EXPRESSWALA ACADEMY

Maths 1B: Change of Axes

MCQ Test, Set 07

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

1. When the origin is shifted to $(-1, -4)$, the new coordinates of $(-2, -4)$ are

A. $(-1, 0)$ **B.** $(-3, -8)$ **C.** $(1, 0)$ **D.** $(2, -3)$

2. After the origin is shifted to $(5, 7)$, a point has new coordinates $(-4, 7)$. Its original coordinates are

A. $(9, 0)$ **B.** $(-9, 0)$ **C.** $(3, 12)$ **D.** $(1, 14)$

3. When the origin is shifted to $(3, -4)$, the equation $y^2 - 4x + 8y + 28 = 0$ becomes

A. $Y^2 - 4X = 0$ **B.** $Y^2 - 4X + 16Y + 88 = 0$ **C.** $Y^2 - 4X + 14Y + 77 = 0$ **D.** $Y^2 - 4X + 2 = 0$

4. When the origin is shifted to $(-4, -3)$, the equation $x^2 + y^2 + 8x + 6y + 9 = 0$ becomes

A. $X^2 + Y^2 + 2X - 2Y - 14 = 0$ **B.** $X^2 + Y^2 + 16X + 12Y + 84 = 0$

C. $X^2 + Y^2 - 14 = 0$ **D.** $X^2 + Y^2 - 16 = 0$

5. When the first degree terms of $3x^2 + 6xy + 2y^2 + 24x + 22y - 12 = 0$ are removed by a translation, the constant term of the new equation is

A. 59 **B.** -71 **C.** -59 **D.** -12

6. To remove the first degree terms from $x^2 + 4xy + y^2 + 18x + 12y - 3 = 0$, the origin must be shifted to

A. $(4, 1)$ **B.** $(1, 4)$ **C.** $(-1, -4)$ **D.** $(-4, -1)$

7. When the first degree terms of $x^2 - 6xy + 3y^2 - 2x + 18y - 2 = 0$ are removed by a translation, the constant term of the new equation is

A. 1 **B.** 3 **C.** -3 **D.** -2

8. When the origin is shifted to $(4, 3)$, the coefficient of X in the new form of $-x^2 - 2xy + 2y^2 + 2x + 6y - 7 = 0$ is

A. -12 **B.** 2 **C.** -6 **D.** 12

9. When the origin is shifted to $(1, -2)$, the coefficient of Y in the new form of $3x^2 + 3xy - y^2 + 5x + y + 5 = 0$ is

A. 1 **B.** 4 **C.** 8 **D.** -8

10. For $4x^2 + 2xy + 6y^2 + 26x - 28y + 4 = 0$, the value of $abc + 2fgh - af^2 - bg^2 - ch^2$ is

A. -2070 **B.** 96 **C.** -2434 **D.** 2070

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11. The equation $2x^2 + 2y^2 - 8x - 8y + k = 0$ can be reduced to the form $aX^2 + 2hXY + bY^2 = 0$ by a translation when $k =$

A. 17 **B.** 32 **C.** 16 **D.** -16

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12. The axes are rotated through an angle θ with $\cos\theta = \frac{5}{13}$, $\sin\theta = \frac{12}{13}$. The new coordinates of $(1, -4)$ are

A. $(-\frac{43}{13}, -\frac{32}{13})$ **B.** $(\frac{43}{13}, \frac{32}{13})$
C. $(\frac{53}{13}, -\frac{8}{13})$ **D.** $(-\frac{32}{13}, -\frac{43}{13})$

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13. The axes are rotated through $60°$. The new coordinates of $(2, 4)$ are

A. $(1 + 2\sqrt{3}, 2 - \sqrt{3})$ **B.** $(2 - \sqrt{3}, 1 + 2\sqrt{3})$
C. $(1 - 2\sqrt{3}, \sqrt{3} + 2)$ **D.** $(-2\sqrt{3} - 1, -2 + \sqrt{3})$

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14. When the axes are rotated through $45°$, the equation $2x^2 + 2xy + y^2 = 4$ becomes

A. $X^2 - 2XY + 5Y^2 = 8$ **B.** $5X^2 - 2XY + Y^2 = -8$ **C.** $5X^2 - 2XY + Y^2 = 8$ **D.** $X^2 + 2XY + 5Y^2 = 8$

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15. When the axes are rotated through $45°$, the equation $x^2 + y^2 + 7x + 2y - 3 = 0$ becomes

A. $2X^2 + 2Y^2 + 5\sqrt{2}X + 9\sqrt{2}Y - 6 = 0$ **B.** $2X^2 + 2Y^2 - 5\sqrt{2}X + 9\sqrt{2}Y - 6 = 0$
C. $2X^2 + 2Y^2 + 9\sqrt{2}X - 5\sqrt{2}Y + 6 = 0$ **D.** $2X^2 + 2Y^2 + 9\sqrt{2}X - 5\sqrt{2}Y - 6 = 0$

