EXPRESSWALA ACADEMY

Maths 1A: Addition of Vectors

Question Paper, Set 08

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Find the unit vector in the direction opposite to $\bar{a} = 12\bar{i} - 15\bar{j} + 16\bar{k}$. [1 mark]

2. If $\bar{a} = -3\bar{i} + \bar{k}$ and $\bar{b} = 7\bar{i} - 4\bar{k}$, find the vector of magnitude 15 in the direction of $\bar{a} + \bar{b}$. [2 marks]

3. If $\overline{OC} = -2\bar{i} + 5\bar{j} - 3\bar{k}$ and $\overline{OD} = 2\bar{i} - \bar{j} + 3\bar{k}$, find $\overline{CD}$. [1 mark]

4. If $(\bar{a}, \bar{b}) = 110^\circ$, find $(\bar{a}, 4\bar{b})$. [1 mark]

5. Find the distance $|\overline{AB}|$ between $A(-4, 2, -4)$ and $B(5, 3, 2)$. [1 mark]

6. A vector of length 51 has direction cosines $(-\frac{15}{17}, \frac{8}{17}, 0)$. Find the vector. [2 marks]

7. Find p if the vectors $-18\bar{i} + p\bar{j} - 12\bar{k}$ and $-6\bar{i} - 3\bar{j} - 4\bar{k}$ are collinear. [2 marks]

8. Find p so that the points $A(-4, 0, -1)$, $B(-1, 1, -1)$ and $C(2, p, -1)$ are collinear. [3 marks]

9. If $\bar{a} = 2\bar{j} - \bar{k}$, $\bar{b} = -2\bar{i} - 2\bar{j} - 2\bar{k}$ and $\bar{c} = 3\bar{i} + 3\bar{j} + \bar{k}$, find scalars x, y, z such that $-7\bar{i} - 11\bar{j} + \bar{k} = x\bar{a} + y\bar{b} + z\bar{c}$. [5 marks]

10. Find p if the points $A(4, 3, 2)$, $B(6, 3, -1)$, $C(7, 6, 2)$ and $D(14, p, -4)$ are coplanar. [5 marks]

11. The midpoint of AB is $(5, \frac{5}{2}, -\frac{3}{2})$ and $A = (7, 1, 1)$. Find B. [1 mark]

12. Find the point that divides the segment joining $A(2, -1, 4)$ and $B(5, 4, 6)$ externally in the ratio $4 : 3$. [2 marks]

13. In what ratio does the point $C(6, -25, -19)$ divide the segment joining $A(2, -5, -5)$ and $B(0, 5, 2)$? [3 marks]

14. Find the incentre of the triangle whose vertices are $A(1, 7, 4)$, $B(1, 1, 4)$ and $C(5, 4, 4)$. [5 marks]

15. Find the vector and cartesian equations of the line through $A(-2, 2, 3)$ parallel to $-\bar{i} + 3\bar{j} - \bar{k}$. Also find the point of the line for $t = 3$. [3 marks]

16. Write the vector equation of the plane through $-\bar{i} - \bar{j} + 4\bar{k}$ parallel to $2\bar{i} - \bar{j} - 2\bar{k}$ and $2\bar{i} + 4\bar{j} - 2\bar{k}$, and find its point with $s = 1$, $t = 2$. [2 marks]

17. Find a vector along the external bisector of the angle between $\bar{a} = -19\bar{i} + 8\bar{j} - 4\bar{k}$ and $\bar{b} = -8\bar{i} + 6\bar{j}$. [2 marks]

18. Using vectors, prove that the midpoints of the sides of any quadrilateral form a parallelogram. [5 marks]

19. Find the components of $\bar{a} = 3\bar{j} - 4\bar{k}$ when the coordinate axes are rotated about the y-axis through 30°. [3 marks]

20. A straight line cuts the lines BC, CA, AB of triangle ABC at D, E, F. If $\frac{BD}{DC} = -\frac{2}{5}$ and $\frac{CE}{EA} = \frac{3}{5}$ (signed ratios), find $\frac{AF}{FB}$. [1 mark]

