EXPRESSWALA ACADEMY

Maths 1B: Change of Axes

MCQ Test, Set 08

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

1. When the origin is shifted to $(-6, -5)$, the new coordinates of $(8, 6)$ are

A. $(14, 11)$ **B.** $(13, 12)$ **C.** $(2, 1)$ **D.** $(-14, -11)$

2. When the origin is shifted to $(3, 6)$, the new coordinates of $(8, 0)$ are

A. $(11, 6)$ **B.** $(2, -3)$ **C.** $(-5, 6)$ **D.** $(5, -6)$

3. When the origin is shifted to $(4, 1)$, the equation $-2x^2 + 3xy + 3y^2 - 3x + y + 9 = 0$ becomes

A. $-2X^2 + 3XY + 3Y^2 - 16X + 19Y - 17 = 0$ **B.** $-2X^2 + 3XY + 3Y^2 + 5X + 28Y + 68 = 0$

C. $-2X^2 + 3XY + 3Y^2 + 10X - 17Y + 3 = 0$ **D.** $-2X^2 + 3XY + 3Y^2 - 16X + 19Y - 19 = 0$

4. When the origin is shifted to $(1, 2)$, the equation $3x^2 + 2xy - 3y^2 + 2x + 5y - 4 = 0$ becomes

A. $3X^2 + 2XY - 3Y^2 + 16X + 3Y + 18 = 0$ **B.** $3X^2 + 2XY - 3Y^2 - 8X + 15Y - 21 = 0$

C. $3X^2 + 2XY - 3Y^2 + 12X - 5Y + 3 = 0$ **D.** $3X^2 + 2XY - 3Y^2 + 12X - 5Y + 5 = 0$

5. When the first degree terms of $4x^2 + 4xy + 2y^2 + 4x - 4y + 11 = 0$ are removed by a translation, the constant term of the new equation is

A. -1 **B.** 1 **C.** 11 **D.** 12

6. When the first degree terms of $-3x^2 - 2xy - 2y^2 - 20x - 10y + 8 = 0$ are removed by a translation, the constant term of the new equation is

A. 43 **B.** -43 **C.** 8 **D.** 51

7. When the first degree terms of $3x^2 + 6xy + 5y^2 - 12x - 16y - 5 = 0$ are removed by a translation, the constant term of the new equation is

A. -24 **B.** 19 **C.** -5 **D.** -19

8. When the origin is shifted to $(-1, 1)$, the coefficient of X in the new form of $x^2 - 2xy + 3y^2 - 4y - 2 = 0$ is

A. -2 **B.** -4 **C.** 4 **D.** 0

9. When the origin is shifted to $(-3, -2)$, the coefficient of X in the new form of $-x^2 - 2xy + 4y^2 + x - 5y - 1 = 0$ is

A. 1 **B.** 11 **C.** 12 **D.** -11

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10. The equation $2x^2 + 2xy + 4y^2 - 6x + 18y + k = 0$ can be reduced to the form $aX^2 + 2hXY + bY^2 = 0$ by a translation when $k =$

A. -36 **B.** 72 **C.** 36 **D.** 37

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11. The equation $-2x^2 - 6xy + 3y^2 - 22x + 12y + k = 0$ can be reduced to the form $aX^2 + 2hXY + bY^2 = 0$ by a translation when $k =$

A. 7 **B.** -7 **C.** 14 **D.** 8

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12. The axes are rotated through 135°. The new coordinates of $(6, -3)$ are

A. $(-\frac{9\sqrt{2}}{2}, -\frac{3\sqrt{2}}{2})$ **B.** $(-\frac{3\sqrt{2}}{2}, -\frac{9\sqrt{2}}{2})$
C. $(\frac{9\sqrt{2}}{2}, \frac{3\sqrt{2}}{2})$ **D.** $(-\frac{3\sqrt{2}}{2}, \frac{9\sqrt{2}}{2})$

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13. The axes are rotated through 120°. The new coordinates of $(-2, -2)$ are

