EXPRESSWALA ACADEMY

# Maths 1B: Change of Axes

## MCQ Test, Set 02

Time: 30 minutes | Maximum marks: 25

Name: _____ Class: _____ Date: _____

1. There are 25 questions. Each has exactly one correct answer: choose A, B, C or D.
2. Each correct answer carries 1 mark. There is no negative marking.
3. The key sheet and worked solutions are at the end of this test.

**1.** After the origin is shifted to $(-6, -4)$, a point has new coordinates $(-5, 9)$. Its original coordinates are

**A.** $(-1, -13)$ **B.** $(1, 13)$ **C.** $(-11, 5)$ **D.** $(-9, 3)$

**2.** When the origin is shifted to $(-5, 6)$, the new coordinates of $(1, -3)$ are

**A.** $(-6, 9)$ **B.** $(-5, 2)$ **C.** $(6, -9)$ **D.** $(-4, 3)$

**3.** When the origin is shifted to $(4, 1)$, the equation $x^2 + y^2 - 8x - 2y + 4 = 0$ becomes

**A.** $X^2 + Y^2 - 16X - 4Y + 55 = 0$ **B.** $X^2 + Y^2 - 6X + 6Y + 5 = 0$

**C.** $X^2 + Y^2 - 11 = 0$ **D.** $X^2 + Y^2 - 13 = 0$

**4.** When the origin is shifted to $(3, -2)$, the equation $x^2 + y^2 - 6x + 4y + 9 = 0$ becomes

**A.** $X^2 + Y^2 - 12X + 8Y + 48 = 0$ **B.** $X^2 + Y^2 - 10X + 10Y + 46 = 0$

**C.** $X^2 + Y^2 - 4 = 0$ **D.** $X^2 + Y^2 - 2 = 0$

**5.** When the first degree terms of $4x^2 - 2xy - 2y^2 - 8x + 20y + 3 = 0$ are removed by a translation, the constant term of the new equation is

**A.** $38$ **B.** $3$ **C.** $-35$ **D.** $35$

**6.** To remove the first degree terms from $-x^2 - 3y^2 - 6x - 12y + 2 = 0$, the origin must be shifted to

**A.** $(-3, -2)$ **B.** $(3, 2)$ **C.** $(-2, -3)$ **D.** $(2, 3)$

**7.** To remove the first degree terms from $-x^2 + 2y^2 + 6x + 8y + 1 = 0$, the origin must be shifted to

**A.** $(2, -3)$ **B.** $(3, -2)$ **C.** $(-3, 2)$ **D.** $(-2, 3)$

**8.** When the origin is shifted to $(-2, 1)$, the coefficient of $X$ in the new form of $2x^2 + y^2 - 5x + 8y - 5 = 0$ is

**A.** $-13$ **B.** $-5$ **C.** $-12$ **D.** $13$

**9.** When the origin is shifted to $(-2, 1)$, the constant term of $5x^2 + 4y^2 - 7x - 3y - 1 = 0$ becomes

**A.** $-1$ **B.** $34$ **C.** $32$ **D.** $-34$

**10.** The equation $x^2 + 4xy + 2y^2 + 2x - 8y + k = 0$ can be reduced to the form $aX^2 + 2hXY + bY^2 = 0$ by a translation when $k =$

**A.** $-17$ **B.** $17$ **C.** $-16$ **D.** $-34$

---

**11.** For $2x^2 - 2xy - 3y^2 - 20x - 4y - 11 = 0$, the value of $abc + 2fgh - af^2 - bg^2 - ch^2$ is

**A.** $329$ **B.** $-329$ **C.** $66$ **D.** $289$

---

**12.** After the axes are rotated through $30^\circ$, a point has new coordinates $(2, 4)$. Its original coordinates are

**A.** $(\sqrt{3} + 2, -1 + 2\sqrt{3})$ **B.** $(2 - \sqrt{3}, -2\sqrt{3} - 1)$
**C.** $(1 + 2\sqrt{3}, -2 + \sqrt{3})$ **D.** $(-2 + \sqrt{3}, 1 + 2\sqrt{3})$