Key sheet

Q	Answer
1	$-\frac{1}{25}(12\bar{i}-15\bar{j}+16\bar{k})$
2	$12\bar{i}-9\bar{k}$
3	$4\bar{i}-6\bar{j}+6\bar{k}$
4	110°
5	$\sqrt{118}$
6	$-45\bar{i}+24\bar{j}$
7	$p=-9$
8	$p=2$
9	$\bar{r}=-2\bar{a}-\bar{b}-3\bar{c}$ ($x=-2$, $y=-1$, $z=-3$)
10	$p=9$
11	$(3,4,-4)$
12	$(14,19,12)$
13	Externally in the ratio $2:3$
14	$(\frac{5}{2},4,4)$
15	$\bar{r}=(-2\bar{i}+2\bar{j}+3\bar{k})+t(-\bar{i}+3\bar{j}-\bar{k})$; $\frac{x+2}{-1}=\frac{y-2}{3}=\frac{z-3}{-1}$; point $(-5,11,0)$
16	$\bar{r}=(-\bar{i}-\bar{j}+4\bar{k})+s(2\bar{i}-\bar{j}-2\bar{k})+t(2\bar{i}+4\bar{j}-2\bar{k})$; point $(5,6,-2)$
17	$-\frac{11}{105}\bar{i}-\frac{23}{105}\bar{j}-\frac{4}{21}\bar{k}$ (or any multiple)
18	Proved (see the working)
19	$(2,3,-2\sqrt{3})$
20	$\frac{25}{6}$

Solutions

1. Find the unit vector in the direction opposite to $\bar{a} = 12\bar{i} - 15\bar{j} + 16\bar{k}$. [1]

$|\bar{a}| = \sqrt{144 + 225 + 256} = \sqrt{625} = 25$

Unit vector opposite to $\bar{a} = -\frac{\bar{a}}{|\bar{a}|} = -\frac{1}{25}(12\bar{i} - 15\bar{j} + 16\bar{k})$

Ans: $-\frac{1}{25}(12\bar{i} - 15\bar{j} + 16\bar{k})$

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2. If $\bar{a} = -3\bar{i} + \bar{k}$ and $\bar{b} = 7\bar{i} - 4\bar{k}$, find the vector of magnitude 15 in the direction of $\bar{a} + \bar{b}$. [2]

$\bar{a} + \bar{b} = 4\bar{i} - 3\bar{k}$

$|\bar{a} + \bar{b}| = \sqrt{16 + 0 + 9} = 5$

Required vector $= \frac{15(\bar{a}+\bar{b})}{|\bar{a}+\bar{b}|} = \frac{15}{5}(4\bar{i} - 3\bar{k}) = 12\bar{i} - 9\bar{k}$

Ans: $12\bar{i} - 9\bar{k}$

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3. If $\overline{OC} = -2\bar{i} + 5\bar{j} - 3\bar{k}$ and $\overline{OD} = 2\bar{i} - \bar{j} + 3\bar{k}$, find $\overline{CD}$. [1]

$\overline{CD} = \overline{OD} - \overline{OC}$

$= (2\bar{i} - \bar{j} + 3\bar{k}) - (-2\bar{i} + 5\bar{j} - 3\bar{k}) = 4\bar{i} - 6\bar{j} + 6\bar{k}$

Ans: $4\bar{i} - 6\bar{j} + 6\bar{k}$

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4. If $(\bar{a}, \bar{b}) = 110^\circ$, find $(\bar{a}, 4\bar{b})$. [1]

The scalars 1 and 4 have the same sign.

$(\bar{a}, 4\bar{b}) = (\bar{a}, \bar{b}) = 110^\circ$

Ans: 110°

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5. Find the distance $|\overline{AB}|$ between $A(-4, 2, -4)$ and $B(5, 3, 2)$. [1]

$\overline{AB} = 9\bar{i} + \bar{j} + 6\bar{k}$

$|\overline{AB}| = \sqrt{81 + 1 + 36} = \sqrt{118}$

Ans: $\sqrt{118}$

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6. A vector of length 51 has direction cosines $(-\frac{15}{17}, \frac{8}{17}, 0)$. Find the vector. [2]

$x = lr,\ y = mr,\ z = nr$ with $r = 51$

$x = -45,\ y = 24,\ z = 0$

Vector $= -45\bar{i} + 24\bar{j}$

Ans: $-45\bar{i} + 24\bar{j}$

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7. Find p if the vectors $-18\bar{i} + p\bar{j} - 12\bar{k}$ and $-6\bar{i} - 3\bar{j} - 4\bar{k}$ are collinear. [2]

Collinear vectors have proportional components.

$\frac{-12}{-4} = \frac{p}{-3}$

$\Rightarrow p = -9$

Ans: $p = -9$

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8. Find p so that the points $A(-4, 0, -1)$, $B(-1, 1, -1)$ and $C(2, p, -1)$ are collinear. [3]

$\overline{AB} = (3, 1, 0)$

$\overline{AC} = (6, p, 0)$

For collinear points the components are proportional:

$\frac{6}{3} = \frac{p}{1}$

$\Rightarrow p = 2$

Ans: $p = 2$

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9. If $\bar{a} = 2\bar{j} - \bar{k}$, $\bar{b} = -2\bar{i} - 2\bar{j} - 2\bar{k}$ and $\bar{c} = 3\bar{i} + 3\bar{j} + \bar{k}$, find scalars x, y, z such that $-7\bar{i} - 11\bar{j} + \bar{k} = x\bar{a} + y\bar{b} + z\bar{c}$. [5]

Let $-7\bar{i} - 11\bar{j} + \bar{k} = x(2\bar{j} - \bar{k}) + y(-2\bar{i} - 2\bar{j} - 2\bar{k}) + z(3\bar{i} + 3\bar{j} + \bar{k})$.