A. $(1 - \sqrt{3}, -\sqrt{3} - 1)$ **B.** $(1 - \sqrt{3}, 1 + \sqrt{3})$
C. $(1 + \sqrt{3}, 1 - \sqrt{3})$ **D.** $(-1 + \sqrt{3}, -\sqrt{3} - 1)$

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14. When the axes are rotated through 45°, the equation $x^2 + y^2 + 3x + 2y + 8 = 0$ becomes

A. $2X^2 + 2Y^2 + \sqrt{2}X + 5\sqrt{2}Y + 16 = 0$ **B.** $2X^2 + 2Y^2 + 5\sqrt{2}X - \sqrt{2}Y - 16 = 0$
C. $2X^2 + 2Y^2 + 5\sqrt{2}X - \sqrt{2}Y + 16 = 0$ **D.** $2X^2 + 2Y^2 - \sqrt{2}X + 5\sqrt{2}Y + 16 = 0$

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15. When the axes are rotated through an angle θ with $\cos\theta = \frac{3}{5}$, $\sin\theta = \frac{4}{5}$, the equation $x^2 + y^2 + x - y - 2 = 0$ becomes

A. $5X^2 + 5Y^2 - X - 7Y + 10 = 0$ **B.** $5X^2 + 5Y^2 + X - 7Y - 10 = 0$
C. $5X^2 + 5Y^2 - X - 7Y - 10 = 0$ **D.** $5X^2 + 5Y^2 + 7X + Y - 10 = 0$

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16. The angle of rotation that removes the xy term from $6x^2 - 2\sqrt{3}xy + 8y^2 = -3$ is

A. $\frac{\pi}{3}$ **B.** $\tan^{-1}\frac{1}{2}$
C. $\tan^{-1} 2$ **D.** $\frac{\pi}{6}$

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17. The angle of rotation that removes the xy term from $-2x^2 - 6xy + 6y^2 = 16$ is

A. $\frac{\pi}{3}$ **B.** $\tan^{-1}\frac{1}{2}$
C. $\frac{\pi}{4}$ **D.** $\tan^{-1}\frac{1}{3}$

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18. To reduce the line $6x + 8y + 7 = 0$ to the form $Y = \text{constant}$, the axes must be rotated through

A. $\tan^{-1}\left(-\frac{3}{4}\right)$ **B.** $\tan^{-1}\left(\frac{4}{3}\right)$
C. $\tan^{-1}\left(\frac{3}{4}\right)$ **D.** $\tan^{-1}\left(-\frac{4}{3}\right)$

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19. To reduce the line $x + \sqrt{3}y - 13 = 0$ to the form $X = \text{constant}$, the axes must be rotated through

A. -30° **B.** 30° **C.** 60° **D.** -60°

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20. When the axes are rotated, which of the following is NOT an invariant of $-2x^2 - 2xy - 4y^2 + 2y - 2 = 0$?

A. $(a-b)^2 + 4h^2$ **B.** $a^2 + b^2$

C. c **D.** $abc + 2fgh - af^2 - bg^2 - ch^2$

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21. The axes are rotated through 60° and $-4x^2 + 2xy - 4y^2 - 3x - 3y = 0$ becomes $AX^2 + 2HXY + BY^2 + 2GX + 2FY + C = 0$. Then $A + B =$

A. 16 **B.** 0 **C.** -8 **D.** -7

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22. The origin is shifted to $(-2, -1)$ and the axes are then rotated through an angle θ with $\cos\theta = \frac{3}{5}$, $\sin\theta = \frac{4}{5}$. The new coordinates of $(-1, -5)$ are

A. $(-\frac{13}{5}, -\frac{16}{5})$ **B.** $(\frac{19}{5}, -\frac{8}{5})$

C. $(-\frac{23}{5}, -\frac{11}{5})$ **D.** $(-\frac{16}{5}, -\frac{13}{5})$

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23. The origin is shifted to $(4, -1)$ and the axes are then rotated through an angle θ with $\cos\theta = \frac{4}{5}$, $\sin\theta = \frac{3}{5}$. The new coordinates of $(-1, 7)$ are