---

**13.** After the axes are rotated through an angle $\theta$ with $\cos\theta = \frac{3}{5}$, $\sin\theta = \frac{4}{5}$, a point has new coordinates $(-4, -5)$. Its original coordinates are

**A.** $(-\frac{31}{5}, \frac{8}{5})$ **B.** $(\frac{8}{5}, -\frac{31}{5})$
**C.** $(-\frac{32}{5}, \frac{1}{5})$ **D.** $(-\frac{8}{5}, \frac{31}{5})$

---

**14.** When the axes are rotated through $90^\circ$, the equation $x^2 + y^2 - 5x + 2y = 0$ becomes

**A.** $X^2 + Y^2 - 5X + 2Y = 0$ **B.** $X^2 + Y^2 + 2X + 5Y + 1 = 0$
**C.** $X^2 + Y^2 + 2X + 5Y = 0$ **D.** $X^2 + Y^2 - 2X - 5Y = 0$

---

**15.** When the axes are rotated through $45^\circ$, the equation $4x + 2y - 2 = 0$ becomes

**A.** $-\sqrt{2}X + 3\sqrt{2}Y - 2 = 0$ **B.** $\sqrt{2}X + 3\sqrt{2}Y - 2 = 0$
**C.** $3\sqrt{2}X - \sqrt{2}Y + 2 = 0$ **D.** $3\sqrt{2}X - \sqrt{2}Y - 2 = 0$

---

**16.** The angle of rotation that removes the $xy$ term from $6x^2 + 6xy - 2y^2 = 12$ is

**A.** $\frac{\pi}{4}$ **B.** $\frac{\pi}{6}$
**C.** $\tan^{-1}\frac{1}{3}$ **D.** $\frac{\pi}{3}$

---

**17.** The angle of rotation that removes the $xy$ term from $-2x^2 - 6xy + 6y^2 = 2$ is

**A.** $\frac{\pi}{6}$ **B.** $\tan^{-1}\frac{1}{3}$
**C.** $\tan^{-1}2$ **D.** $\tan^{-1}\frac{1}{2}$

---

**18.** To reduce the line $12x + 5y + 10 = 0$ to the form $X = \text{constant}$, the axes must be rotated through

**A.** $\tan^{-1}\left(-\frac{12}{5}\right)$ **B.** $\tan^{-1}\left(\frac{12}{5}\right)$
**C.** $\tan^{-1}\left(-\frac{5}{12}\right)$ **D.** $\tan^{-1}\left(\frac{5}{12}\right)$

---

**19.** To reduce the line $6x + 8y - 15 = 0$ to the form $Y = \text{constant}$, the axes must be rotated through

**A.** $\tan^{-1}\left(-\frac{3}{4}\right)$ **B.** $\tan^{-1}\left(-\frac{4}{3}\right)$
**C.** $\tan^{-1}\left(\frac{4}{3}\right)$ **D.** $\tan^{-1}\left(\frac{3}{4}\right)$

---

**20.** When the axes are rotated, which of the following is NOT an invariant of $2x^2 + xy - y^2 - 3x + 4y + 9 = 0$?

**A.** $ab$ **B.** $a + b$ **C.** $h^2 - ab$ **D.** $(a - b)^2 + 4h^2$

---

**21.** When the axes are rotated, which of the following is NOT an invariant of $-2x^2 - 3xy + 3y^2 + 4x + 5y + 8 = 0$?

**A.** $c$ **B.** $ab$ **C.** $h^2 - ab$ **D.** $(a-b)^2 + 4h^2$

**22.** The origin is shifted to $(-1, -2)$ and the axes are then rotated through an angle $\theta$ with $\cos\theta = \frac{4}{5}$, $\sin\theta = \frac{3}{5}$. The new coordinates of $(-5, 4)$ are

**A.** $(\frac{36}{5}, \frac{2}{5})$
**B.** $(\frac{2}{5}, \frac{36}{5})$
**C.** $(-\frac{34}{5}, \frac{12}{5})$
**D.** $(-\frac{8}{5}, \frac{31}{5})$