Comparing the coefficients of $\bar{i}, \bar{j}, \bar{k}$:

$-2y + 3z = -7$

$2x - 2y + 3z = -11$

$-x - 2y + z = 1$

Solving by Cramer's rule:

$$\Delta = \begin{vmatrix} 0 & -2 & 3 \\ 2 & -2 & 3 \\ -1 & -2 & 1 \end{vmatrix} = -8$$

$$\Delta_1 = \begin{vmatrix} -7 & -2 & 3 \\ -11 & -2 & 3 \\ 1 & -2 & 1 \end{vmatrix} = 16, \quad x = \frac{\Delta_1}{\Delta} = -2$$

$$\Delta_2 = \begin{vmatrix} 0 & -7 & 3 \\ 2 & -11 & 3 \\ -1 & 1 & 1 \end{vmatrix} = 8, \quad y = \frac{\Delta_2}{\Delta} = -1$$

$$\Delta_3 = \begin{vmatrix} 0 & -2 & -7 \\ 2 & -2 & -11 \\ -1 & -2 & 1 \end{vmatrix} = 24, \quad z = \frac{\Delta_3}{\Delta} = -3$$

$\therefore \bar{r} = -2\bar{a} - \bar{b} - 3\bar{c}$

Ans: $\bar{r} = -2\bar{a} - \bar{b} - 3\bar{c}$ **(**$x = -2$**,** $y = -1$**,** $z = -3$**)**

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10. Find p if the points $A(4, 3, 2)$, $B(6, 3, -1)$, $C(7, 6, 2)$ and $D(14, p, -4)$ are coplanar. [5]

$\overline{AB} = (2, 0, -3)$, $\overline{AC} = (3, 3, 0)$, $\overline{AD} = (10, p - 3, -6)$

For coplanar points these vectors are linearly dependent:

$$\begin{vmatrix} 2 & 0 & -3 \\ 3 & 3 & 0 \\ 10 & p-3 & -6 \end{vmatrix} = 0$$

$\Rightarrow 81 - 9p = 0$

$\Rightarrow p = 9$

Ans: $p = 9$

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11. The midpoint of AB is $(5, \frac{5}{2}, -\frac{3}{2})$ and $A = (7, 1, 1)$. Find B. [1]

$\frac{\bar{a} + \bar{b}}{2} = \bar{m} \Rightarrow \bar{b} = 2\bar{m} - \bar{a}$

$= (10, 5, -3) - (7, 1, 1) = (3, 4, -4)$

Ans: $(3, 4, -4)$

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12. Find the point that divides the segment joining $A(2,-1,4)$ and $B(5,4,6)$ externally in the ratio $4:3$. [2]

$\bar{r} = \frac{m\bar{b}-n\bar{a}}{m-n}$
$= \frac{4(5,4,6)-3(2,-1,4)}{4-3}$
$= \frac{(14,19,12)}{1} = (14,19,12)$
Ans: $(14,19,12)$

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13. In what ratio does the point $C(6,-25,-19)$ divide the segment joining $A(2,-5,-5)$ and $B(0,5,2)$? [3]

Let C divide AB in the ratio $\lambda:1$. Then $\bar{c} = \frac{\lambda\bar{b}+\bar{a}}{\lambda+1}$.
Comparing one coordinate: $6 = \frac{0\lambda+2}{\lambda+1}$
$\Rightarrow \lambda = -\frac{2}{3}$
$\lambda < 0$: C divides AB externally in the ratio $2:3$.
Ans: Externally in the ratio $2:3$

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14. Find the incentre of the triangle whose vertices are $A(1,7,4)$, $B(1,1,4)$ and $C(5,4,4)$. [5]

$a = BC = \sqrt{16+9+0} = 5$
$b = CA = \sqrt{16+9+0} = 5$
$c = AB = \sqrt{0+36+0} = 6$
$\text{Incentre } I = \frac{a\bar{a}+b\bar{b}+c\bar{c}}{a+b+c}$
$= \frac{5(1,7,4)+5(1,1,4)+6(5,4,4)}{16}$
$= \frac{(40,64,64)}{16} = (\frac{5}{2},4,4)$
Ans: $(\frac{5}{2},4,4)$

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15. Find the vector and cartesian equations of the line through $A(-2,2,3)$ parallel to $-\bar{i}+3\bar{j}-\bar{k}$. Also find the point of the line for $t=3$. [3]

