EXPRESSWALA ACADEMY

Maths 1B: Change of Axes

MCQ Test, Set 03

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

1. When the origin is shifted to $(6, -4)$, the new coordinates of $(-4, -7)$ are

A. $(-10, -3)$ **B.** $(0, -13)$ **C.** $(2, -11)$ **D.** $(10, 3)$

2. After the origin is shifted to $(2, 7)$, a point has new coordinates $(1, -6)$. Its original coordinates are

A. $(1, 13)$ **B.** $(8, -4)$ **C.** $(3, 1)$ **D.** $(-1, -13)$

3. When the origin is shifted to $(3, 1)$, the equation $x^2 + y^2 - 6x - 2y - 3 = 0$ becomes

A. $X^2 + Y^2 - 13 = 0$ **B.** $X^2 + Y^2 - 4X + 4Y - 5 = 0$

C. $X^2 + Y^2 - 12X - 4Y + 27 = 0$ **D.** $X^2 + Y^2 - 11 = 0$

4. When the origin is shifted to $(-2, -4)$, the equation $-2x^2 + 2xy - 4y^2 - 2x - 7y + 6 = 0$ becomes

A. $-2X^2 + 2XY - 4Y^2 - 2X - 35Y - 82 = 0$ **B.** $-2X^2 + 2XY - 4Y^2 - 2X + 21Y - 16 = 0$

C. $-2X^2 + 2XY - 4Y^2 + 10X + Y - 4 = 0$ **D.** $-2X^2 + 2XY - 4Y^2 - 2X + 21Y - 18 = 0$

5. When the first degree terms of $-2x^2 + 6xy - 3y^2 + 10x - 12y - 10 = 0$ are removed by a translation, the constant term of the new equation is

A. -1 **B.** 1 **C.** -10 **D.** -9

6. When the first degree terms of $2x^2 - 2xy - 2y^2 - 20x - 10y + 12 = 0$ are removed by a translation, the constant term of the new equation is

A. 2 **B.** -2 **C.** 12 **D.** 14

7. When the first degree terms of $-2x^2 + 6xy - y^2 + 4x - 20y - 8 = 0$ are removed by a translation, the constant term of the new equation is

A. -28 **B.** 20 **C.** -8 **D.** -20

8. When the origin is shifted to $(2, 1)$, the coefficient of Y in the new form of $5x^2 - 4xy - 2y^2 + 8x - 6y - 8 = 0$ is

A. 18 **B.** -18 **C.** -9 **D.** -6

9. When the origin is shifted to $(4, -1)$, the coefficient of Y in the new form of $6x^2 + 4xy + 3y^2 - 4x - 5 = 0$ is

A. 10 **B.** 0 **C.** -10 **D.** 5

10. The equation $-3x^2 + 2xy - 3y^2 - 24x + 24y + k = 0$ can be reduced to the form $aX^2 + 2hXY + bY^2 = 0$ by a translation when $k =$

A. -144 **B.** 72 **C.** -72 **D.** -71

11. The equation $3x^2 + 2xy + 4y^2 - 22x + 22y + k = 0$ can be reduced to the form $aX^2 + 2hXY + bY^2 = 0$ by a translation when $k =$

A. 100 **B.** 198 **C.** -99 **D.** 99

12. The axes are rotated through an angle θ with $\cos\theta = \frac{5}{13}$, $\sin\theta = \frac{12}{13}$. The new coordinates of $(-1, 2)$ are

A. $(-\frac{29}{13}, -\frac{2}{13})$ **B.** $(-\frac{19}{13}, -\frac{22}{13})$
C. $(\frac{19}{13}, \frac{22}{13})$ **D.** $(\frac{22}{13}, \frac{19}{13})$

13. The axes are rotated through 90° in the clockwise direction. The new coordinates of $(5, 2)$ are

A. $(2, -5)$ **B.** $(-2, 5)$ **C.** $(5, -2)$ **D.** $(-2, -5)$

14. When the axes are rotated through 45°, the equation $2x^2 + 2xy + 3y^2 = 8$ becomes

A. $3X^2 - 2XY + 7Y^2 = 16$ **B.** $3X^2 + 2XY + 7Y^2 = 16$ **C.** $7X^2 + 2XY + 3Y^2 = -16$ **D.** $7X^2 + 2XY + 3Y^2 = 16$