**23.** The origin is shifted to $(5, 1)$ and the axes are then rotated through $30^\circ$. The new coordinates of $(-3, 5)$ are

**A.** $(2\sqrt{3} + 4, 2 - 4\sqrt{3})$
**B.** $(-4\sqrt{3} - 2, -4 + 2\sqrt{3})$
**C.** $(2 - 4\sqrt{3}, 2\sqrt{3} + 4)$
**D.** $(\frac{5}{2} - \frac{3\sqrt{3}}{2}, \frac{3}{2} + \frac{5\sqrt{3}}{2})$

**24.** When the origin is shifted to $(4, 1)$ and the axes are then rotated through $45^\circ$, the equation $-2x^2 + 6xy - 2y^2 + 10x - 20y - 12 = 0$ becomes

**A.** $-2X^2 - 2Y^2 = 2$ **B.** $X^2 - 5Y^2 = 2$ **C.** $X^2 - 5Y^2 = -2$ **D.** $-5X^2 + Y^2 = 2$

**25.** When the origin is shifted to $(1, 2)$ and the axes are then rotated through $\tan^{-1}\frac{1}{3}$, the equation $4x^2 + 6xy - 4y^2 - 20x + 10y - 2 = 0$ becomes

**A.** $5X^2 - 5Y^2 = -2$ **B.** $-5X^2 + 5Y^2 = 2$ **C.** $4X^2 - 4Y^2 = 2$ **D.** $5X^2 - 5Y^2 = 2$

## Key sheet

| | | | | |
|---|---|---|---|---|
| **1.** C | **2.** C | **3.** D | **4.** C | **5.** D |
| **6.** A | **7.** B | **8.** A | **9.** B | **10.** A |
| **11.** A | **12.** D | **13.** B | **14.** C | **15.** D |
| **16.** C | **17.** B | **18.** D | **19.** A | **20.** A |
| **21.** B | **22.** B | **23.** C | **24.** B | **25.** D |

## Solutions

**1.** After the origin is shifted to $(-6, -4)$, a point has new coordinates $(-5, 9)$. Its original coordinates are

$x = X + h,\ y = Y + k$

$x = -5 + (-6) = -11,\ y = 9 + (-4) = 5$

**Ans: (C)** $(-11, 5)$

---

**2.** When the origin is shifted to $(-5, 6)$, the new coordinates of $(1, -3)$ are

$X = x - h,\ Y = y - k$

$X = 1 - (-5) = 6,\ Y = -3 - (6) = -9$

**Ans: (C)** $(6, -9)$

---

**3.** When the origin is shifted to $(4, 1)$, the equation $x^2 + y^2 - 8x - 2y + 4 = 0$ becomes

Put $x = X + 4,\ y = Y + 1$ in the equation.

Expanding and collecting terms: $X^2 + Y^2 - 13 = 0$

**Ans: (D)** $X^2 + Y^2 - 13 = 0$

---

**4.** When the origin is shifted to $(3, -2)$, the equation $x^2 + y^2 - 6x + 4y + 9 = 0$ becomes

Put $x = X + 3,\ y = Y - 2$ in the equation.

Expanding and collecting terms: $X^2 + Y^2 - 4 = 0$

**Ans: (C)** $X^2 + Y^2 - 4 = 0$

---

**5.** When the first degree terms of $4x^2 - 2xy - 2y^2 - 8x + 20y + 3 = 0$ are removed by a translation, the constant term of the new equation is

$a = 4,\ h = -1,\ b = -2,\ g = -4,\ f = 10,\ c = 3$

Solve $ax + hy + g = 0$ and $hx + by + f = 0$: $4x - y - 4 = 0$, $-x - 2y + 10 = 0$

So $(x_1, y_1) = (2, 4)$.

Constant $= gx_1 + fy_1 + c = (-4)(2) + (10)(4) + (3) = 35$

**Ans: (D)** $35$

---

**6.** To remove the first degree terms from $-x^2 - 3y^2 - 6x - 12y + 2 = 0$, the origin must be shifted to

$a = -1,\ h = 0,\ b = -3,\ g = -3,\ f = -6,\ c = 2$

Solve $ax + hy + g = 0$ and $hx + by + f = 0$: $-x - 3 = 0$, $-3y - 6 = 0$

So $(x_1, y_1) = (-3, -2)$.