A. $(-\frac{44}{5}, \frac{17}{5})$ **B.** $(\frac{47}{5}, \frac{4}{5})$

C. $(\frac{17}{5}, \frac{31}{5})$ **D.** $(\frac{4}{5}, \frac{47}{5})$

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24. When the origin is shifted to $(-3, 4)$ and the axes are then rotated through $\tan^{-1}\frac{1}{3}$, the equation $6x^2 + 6xy - 2y^2 + 12x + 34y - 74 = 0$ becomes

A. $7X^2 - 3Y^2 = 24$ **B.** $-3X^2 + 7Y^2 = 24$ **C.** $6X^2 - 2Y^2 = 24$ **D.** $7X^2 - 3Y^2 = -24$

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25. When the origin is shifted to $(3, 2)$ and the axes are then rotated through $\tan^{-1}\frac{1}{2}$, the equation $4x^2 - 4xy + 7y^2 - 16x - 16y + 34 = 0$ becomes

A. $4X^2 + 7Y^2 = 6$ **B.** $3X^2 + 8Y^2 = 6$ **C.** $8X^2 + 3Y^2 = 6$ **D.** $3X^2 + 8Y^2 = -6$

Key sheet

1. A	**2.** D	**3.** D	**4.** C	**5.** B
6. A	**7.** D	**8.** B	**9.** B	**10.** C
11. A	**12.** A	**13.** B	**14.** C	**15.** C
16. D	**17.** D	**18.** A	**19.** C	**20.** B
21. C	**22.** A	**23.** D	**24.** A	**25.** B

Solutions

1. When the origin is shifted to $(-6, -5)$, the new coordinates of $(8, 6)$ are

$X = x - h,\ Y = y - k$

$X = 8 - (-6) = 14,\ Y = 6 - (-5) = 11$

Ans: (A) $(14, 11)$

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2. When the origin is shifted to $(3, 6)$, the new coordinates of $(8, 0)$ are

$X = x - h,\ Y = y - k$

$X = 8 - (3) = 5,\ Y = 0 - (6) = -6$

Ans: (D) $(5, -6)$

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3. When the origin is shifted to $(4, 1)$, the equation $-2x^2 + 3xy + 3y^2 - 3x + y + 9 = 0$ becomes

Put $x = X + 4,\ y = Y + 1$ in the equation.

Expanding and collecting terms: $-2X^2 + 3XY + 3Y^2 - 16X + 19Y - 19 = 0$

Ans: (D) $-2X^2 + 3XY + 3Y^2 - 16X + 19Y - 19 = 0$

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4. When the origin is shifted to $(1, 2)$, the equation $3x^2 + 2xy - 3y^2 + 2x + 5y - 4 = 0$ becomes

Put $x = X + 1,\ y = Y + 2$ in the equation.

Expanding and collecting terms: $3X^2 + 2XY - 3Y^2 + 12X - 5Y + 3 = 0$

Ans: (C) $3X^2 + 2XY - 3Y^2 + 12X - 5Y + 3 = 0$

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5. When the first degree terms of $4x^2 + 4xy + 2y^2 + 4x - 4y + 11 = 0$ are removed by a translation, the constant term of the new equation is

$a = 4,\ h = 2,\ b = 2,\ g = 2,\ f = -2,\ c = 11$

Solve $ax + hy + g = 0$ and $hx + by + f = 0$: $4x + 2y + 2 = 0$, $2x + 2y - 2 = 0$

So $(x_1, y_1) = (-2, 3)$.

Constant $= gx_1 + fy_1 + c = (2)(-2) + (-2)(3) + (11) = 1$

Ans: (B) 1

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6. When the first degree terms of $-3x^2 - 2xy - 2y^2 - 20x - 10y + 8 = 0$ are removed by a translation, the constant term of the new equation is

$a = -3,\ h = -1,\ b = -2,\ g = -10,\ f = -5,\ c = 8$

Solve $ax + hy + g = 0$ and $hx + by + f = 0$: $-3x - y - 10 = 0$, $-x - 2y - 5 = 0$

So $(x_1, y_1) = (-3, -1)$.