15. When the axes are rotated through 90°, the equation $2x - 4y + 5 = 0$ becomes

A. $4X + 2Y + 5 = 0$ **B.** $4X + 2Y - 5 = 0$ **C.** $-4X - 2Y - 5 = 0$ **D.** $-2X + 4Y - 5 = 0$

16. The angle of rotation that removes the xy term from $5x^2 + 6xy - 3y^2 = 1$ is

A. $\tan^{-1}\frac{1}{2}$ **B.** $\frac{\pi}{4}$
C. $\tan^{-1}\frac{1}{3}$ **D.** $\frac{\pi}{3}$

17. The angle of rotation that removes the xy term from $3x^2 + 4\sqrt{3}xy - y^2 = 20$ is

A. $\frac{\pi}{6}$ **B.** $\frac{\pi}{4}$
C. $\frac{\pi}{3}$ **D.** $\tan^{-1}\frac{1}{3}$

18. To reduce the line $6x + 8y + 7 = 0$ to the form $X = \text{constant}$, the axes must be rotated through

A. $\tan^{-1}\left(-\frac{3}{4}\right)$ **B.** $\tan^{-1}\left(\frac{3}{4}\right)$
C. $\tan^{-1}\left(\frac{4}{3}\right)$ **D.** $\tan^{-1}\left(-\frac{4}{3}\right)$

19. To reduce the line $x - \sqrt{3}y - 1 = 0$ to the form $X = \text{constant}$, the axes must be rotated through

A. 60° **B.** -60° **C.** 30° **D.** -30°

20. The axes are rotated through 30° and $3x^2 - 4xy + 4y^2 - 4x - 7y - 3 = 0$ becomes $AX^2 + 2HXY + BY^2 + 2GX + 2FY + C = 0$. Then $(A - B)^2 + 4H^2 =$

A. 65 **B.** 9 **C.** 17 **D.** 1

21. The axes are rotated through an angle θ with $\cos\theta = \frac{4}{5}$, $\sin\theta = \frac{3}{5}$ and $-x^2 - xy + 3y^2 + 6x - 5y + 6 = 0$ becomes $AX^2 + 2HXY + BY^2 + 2GX + 2FY + C = 0$. Then $C =$

A. 12 **B.** -6 **C.** 8 **D.** 6

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22. The origin is shifted to $(4, -3)$ and the axes are then rotated through 30°. The new coordinates of $(-6, -6)$ are

A. $(-3\sqrt{3} - 3, 3 - 3\sqrt{3})$ **B.** $(5 - \frac{3\sqrt{3}}{2}, -5\sqrt{3} - \frac{3}{2})$

C. $(\frac{3}{2} - 5\sqrt{3}, -5 - \frac{3\sqrt{3}}{2})$ **D.** $(-5\sqrt{3} - \frac{3}{2}, 5 - \frac{3\sqrt{3}}{2})$

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23. The origin is shifted to $(-3, -4)$ and the axes are then rotated through an angle θ with $\cos\theta = \frac{4}{5}$, $\sin\theta = \frac{3}{5}$. The new coordinates of $(-3, 6)$ are

A. $(6, 8)$ **B.** $(8, 6)$

C. $(\frac{6}{5}, \frac{33}{5})$ **D.** $(-6, 8)$

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24. When the origin is shifted to $(-3, -2)$ and the axes are then rotated through $\tan^{-1}\frac{1}{3}$, the equation $-2x^2 - 6xy + 6y^2 - 24x + 6y - 46 = 0$ becomes

A. $-3X^2 + 7Y^2 = -16$ **B.** $-2X^2 + 6Y^2 = 16$ **C.** $-3X^2 + 7Y^2 = 16$ **D.** $7X^2 - 3Y^2 = 16$

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25. When the origin is shifted to $(-2, 1)$ and the axes are then rotated through $\tan^{-1}\frac{1}{2}$, the equation $-x^2 - 8xy + 5y^2 + 4x - 26y + 16 = 0$ becomes

A. $-3X^2 + 7Y^2 = -1$ **B.** $-3X^2 + 7Y^2 = 1$ **C.** $7X^2 - 3Y^2 = 1$ **D.** $-X^2 + 5Y^2 = 1$

Key sheet

1. A	**2.** C	**3.** A	**4.** D	**5.** B
6. A	**7.** D	**8.** B	**9.** A	**10.** C
11. D	**12.** C	**13.** B	**14.** D	**15.** B
16. C	**17.** A	**18.** C	**19.** B	**20.** C
21. D	**22.** D	**23.** A	**24.** C	**25.** B

Solutions

1. When the origin is shifted to $(6, -4)$, the new coordinates of $(-4, -7)$ are

$X = x - h,\ Y = y - k$

$X = -4 - (6) = -10,\ Y = -7 - (-4) = -3$

Ans: (A) $(-10, -3)$

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2. After the origin is shifted to $(2, 7)$, a point has new coordinates $(1, -6)$. Its original coordinates are

$x = X + h,\ y = Y + k$

$x = 1 + (2) = 3,\ y = -6 + (7) = 1$

Ans: (C) $(3, 1)$

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3. When the origin is shifted to $(3, 1)$, the equation $x^2 + y^2 - 6x - 2y - 3 = 0$ becomes

Put $x = X + 3,\ y = Y + 1$ in the equation.