**Ans: (A)** $(-3, -2)$

---

**7.** To remove the first degree terms from $-x^2 + 2y^2 + 6x + 8y + 1 = 0$, the origin must be shifted to

$a = -1,\ h = 0,\ b = 2,\ g = 3,\ f = 4,\ c = 1$

Solve $ax + hy + g = 0$ and $hx + by + f = 0$: $-x + 3 = 0$, $2y + 4 = 0$

So $(x_1, y_1) = (3, -2)$.

**Ans: (B)** $(3, -2)$

---

**8.** When the origin is shifted to $(-2, 1)$, the coefficient of $X$ in the new form of $2x^2 + y^2 - 5x + 8y - 5 = 0$ is

$2G = \left(\frac{\partial S}{\partial x}\right)_{(-2,1)} = 2(2)(-2) + (0)(1) + (-5) = -13$

**Ans: (A)** $-13$

---

**9.** When the origin is shifted to $(-2, 1)$, the constant term of $5x^2 + 4y^2 - 7x - 3y - 1 = 0$ becomes

$C = S(-2, 1) = 34$

**Ans: (B)** $34$

---

**10.** The equation $x^2 + 4xy + 2y^2 + 2x - 8y + k = 0$ can be reduced to the form $aX^2 + 2hXY + bY^2 = 0$ by a translation when $k =$

Need $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ with $c = k$: $-2k - 34 = 0$

$k = -17$

**Ans: (A)** $-17$

---

**11.** For $2x^2 - 2xy - 3y^2 - 20x - 4y - 11 = 0$, the value of $abc + 2fgh - af^2 - bg^2 - ch^2$ is

$a = 2,\ h = -1,\ b = -3,\ g = -10,\ f = -2,\ c = -11$

$abc + 2fgh - af^2 - bg^2 - ch^2 = 329$

**Ans: (A)** $329$

---

**12.** After the axes are rotated through $30^\circ$, a point has new coordinates $(2, 4)$. Its original coordinates are

$x = X\cos\theta - Y\sin\theta = -2 + \sqrt{3},\ y = X\sin\theta + Y\cos\theta = 1 + 2\sqrt{3}$

**Ans: (D)** $(-2 + \sqrt{3}, 1 + 2\sqrt{3})$

---

**13.** After the axes are rotated through an angle $\theta$ with $\cos\theta = \frac{3}{5}$, $\sin\theta = \frac{4}{5}$, a point has new coordinates $(-4, -5)$. Its original coordinates are

$x = X\cos\theta - Y\sin\theta = \frac{8}{5},\ y = X\sin\theta + Y\cos\theta = -\frac{31}{5}$

**Ans: (B)** $\left(\frac{8}{5}, -\frac{31}{5}\right)$

---

**14.** When the axes are rotated through $90^\circ$, the equation $x^2 + y^2 - 5x + 2y = 0$ becomes

Put $x = X\cos\theta - Y\sin\theta = -Y,\ y = X\sin\theta + Y\cos\theta = X.$

Simplifying: $X^2 + Y^2 + 2X + 5Y = 0$

**Ans: (C)** $X^2 + Y^2 + 2X + 5Y = 0$

---

**15.** When the axes are rotated through $45^\circ$, the equation $4x + 2y - 2 = 0$ becomes

Put $x = X\cos\theta - Y\sin\theta = \frac{\sqrt{2}X}{2} - \frac{\sqrt{2}Y}{2},\ y = X\sin\theta + Y\cos\theta = \frac{\sqrt{2}X}{2} + \frac{\sqrt{2}Y}{2}.$