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16. The angle of rotation that removes the xy term from $3x^2 + 8xy - 3y^2 = 24$ is

A. $\frac{\pi}{4}$ **B.** $\frac{\pi}{3}$
C. $\tan^{-1} 2$ **D.** $\tan^{-1} \frac{1}{2}$

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17. The angle of rotation that removes the xy term from $3x^2 - 4\sqrt{3}xy + 7y^2 = 8$ is

A. $\tan^{-1} \frac{1}{2}$ **B.** $\frac{\pi}{6}$
C. $\frac{\pi}{3}$ **D.** $\tan^{-1} 2$

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18. To reduce the line $-4x + 3y + 9 = 0$ to the form $Y = \text{constant}$, the axes must be rotated through

A. $\tan^{-1}\left(\frac{3}{4}\right)$ **B.** $\tan^{-1}\left(\frac{4}{3}\right)$
C. $\tan^{-1}\left(-\frac{4}{3}\right)$ **D.** $\tan^{-1}\left(-\frac{3}{4}\right)$

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19. To reduce the line $5x + 12y - 5 = 0$ to the form $Y = \text{constant}$, the axes must be rotated through

A. $\tan^{-1}\left(-\frac{12}{5}\right)$ **B.** $\tan^{-1}\left(\frac{12}{5}\right)$
C. $\tan^{-1}\left(-\frac{5}{12}\right)$ **D.** $\tan^{-1}\left(\frac{5}{12}\right)$

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20. The axes are rotated through $60°$ and $-4x^2 + xy + y^2 - x - 4y = 0$ becomes $AX^2 + 2HXY + BY^2 + 2GX + 2FY + C = 0$. Then $H^2 - AB =$

A. $-\frac{17}{4}$ **B.** $\frac{9}{2}$ **C.** $-\frac{15}{4}$ **D.** $\frac{17}{4}$

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21. The axes are rotated through 45° and $x^2 - 2xy - y^2 - 7x - y - 4 = 0$ becomes $AX^2 + 2HXY + BY^2 + 2GX + 2FY + C = 0$. Then $A + B =$

A. 2 **B.** -1 **C.** 1 **D.** 0

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22. The origin is shifted to $(-4, -2)$ and the axes are then rotated through an angle θ with $\cos\theta = \frac{4}{5}$, $\sin\theta = \frac{3}{5}$. The new coordinates of $(-1, -2)$ are

A. $(-2, -1)$ **B.** $(\frac{12}{5}, -\frac{9}{5})$

C. $(-\frac{9}{5}, \frac{12}{5})$ **D.** $(\frac{12}{5}, \frac{9}{5})$

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23. The origin is shifted to $(4, -1)$ and the axes are then rotated through 180°. The new coordinates of $(1, -2)$ are

A. $(-3, -1)$ **B.** $(3, 1)$ **C.** $(1, 3)$ **D.** $(-1, 2)$

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24. When the origin is shifted to $(-2, -3)$ and the axes are then rotated through $\tan^{-1}\frac{1}{3}$, the equation $4x^2 + 6xy - 4y^2 + 34x - 12y + 8 = 0$ becomes

A. $5X^2 - 5Y^2 = 8$ **B.** $5X^2 - 5Y^2 = -8$ **C.** $4X^2 - 4Y^2 = 8$ **D.** $-5X^2 + 5Y^2 = 8$

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25. When the origin is shifted to $(4, -3)$ and the axes are then rotated through 45°, the equation $-2x^2 - 6xy - 2y^2 - 2x + 12y + 20 = 0$ becomes

A. $X^2 - 5Y^2 = 2$ **B.** $-5X^2 + Y^2 = 2$ **C.** $-5X^2 + Y^2 = -2$ **D.** $-2X^2 - 2Y^2 = 2$

Key sheet

1. A	**2.** D	**3.** A	**4.** D	**5.** C
6. C	**7.** B	**8.** A	**9.** C	**10.** A
11. C	**12.** A	**13.** A	**14.** C	**15.** D
16. D	**17.** B	**18.** B	**19.** C	**20.** D
21. D	**22.** B	**23.** B	**24.** A	**25.** B

Solutions

1. When the origin is shifted to $(-1, -4)$, the new coordinates of $(-2, -4)$ are

$X = x - h,\ Y = y - k$

$X = -2 - (-1) = -1,\ Y = -4 - (-4) = 0$

Ans: (A) $(-1, 0)$

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2. After the origin is shifted to $(5, 7)$, a point has new coordinates $(-4, 7)$. Its original coordinates are

$x = X + h,\ y = Y + k$

$x = -4 + (5) = 1,\ y = 7 + (7) = 14$

Ans: (D) $(1, 14)$

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3. When the origin is shifted to $(3, -4)$, the equation $y^2 - 4x + 8y + 28 = 0$ becomes

Put $x = X + 3,\ y = Y - 4$ in the equation.