Expanding and collecting terms: $X^2 + Y^2 - 13 = 0$

Ans: (A) $X^2 + Y^2 - 13 = 0$

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4. When the origin is shifted to $(-2, -4)$, the equation $-2x^2 + 2xy - 4y^2 - 2x - 7y + 6 = 0$ becomes

Put $x = X - 2,\ y = Y - 4$ in the equation.

Expanding and collecting terms: $-2X^2 + 2XY - 4Y^2 - 2X + 21Y - 18 = 0$

Ans: (D) $-2X^2 + 2XY - 4Y^2 - 2X + 21Y - 18 = 0$

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5. When the first degree terms of $-2x^2 + 6xy - 3y^2 + 10x - 12y - 10 = 0$ are removed by a translation, the constant term of the new equation is

$a = -2,\ h = 3,\ b = -3,\ g = 5,\ f = -6,\ c = -10$

Solve $ax + hy + g = 0$ and $hx + by + f = 0$: $-2x + 3y + 5 = 0$, $3x - 3y - 6 = 0$

So $(x_1, y_1) = (1, -1)$.

Constant $= gx_1 + fy_1 + c = (5)(1) + (-6)(-1) + (-10) = 1$

Ans: (B) 1

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6. When the first degree terms of $2x^2 - 2xy - 2y^2 - 20x - 10y + 12 = 0$ are removed by a translation, the constant term of the new equation is

$a = 2,\ h = -1,\ b = -2,\ g = -10,\ f = -5,\ c = 12$

Solve $ax + hy + g = 0$ and $hx + by + f = 0$: $2x - y - 10 = 0$, $-x - 2y - 5 = 0$

So $(x_1, y_1) = (3, -4)$.

Constant $= gx_1 + fy_1 + c = (-10)(3) + (-5)(-4) + (12) = 2$

Ans: (A) 2

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7. When the first degree terms of $-2x^2+6xy-y^2+4x-20y-8=0$ are removed by a translation, the constant term of the new equation is

$a=-2,\ h=3,\ b=-1,\ g=2,\ f=-10,\ c=-8$

Solve $ax+hy+g=0$ and $hx+by+f=0$: $-2x+3y+2=0$, $3x-y-10=0$

So $(x_1,y_1)=(4,2)$.

Constant $=gx_1+fy_1+c=(2)(4)+(-10)(2)+(-8)=-20$

Ans: (D) -20

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8. When the origin is shifted to $(2,1)$, the coefficient of Y in the new form of $5x^2-4xy-2y^2+8x-6y-8=0$ is

$2F=\left(\frac{\partial S}{\partial y}\right)_{(2,1)}=(-4)(2)+2(-2)(1)+(-6)=-18$

Ans: (B) -18

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9. When the origin is shifted to $(4,-1)$, the coefficient of Y in the new form of $6x^2+4xy+3y^2-4x-5=0$ is

$2F=\left(\frac{\partial S}{\partial y}\right)_{(4,-1)}=(4)(4)+2(3)(-1)+(0)=10$

Ans: (A) 10

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10. The equation $-3x^2+2xy-3y^2-24x+24y+k=0$ can be reduced to the form $aX^2+2hXY+bY^2=0$ by a translation when $k=$

Need $abc+2fgh-af^2-bg^2-ch^2=0$ with $c=k$: $8k+576=0$

$k=-72$

Ans: (C) -72

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11. The equation $3x^2+2xy+4y^2-22x+22y+k=0$ can be reduced to the form $aX^2+2hXY+bY^2=0$ by a translation when $k=$

Need $abc+2fgh-af^2-bg^2-ch^2=0$ with $c=k$: $11k-1089=0$

$k=99$

Ans: (D) 99

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12. The axes are rotated through an angle θ with $\cos\theta=\frac{5}{13}$, $\sin\theta=\frac{12}{13}$. The new coordinates of $(-1,2)$ are

$X=x\cos\theta+y\sin\theta=\frac{19}{13},\ Y=-x\sin\theta+y\cos\theta=\frac{22}{13}$

Ans: (C) $\left(\frac{19}{13},\frac{22}{13}\right)$

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13. The axes are rotated through 90° in the clockwise direction. The new coordinates of $(5,2)$ are

Clockwise: use $\theta=-90^\circ$.

$X=x\cos\theta+y\sin\theta=-2,\ Y=-x\sin\theta+y\cos\theta=5$

Ans: (B) $(-2,5)$

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14. When the axes are rotated through 45°, the equation $2x^2 + 2xy + 3y^2 = 8$ becomes

Put $x = X\cos\theta - Y\sin\theta = \frac{\sqrt{2}X}{2} - \frac{\sqrt{2}Y}{2},\ y = X\sin\theta + Y\cos\theta = \frac{\sqrt{2}X}{2} + \frac{\sqrt{2}Y}{2}$.