Simplifying: $3\sqrt{2}X - \sqrt{2}Y - 2 = 0$

**Ans: (D)** $3\sqrt{2}X - \sqrt{2}Y - 2 = 0$

---

**16.** The angle of rotation that removes the $xy$ term from $6x^2 + 6xy - 2y^2 = 12$ is

$\tan 2\theta = \frac{2h}{a-b} = \frac{6}{6-(-2)} = \frac{3}{4}$

$\theta = \frac{1}{2}\tan^{-1}\left(\frac{3}{4}\right) = \tan^{-1}\frac{1}{3}$

**Ans: (C)** $\tan^{-1}\frac{1}{3}$

---

**17.** The angle of rotation that removes the $xy$ term from $-2x^2 - 6xy + 6y^2 = 2$ is

$\tan 2\theta = \frac{2h}{a-b} = \frac{-6}{-2-(6)} = \frac{3}{4}$

$\theta = \frac{1}{2}\tan^{-1}\left(\frac{3}{4}\right) = \tan^{-1}\frac{1}{3}$

**Ans: (B)** $\tan^{-1}\frac{1}{3}$

---

**18.** To reduce the line $12x + 5y + 10 = 0$ to the form $X = \text{constant}$, the axes must be rotated through

Angle $= \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{5}{12}\right)$.

**Ans: (D)** $\tan^{-1}\left(\frac{5}{12}\right)$

---

**19.** To reduce the line $6x + 8y - 15 = 0$ to the form $Y = \text{constant}$, the axes must be rotated through

Angle $= \tan^{-1}\left(-\frac{a}{b}\right) = \tan^{-1}\left(-\frac{3}{4}\right)$.

**Ans: (A)** $\tan^{-1}\left(-\frac{3}{4}\right)$

---

**20.** When the axes are rotated, which of the following is NOT an invariant of $2x^2 + xy - y^2 - 3x + 4y + 9 = 0$?

The invariants are $a + b$, $h^2 - ab$, $c$, $(a-b)^2 + 4h^2$, $2fgh - af^2 - bg^2$ and $abc + 2fgh - af^2 - bg^2 - ch^2$.

$ab$ is not one of them.

**Ans: (A)** $ab$

---

**21.** When the axes are rotated, which of the following is NOT an invariant of $-2x^2 - 3xy + 3y^2 + 4x + 5y + 8 = 0$?

The invariants are $a + b$, $h^2 - ab$, $c$, $(a-b)^2 + 4h^2$, $2fgh - af^2 - bg^2$ and $abc + 2fgh - af^2 - bg^2 - ch^2$.

$ab$ is not one of them.

**Ans: (B)** $ab$

---

**22.** The origin is shifted to $(-1, -2)$ and the axes are then rotated through an angle $\theta$ with $\cos\theta = \frac{4}{5}$, $\sin\theta = \frac{3}{5}$. The new coordinates of $(-5, 4)$ are

$x - h = -4,\ y - k = 6$

$X = (x-h)\cos\theta + (y-k)\sin\theta = \frac{2}{5}$

$Y = -(x-h)\sin\theta + (y-k)\cos\theta = \frac{36}{5}$

**Ans: (B)** $\left(\frac{2}{5}, \frac{36}{5}\right)$

---

**23.** The origin is shifted to $(5, 1)$ and the axes are then rotated through $30^\circ$. The new coordinates of $(-3, 5)$ are

$x - h = -8,\ y - k = 4$

$X = (x-h)\cos\theta + (y-k)\sin\theta = 2 - 4\sqrt{3}$

$Y = -(x-h)\sin\theta + (y-k)\cos\theta = 2\sqrt{3} + 4$

**Ans: (C)** $(2 - 4\sqrt{3}, 2\sqrt{3} + 4)$

---

**24.** When the origin is shifted to $(4, 1)$ and the axes are then rotated through $45^\circ$, the equation $-2x^2 + 6xy - 2y^2 + 10x - 20y - 12 = 0$ becomes

The $XY$ terms cancel: $X^2 - 5Y^2 = 2$

**Ans: (B)** $X^2 - 5Y^2 = 2$

---

**25.** When the origin is shifted to $(1, 2)$ and the axes are then rotated through $\tan^{-1}\frac{1}{3}$, the equation $4x^2 + 6xy - 4y^2 - 20x + 10y - 2 = 0$ becomes

The $XY$ terms cancel: $5X^2 - 5Y^2 = 2$

**Ans: (D)** $5X^2 - 5Y^2 = 2$

---