Expanding and collecting terms: $Y^2 - 4X = 0$

Ans: (A) $Y^2 - 4X = 0$

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4. When the origin is shifted to $(-4, -3)$, the equation $x^2 + y^2 + 8x + 6y + 9 = 0$ becomes

Put $x = X - 4,\ y = Y - 3$ in the equation.

Expanding and collecting terms: $X^2 + Y^2 - 16 = 0$

Ans: (D) $X^2 + Y^2 - 16 = 0$

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5. When the first degree terms of $3x^2 + 6xy + 2y^2 + 24x + 22y - 12 = 0$ are removed by a translation, the constant term of the new equation is

$a = 3,\ h = 3,\ b = 2,\ g = 12,\ f = 11,\ c = -12$

Solve $ax + hy + g = 0$ and $hx + by + f = 0$: $3x + 3y + 12 = 0$, $3x + 2y + 11 = 0$

So $(x_1, y_1) = (-3, -1)$.

Constant $= gx_1 + fy_1 + c = (12)(-3) + (11)(-1) + (-12) = -59$

Ans: (C) -59

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6. To remove the first degree terms from $x^2 + 4xy + y^2 + 18x + 12y - 3 = 0$, the origin must be shifted to

$a = 1,\ h = 2,\ b = 1,\ g = 9,\ f = 6,\ c = -3$

Solve $ax + hy + g = 0$ and $hx + by + f = 0$: $x + 2y + 9 = 0$, $2x + y + 6 = 0$

So $(x_1, y_1) = (-1, -4)$.

Ans: (C) $(-1, -4)$

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7. When the first degree terms of $x^2 - 6xy + 3y^2 - 2x + 18y - 2 = 0$ are removed by a translation, the constant term of the new equation is

$a = 1,\ h = -3,\ b = 3,\ g = -1,\ f = 9,\ c = -2$

Solve $ax + hy + g = 0$ and $hx + by + f = 0$: $x - 3y - 1 = 0$, $-3x + 3y + 9 = 0$

So $(x_1, y_1) = (4, 1)$.

Constant $= gx_1 + fy_1 + c = (-1)(4) + (9)(1) + (-2) = 3$

Ans: (B) 3

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8. When the origin is shifted to $(4, 3)$, the coefficient of X in the new form of $-x^2 - 2xy + 2y^2 + 2x + 6y - 7 = 0$ is

$2G = \left(\frac{\partial S}{\partial x}\right)_{(4,3)} = 2(-1)(4) + (-2)(3) + (2) = -12$

Ans: (A) -12

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9. When the origin is shifted to $(1, -2)$, the coefficient of Y in the new form of $3x^2 + 3xy - y^2 + 5x + y + 5 = 0$ is

$2F = \left(\frac{\partial S}{\partial y}\right)_{(1,-2)} = (3)(1) + 2(-1)(-2) + (1) = 8$

Ans: (C) 8

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10. For $4x^2 + 2xy + 6y^2 + 26x - 28y + 4 = 0$, the value of $abc + 2fgh - af^2 - bg^2 - ch^2$ is

$a = 4,\ h = 1,\ b = 6,\ g = 13,\ f = -14,\ c = 4$

$abc + 2fgh - af^2 - bg^2 - ch^2 = -2070$

Ans: (A) -2070

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11. The equation $2x^2 + 2y^2 - 8x - 8y + k = 0$ can be reduced to the form $aX^2 + 2hXY + bY^2 = 0$ by a translation when $k =$

Need $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ with $c = k$: $4k - 64 = 0$

$k = 16$

Ans: (C) 16

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12. The axes are rotated through an angle θ with $\cos\theta = \frac{5}{13}$, $\sin\theta = \frac{12}{13}$. The new coordinates of $(1, -4)$ are

$X = x\cos\theta + y\sin\theta = -\frac{43}{13},\ Y = -x\sin\theta + y\cos\theta = -\frac{32}{13}$

Ans: (A) $(-\frac{43}{13}, -\frac{32}{13})$

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13. The axes are rotated through 60°. The new coordinates of $(2, 4)$ are

$X = x\cos\theta + y\sin\theta = 1 + 2\sqrt{3},\ Y = -x\sin\theta + y\cos\theta = 2 - \sqrt{3}$

Ans: (A) $(1 + 2\sqrt{3}, 2 - \sqrt{3})$

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14. When the axes are rotated through 45°, the equation $2x^2 + 2xy + y^2 = 4$ becomes

Put $x = X\cos\theta - Y\sin\theta = \frac{\sqrt{2}X}{2} - \frac{\sqrt{2}Y}{2},\ y = X\sin\theta + Y\cos\theta = \frac{\sqrt{2}X}{2} + \frac{\sqrt{2}Y}{2}$.

