = 0$: -5 and 5 (in some order).

Ans: -5 **and** 5 **(in some order)**

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18. The origin is shifted to $(-1, 3)$ and the axes are then rotated through an angle θ with $\cos\theta = \frac{3}{5}$, $\sin\theta = \frac{4}{5}$. Find the new coordinates of $(-4, 1)$. [1]

$x - h = -3,\ y - k = -2$

$X = (x-h)\cos\theta + (y-k)\sin\theta = -\frac{17}{5}$

$Y = -(x-h)\sin\theta + (y-k)\cos\theta = \frac{6}{5}$

Ans: $\left(-\frac{17}{5}, \frac{6}{5}\right)$

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19. The origin is shifted to $(2, 1)$ and the axes are then rotated through 30°. A point has new coordinates $(-2, 3)$. Find its original coordinates. [2]

$x - h = X\cos\theta - Y\sin\theta = -\sqrt{3} - \frac{3}{2}$

$y - k = X\sin\theta + Y\cos\theta = -1 + \frac{3\sqrt{3}}{2}$

So $(x, y) = \left(\frac{1}{2} - \sqrt{3}, \frac{3\sqrt{3}}{2}\right)$.

Ans: $\left(\frac{1}{2} - \sqrt{3}, \frac{3\sqrt{3}}{2}\right)$

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20. Shift the origin to $(1, 1)$ and then rotate the axes through $45°$. Find the transformed equation of $-4x^2 - 2xy - 4y^2 + 10x + 10y - 12 = 0$. [5]

Shift the origin: $x - h = x - 1,\ y - k = y - 1$. The equation is $-4(x-1)^2 - 2(x-1)(y-1) - 4(y-1)^2 = 2$.

Rotate: $\cos\theta = \frac{\sqrt{2}}{2},\ \sin\theta = \frac{\sqrt{2}}{2}$; put $x - h = \frac{\sqrt{2}X}{2} - \frac{\sqrt{2}Y}{2},\ y - k = \frac{\sqrt{2}X}{2} + \frac{\sqrt{2}Y}{2}$.

The XY terms cancel: $-5X^2 - 3Y^2 = 2$

Ans: $-5X^2 - 3Y^2 = 2$