Simplifying: $7X^2 + 2XY + 3Y^2 = 16$

Ans: (D) $7X^2 + 2XY + 3Y^2 = 16$

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15. When the axes are rotated through 90°, the equation $2x - 4y + 5 = 0$ becomes

Put $x = X\cos\theta - Y\sin\theta = -Y,\ y = X\sin\theta + Y\cos\theta = X$.

Simplifying: $4X + 2Y - 5 = 0$

Ans: (B) $4X + 2Y - 5 = 0$

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16. The angle of rotation that removes the xy term from $5x^2 + 6xy - 3y^2 = 1$ is

$\tan 2\theta = \frac{2h}{a-b} = \frac{6}{5-(-3)} = \frac{3}{4}$

$\theta = \frac{1}{2}\tan^{-1}\left(\frac{3}{4}\right) = \tan^{-1}\frac{1}{3}$

Ans: (C) $\tan^{-1}\frac{1}{3}$

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17. The angle of rotation that removes the xy term from $3x^2 + 4\sqrt{3}xy - y^2 = 20$ is

$\tan 2\theta = \frac{2h}{a-b} = \frac{4\sqrt{3}}{3-(-1)} = \sqrt{3}$

$\theta = \frac{\pi}{6} = \frac{\pi}{6}$

Ans: (A) $\frac{\pi}{6}$

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18. To reduce the line $6x + 8y + 7 = 0$ to the form $X = \text{constant}$, the axes must be rotated through

Angle $= \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{4}{3}\right)$.

Ans: (C) $\tan^{-1}\left(\frac{4}{3}\right)$

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19. To reduce the line $x - \sqrt{3}y - 1 = 0$ to the form $X = \text{constant}$, the axes must be rotated through

Angle $= \tan^{-1}\left(\frac{b}{a}\right) = -60^\circ$.

Ans: (B) -60°

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20. The axes are rotated through 30° and $3x^2 - 4xy + 4y^2 - 4x - 7y - 3 = 0$ becomes $AX^2 + 2HXY + BY^2 + 2GX + 2FY + C = 0$. Then $(A - B)^2 + 4H^2 =$

$(A - B)^2 + 4H^2 = (a - b)^2 + 4h^2$ (an invariant under rotation).

$= 17$

Ans: (C) 17

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21. The axes are rotated through an angle θ with $\cos\theta = \frac{4}{5}$, $\sin\theta = \frac{3}{5}$ and $-x^2 - xy + 3y^2 + 6x - 5y + 6 = 0$ becomes $AX^2 + 2HXY + BY^2 + 2GX + 2FY + C = 0$. Then $C =$

$C = c$ (an invariant under rotation).

$= 6$

Ans: (D) 6

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22. The origin is shifted to $(4, -3)$ and the axes are then rotated through 30°. The new coordinates of $(-6, -6)$ are

$x - h = -10,\ y - k = -3$

$X = (x-h)\cos\theta + (y-k)\sin\theta = -5\sqrt{3} - \frac{3}{2}$

$Y = -(x-h)\sin\theta + (y-k)\cos\theta = 5 - \frac{3\sqrt{3}}{2}$

Ans: (D) $(-5\sqrt{3} - \frac{3}{2}, 5 - \frac{3\sqrt{3}}{2})$

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23. The origin is shifted to $(-3, -4)$ and the axes are then rotated through an angle θ with $\cos\theta = \frac{4}{5}$, $\sin\theta = \frac{3}{5}$. The new coordinates of $(-3, 6)$ are

$x - h = 0,\ y - k = 10$

$X = (x-h)\cos\theta + (y-k)\sin\theta = 6$

$Y = -(x-h)\sin\theta + (y-k)\cos\theta = 8$

Ans: (A) $(6, 8)$

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24. When the origin is shifted to $(-3, -2)$ and the axes are then rotated through $\tan^{-1}\frac{1}{3}$, the equation $-2x^2 - 6xy + 6y^2 - 24x + 6y - 46 = 0$ becomes

The XY terms cancel: $-3X^2 + 7Y^2 = 16$

Ans: (C) $-3X^2 + 7Y^2 = 16$

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25. When the origin is shifted to $(-2, 1)$ and the axes are then rotated through $\tan^{-1}\frac{1}{2}$, the equation $-x^2 - 8xy + 5y^2 + 4x - 26y + 16 = 0$ becomes

The XY terms cancel: $-3X^2 + 7Y^2 = 1$

Ans: (B) $-3X^2 + 7Y^2 = 1$