Constant $= gx_1 + fy_1 + c = (-10)(-3) + (-5)(-1) + (8) = 43$

Ans: (A) 43

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7. When the first degree terms of $3x^2 + 6xy + 5y^2 - 12x - 16y - 5 = 0$ are removed by a translation, the constant term of the new equation is

$a = 3,\ h = 3,\ b = 5,\ g = -6,\ f = -8,\ c = -5$

Solve $ax + hy + g = 0$ and $hx + by + f = 0$: $3x + 3y - 6 = 0$, $3x + 5y - 8 = 0$

So $(x_1, y_1) = (1, 1)$.

Constant $= gx_1 + fy_1 + c = (-6)(1) + (-8)(1) + (-5) = -19$

Ans: (D) -19

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8. When the origin is shifted to $(-1, 1)$, the coefficient of X in the new form of $x^2 - 2xy + 3y^2 - 4y - 2 = 0$ is

$2G = \left(\frac{\partial S}{\partial x}\right)_{(-1,1)} = 2(1)(-1) + (-2)(1) + (0) = -4$

Ans: (B) -4

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9. When the origin is shifted to $(-3, -2)$, the coefficient of X in the new form of $-x^2 - 2xy + 4y^2 + x - 5y - 1 = 0$ is

$2G = \left(\frac{\partial S}{\partial x}\right)_{(-3,-2)} = 2(-1)(-3) + (-2)(-2) + (1) = 11$

Ans: (B) 11

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10. The equation $2x^2 + 2xy + 4y^2 - 6x + 18y + k = 0$ can be reduced to the form $aX^2 + 2hXY + bY^2 = 0$ by a translation when $k =$

Need $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ with $c = k$: $7k - 252 = 0$

$k = 36$

Ans: (C) 36

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11. The equation $-2x^2 - 6xy + 3y^2 - 22x + 12y + k = 0$ can be reduced to the form $aX^2 + 2hXY + bY^2 = 0$ by a translation when $k =$

Need $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ with $c = k$: $-15k + 105 = 0$

$k = 7$

Ans: (A) 7

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12. The axes are rotated through $135°$. The new coordinates of $(6, -3)$ are

$X = x\cos\theta + y\sin\theta = -\frac{9\sqrt{2}}{2},\ Y = -x\sin\theta + y\cos\theta = -\frac{3\sqrt{2}}{2}$

Ans: (A) $(-\frac{9\sqrt{2}}{2}, -\frac{3\sqrt{2}}{2})$

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13. The axes are rotated through $120°$. The new coordinates of $(-2, -2)$ are

$X = x\cos\theta + y\sin\theta = 1 - \sqrt{3},\ Y = -x\sin\theta + y\cos\theta = 1 + \sqrt{3}$

Ans: (B) $(1 - \sqrt{3}, 1 + \sqrt{3})$

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14. When the axes are rotated through $45°$, the equation $x^2 + y^2 + 3x + 2y + 8 = 0$ becomes

Put $x = X\cos\theta - Y\sin\theta = \frac{\sqrt{2}X}{2} - \frac{\sqrt{2}Y}{2},\ y = X\sin\theta + Y\cos\theta = \frac{\sqrt{2}X}{2} + \frac{\sqrt{2}Y}{2}$.

Simplifying: $2X^2 + 2Y^2 + 5\sqrt{2}X - \sqrt{2}Y + 16 = 0$

Ans: (C) $2X^2 + 2Y^2 + 5\sqrt{2}X - \sqrt{2}Y + 16 = 0$

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15. When the axes are rotated through an angle θ with $\cos\theta = \frac{3}{5}$, $\sin\theta = \frac{4}{5}$, the equation $x^2 + y^2 + x - y - 2 = 0$ becomes

Put $x = X\cos\theta - Y\sin\theta = \frac{3X}{5} - \frac{4Y}{5}$, $y = X\sin\theta + Y\cos\theta = \frac{4X}{5} + \frac{3Y}{5}$.