$\bar{r} = \bar{a} + t\bar{b}$
$\bar{r} = (-2\bar{i}+2\bar{j}+3\bar{k}) + t(-\bar{i}+3\bar{j}-\bar{k})$
$\text{Cartesian: } \frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$
$\frac{x+2}{-1} = \frac{y-2}{3} = \frac{z-3}{-1}$
$t=3:\ \bar{r} = (-5,11,0)$
Ans: $\bar{r} = (-2\bar{i}+2\bar{j}+3\bar{k}) + t(-\bar{i}+3\bar{j}-\bar{k})$; $\frac{x+2}{-1} = \frac{y-2}{3} = \frac{z-3}{-1}$; **point** $(-5,11,0)$

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16. Write the vector equation of the plane through $-\bar{i}-\bar{j}+4\bar{k}$ parallel to $2\bar{i}-\bar{j}-2\bar{k}$ and $2\bar{i}+4\bar{j}-2\bar{k}$, and find its point with $s=1$, $t=2$. [2]

$\bar{r} = \bar{a} + s\bar{b} + t\bar{c}$
$\bar{r} = (-\bar{i}-\bar{j}+4\bar{k}) + s(2\bar{i}-\bar{j}-2\bar{k}) + t(2\bar{i}+4\bar{j}-2\bar{k})$
$s=1,\ t=2:\ \bar{r} = (5,6,-2)$
Ans: $\bar{r} = (-\bar{i}-\bar{j}+4\bar{k}) + s(2\bar{i}-\bar{j}-2\bar{k}) + t(2\bar{i}+4\bar{j}-2\bar{k})$; **point** $(5,6,-2)$

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17. Find a vector along the external bisector of the angle between $\bar{a} = -19\bar{i} + 8\bar{j} - 4\bar{k}$ and $\bar{b} = -8\bar{i} + 6\bar{j}$. [2]

$|\bar{a}| = \sqrt{361 + 64 + 16} = 21, \quad |\bar{b}| = \sqrt{64 + 36 + 0} = 10$
$\frac{\bar{a}}{|\bar{a}|} - \frac{\bar{b}}{|\bar{b}|} = \frac{1}{21}(-19\bar{i} + 8\bar{j} - 4\bar{k}) - \frac{1}{10}(-8\bar{i} + 6\bar{j})$
$= -\frac{11}{105}\bar{i} - \frac{23}{105}\bar{j} - \frac{4}{21}\bar{k}$
Any non-zero multiple of this vector is also along the external bisector.

Ans: $-\frac{11}{105}\bar{i} - \frac{23}{105}\bar{j} - \frac{4}{21}\bar{k}$ **(or any multiple)**

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18. Using vectors, prove that the midpoints of the sides of any quadrilateral form a parallelogram. [5]

Let $ABCD$ have position vectors $\bar{a}, \bar{b}, \bar{c}, \bar{d}$, and P, Q, R, S be the midpoints of AB, BC, CD, DA.
$\overline{PQ} = \frac{\bar{b}+\bar{c}}{2} - \frac{\bar{a}+\bar{b}}{2} = \frac{\bar{c}-\bar{a}}{2}$
$\overline{SR} = \frac{\bar{c}+\bar{d}}{2} - \frac{\bar{d}+\bar{a}}{2} = \frac{\bar{c}-\bar{a}}{2}$
$\therefore \overline{PQ} = \overline{SR}$
Opposite sides PQ and SR are equal and parallel, so $PQRS$ is a parallelogram.

Ans: Proved (see the working)

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19. Find the components of $\bar{a} = 3\bar{j} - 4\bar{k}$ when the coordinate axes are rotated about the y-axis through $30°$. [3]

About the y-axis through α the new components are $(-a_3 \sin\alpha + a_1 \cos\alpha,\ a_2,\ a_3 \cos\alpha + a_1 \sin\alpha)$.
$\cos 30° = \frac{\sqrt{3}}{2}, \quad \sin 30° = \frac{1}{2}$
Substituting $a_1 = 0$, $a_2 = 3$, $a_3 = -4$:
$(-(-4) \cdot \frac{1}{2} + 0 \cdot \frac{\sqrt{3}}{2},\ 3,\ (-4) \cdot \frac{\sqrt{3}}{2} + 0 \cdot \frac{1}{2})$
$= (2, 3, -2\sqrt{3})$

Ans: $(2, 3, -2\sqrt{3})$

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20. A straight line cuts the lines BC, CA, AB of triangle ABC at D, E, F. If $\frac{BD}{DC} = -\frac{2}{5}$ and $\frac{CE}{EA} = \frac{3}{5}$ (signed ratios), find $\frac{AF}{FB}$. [1]

By Menelaus' theorem $\frac{BD}{DC} \cdot \frac{CE}{EA} \cdot \frac{AF}{FB} = -1$
$\frac{AF}{FB} = \frac{-1}{(-\frac{2}{5})(\frac{3}{5})} = \frac{25}{6}$

Ans: $\frac{25}{6}$