Simplifying: $5X^2 - 2XY + Y^2 = 8$

Ans: (C) $5X^2 - 2XY + Y^2 = 8$

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15. When the axes are rotated through 45°, the equation $x^2 + y^2 + 7x + 2y - 3 = 0$ becomes

Put $x = X\cos\theta - Y\sin\theta = \frac{\sqrt{2}X}{2} - \frac{\sqrt{2}Y}{2},\ y = X\sin\theta + Y\cos\theta = \frac{\sqrt{2}X}{2} + \frac{\sqrt{2}Y}{2}$.

Simplifying: $2X^2 + 2Y^2 + 9\sqrt{2}X - 5\sqrt{2}Y - 6 = 0$

Ans: (D) $2X^2 + 2Y^2 + 9\sqrt{2}X - 5\sqrt{2}Y - 6 = 0$

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16. The angle of rotation that removes the xy term from $3x^2 + 8xy - 3y^2 = 24$ is

$\tan 2\theta = \frac{2h}{a-b} = \frac{8}{3-(-3)} = \frac{4}{3}$

$\theta = \frac{1}{2}\tan^{-1}\left(\frac{4}{3}\right) = \tan^{-1}\frac{1}{2}$

Ans: (D) $\tan^{-1}\frac{1}{2}$

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17. The angle of rotation that removes the xy term from $3x^2 - 4\sqrt{3}xy + 7y^2 = 8$ is

$\tan 2\theta = \frac{2h}{a-b} = \frac{-4\sqrt{3}}{3-(7)} = \sqrt{3}$

$\theta = \frac{\pi}{6} = \frac{\pi}{6}$

Ans: (B) $\frac{\pi}{6}$

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18. To reduce the line $-4x + 3y + 9 = 0$ to the form $Y = \text{constant}$, the axes must be rotated through

Angle $= \tan^{-1}\left(-\frac{a}{b}\right) = \tan^{-1}\left(\frac{4}{3}\right)$.

Ans: (B) $\tan^{-1}\left(\frac{4}{3}\right)$

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19. To reduce the line $5x + 12y - 5 = 0$ to the form $Y = \text{constant}$, the axes must be rotated through

Angle $= \tan^{-1}\left(-\frac{a}{b}\right) = \tan^{-1}\left(-\frac{5}{12}\right)$.

Ans: (C) $\tan^{-1}\left(-\frac{5}{12}\right)$

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20. The axes are rotated through 60° and $-4x^2 + xy + y^2 - x - 4y = 0$ becomes $AX^2 + 2HXY + BY^2 + 2GX + 2FY + C = 0$. Then $H^2 - AB =$

$H^2 - AB = h^2 - ab$ (an invariant under rotation).

$= \frac{17}{4}$

Ans: (D) $\frac{17}{4}$

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21. The axes are rotated through 45° and $x^2 - 2xy - y^2 - 7x - y - 4 = 0$ becomes $AX^2 + 2HXY + BY^2 + 2GX + 2FY + C = 0$. Then $A + B =$

$A + B = a + b$ (an invariant under rotation).

$= 0$

Ans: (D) 0

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22. The origin is shifted to $(-4, -2)$ and the axes are then rotated through an angle θ with $\cos\theta = \frac{4}{5}$, $\sin\theta = \frac{3}{5}$. The new coordinates of $(-1, -2)$ are

$x - h = 3,\ y - k = 0$

$X = (x-h)\cos\theta + (y-k)\sin\theta = \frac{12}{5}$

$Y = -(x-h)\sin\theta + (y-k)\cos\theta = -\frac{9}{5}$

Ans: (B) $(\frac{12}{5}, -\frac{9}{5})$

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23. The origin is shifted to $(4, -1)$ and the axes are then rotated through 180°. The new coordinates of $(1, -2)$ are

$x - h = -3,\ y - k = -1$

$X = (x-h)\cos\theta + (y-k)\sin\theta = 3$

$Y = -(x-h)\sin\theta + (y-k)\cos\theta = 1$

Ans: (B) $(3, 1)$

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24. When the origin is shifted to $(-2, -3)$ and the axes are then rotated through $\tan^{-1}\frac{1}{3}$, the equation $4x^2 + 6xy - 4y^2 + 34x - 12y + 8 = 0$ becomes

The XY terms cancel: $5X^2 - 5Y^2 = 8$

Ans: (A) $5X^2 - 5Y^2 = 8$

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25. When the origin is shifted to $(4, -3)$ and the axes are then rotated through 45°, the equation $-2x^2 - 6xy - 2y^2 - 2x + 12y + 20 = 0$ becomes

The XY terms cancel: $-5X^2 + Y^2 = 2$

Ans: (B) $-5X^2 + Y^2 = 2$