Simplifying: $5X^2 + 5Y^2 - X - 7Y - 10 = 0$

Ans: (C) $5X^2 + 5Y^2 - X - 7Y - 10 = 0$

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16. The angle of rotation that removes the xy term from $6x^2 - 2\sqrt{3}xy + 8y^2 = -3$ is

$\tan 2\theta = \frac{2h}{a-b} = \frac{-2\sqrt{3}}{6-(8)} = \sqrt{3}$

$\theta = \frac{\pi}{6} = \frac{\pi}{6}$

Ans: (D) $\frac{\pi}{6}$

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17. The angle of rotation that removes the xy term from $-2x^2 - 6xy + 6y^2 = 16$ is

$\tan 2\theta = \frac{2h}{a-b} = \frac{-6}{-2-(6)} = \frac{3}{4}$

$\theta = \frac{1}{2}\tan^{-1}\left(\frac{3}{4}\right) = \tan^{-1}\frac{1}{3}$

Ans: (D) $\tan^{-1}\frac{1}{3}$

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18. To reduce the line $6x + 8y + 7 = 0$ to the form $Y = \text{constant}$, the axes must be rotated through

Angle $= \tan^{-1}\left(-\frac{a}{b}\right) = \tan^{-1}\left(-\frac{3}{4}\right)$.

Ans: (A) $\tan^{-1}\left(-\frac{3}{4}\right)$

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19. To reduce the line $x + \sqrt{3}y - 13 = 0$ to the form $X = \text{constant}$, the axes must be rotated through

Angle $= \tan^{-1}\left(\frac{b}{a}\right) = 60^\circ$.

Ans: (C) 60°

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20. When the axes are rotated, which of the following is NOT an invariant of $-2x^2 - 2xy - 4y^2 + 2y - 2 = 0$?

The invariants are $a + b$, $h^2 - ab$, c, $(a-b)^2 + 4h^2$, $2fgh - af^2 - bg^2$ and $abc + 2fgh - af^2 - bg^2 - ch^2$.

$a^2 + b^2$ is not one of them.

Ans: (B) $a^2 + b^2$

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21. The axes are rotated through 60° and $-4x^2 + 2xy - 4y^2 - 3x - 3y = 0$ becomes $AX^2 + 2HXY + BY^2 + 2GX + 2FY + C = 0$. Then $A + B =$

$A + B = a + b$ (an invariant under rotation).

$= -8$

Ans: (C) -8

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22. The origin is shifted to $(-2, -1)$ and the axes are then rotated through an angle θ with $\cos\theta = \frac{3}{5}$, $\sin\theta = \frac{4}{5}$. The new coordinates of $(-1, -5)$ are

$x - h = 1,\ y - k = -4$

$X = (x-h)\cos\theta + (y-k)\sin\theta = -\frac{13}{5}$

$Y = -(x-h)\sin\theta + (y-k)\cos\theta = -\frac{16}{5}$

Ans: (A) $(-\frac{13}{5}, -\frac{16}{5})$

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23. The origin is shifted to $(4, -1)$ and the axes are then rotated through an angle θ with $\cos\theta = \frac{4}{5}$, $\sin\theta = \frac{3}{5}$. The new coordinates of $(-1, 7)$ are

$x - h = -5,\ y - k = 8$

$X = (x-h)\cos\theta + (y-k)\sin\theta = \frac{4}{5}$

$Y = -(x-h)\sin\theta + (y-k)\cos\theta = \frac{47}{5}$

Ans: (D) $(\frac{4}{5}, \frac{47}{5})$

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24. When the origin is shifted to $(-3, 4)$ and the axes are then rotated through $\tan^{-1}\frac{1}{3}$, the equation $6x^2 + 6xy - 2y^2 + 12x + 34y - 74 = 0$ becomes

The XY terms cancel: $7X^2 - 3Y^2 = 24$

Ans: (A) $7X^2 - 3Y^2 = 24$

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25. When the origin is shifted to $(3, 2)$ and the axes are then rotated through $\tan^{-1}\frac{1}{2}$, the equation $4x^2 - 4xy + 7y^2 - 16x - 16y + 34 = 0$ becomes

The XY terms cancel: $3X^2 + 8Y^2 = 6$

Ans: (B) $3X^2 + 8Y^2 = 6$
