EXPRESSWALA ACADEMY

Maths 1A: Addition of Vectors

Question Paper, Set 07

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Find the unit vector in the direction of $\bar{a} = -6\bar{i} + 7\bar{j} - 6\bar{k}$. [1 mark]

2. Find the vectors of magnitude 22 that are parallel to $\bar{a} = -6\bar{i} + 6\bar{j} - 7\bar{k}$. [2 marks]

3. The position vectors of L and M are $\bar{i} + 3\bar{j} + 3\bar{k}$ and $\bar{i} - 2\bar{j} + 5\bar{k}$. Find $\overline{LM}$. [1 mark]

4. If $(\bar{a}, \bar{b}) = 140^\circ$, find $(-2\bar{a}, 4\bar{b})$. [1 mark]

5. Find the magnitude of $\overline{OP}$, where $P = (0, -2, -4)$. [1 mark]

6. Find the direction cosines of the vector $6\bar{j} + 8\bar{k}$. [2 marks]

7. Find p if the vectors $-10\bar{i} + 8\bar{j} + p\bar{k}$ and $-5\bar{i} + 4\bar{j} + 3\bar{k}$ are collinear. [2 marks]

8. Show that the points $A(1, -2, -2)$, $B(3, 1, -3)$ and $C(7, 7, -5)$ are collinear. [3 marks]

9. Express $\bar{r} = 14\bar{i} + \bar{j} - 11\bar{k}$ as a linear combination of $\bar{a} = -2\bar{i} + \bar{j} + 2\bar{k}$, $\bar{b} = 2\bar{i} - 2\bar{k}$ and $\bar{c} = -2\bar{i} - \bar{j} + \bar{k}$. [5 marks]

10. Show that the points $A(2, 4, -3)$, $B(4, 2, -2)$, $C(5, 1, -4)$ and $D(3, 3, -5)$ are coplanar. [5 marks]

11. Find the midpoint of the segment joining $A(-1, 5, 1)$ and $B(-4, -4, 1)$. [1 mark]

12. Find the point that divides the segment joining $A(-4, 2, -3)$ and $B(-4, 2, 6)$ internally in the ratio $1 : 2$. [2 marks]

13. Find the points of trisection of the segment joining $A(0, 4, -3)$ and $B(6, 4, 3)$. [3 marks]

14. Find the incentre of the triangle whose vertices are $A(6, 4, 2)$, $B(-2, 4, 2)$ and $C(-2, -11, 2)$. [5 marks]

15. Find the vector and cartesian equations of the line through $A(1, 2, 0)$ and $B(0, 4, -1)$. [3 marks]

16. Write the vector equation of the plane through the points $A(3, -1, 3)$, $B(0, 2, 2)$ and $C(0, 1, -1)$, and find its point with $s = 3$, $t = 1$. [2 marks]

17. Find a vector along the internal bisector of the angle between $\bar{a} = -2\bar{i} - \bar{j} - 2\bar{k}$ and $\bar{b} = -16\bar{i} + 12\bar{j} - 15\bar{k}$. [2 marks]

18. Using vectors, prove that the line joining the midpoints of two sides of a triangle is parallel to the third side and half of it. [5 marks]

19. Find the components of $\bar{a} = \bar{j} + 5\bar{k}$ when the coordinate axes are rotated about the x-axis through 30°. [3 marks]

20. In triangle ABC, the lines joining a point P to A, B, C meet BC, CA, AB at D, E, F. If $\frac{BD}{DC} = \frac{5}{2}$ and $\frac{CE}{EA} = \frac{2}{5}$, find $\frac{AF}{FB}$. [1 mark]

Key sheet

Q	Answer
1	$\frac{1}{11}(-6\bar{i}+7\bar{j}-6\bar{k})$
2	$\pm(-12\bar{i}+12\bar{j}-14\bar{k})$
3	$-5\bar{j}+2\bar{k}$
4	40°
5	$2\sqrt{5}$
6	$(0, \frac{3}{5}, \frac{4}{5})$
7	$p=6$
8	$\overline{AC}=3\,\overline{AB}$, so A, B, C are collinear
9	$\bar{r}=-2\bar{a}+2\bar{b}-3\bar{c}$ ($x=-2$, $y=2$, $z=-3$)
10	The determinant is 0, so A, B, C, D are coplanar
11	$(-\frac{5}{2}, \frac{1}{2}, 1)$
12	$(-4, 2, 0)$
13	$(2, 4, -1)$ and $(4, 4, 1)$
14	$(1, 1, 2)$
15	$\bar{r}=(1-t)(\bar{i}+2\bar{j})+t(4\bar{j}-\bar{k})$; $\frac{x-1}{-1}=\frac{y-2}{2}=\frac{z}{-1}$
16	$\bar{r}=(1-s-t)(3\bar{i}-\bar{j}+3\bar{k})+s(2\bar{j}+2\bar{k})+t(\bar{j}-\bar{k})$; point $(-9, 10, -4)$
17	$-\frac{98}{75}\bar{i}+\frac{11}{75}\bar{j}-\frac{19}{15}\bar{k}$ (or any multiple)
18	Proved (see the working)
19	$(0, \frac{\sqrt{3}}{2}+\frac{5}{2}, -\frac{1}{2}+\frac{5\sqrt{3}}{2})$
20	1

Solutions

1. Find the unit vector in the direction of $\bar{a} = -6\bar{i} + 7\bar{j} - 6\bar{k}$. [1]

$|\bar{a}| = \sqrt{36 + 49 + 36} = \sqrt{121} = 11$

$\text{Unit vector} = \frac{\bar{a}}{|\bar{a}|} = \frac{1}{11}(-6\bar{i} + 7\bar{j} - 6\bar{k})$

Ans: $\frac{1}{11}(-6\bar{i} + 7\bar{j} - 6\bar{k})$

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2. Find the vectors of magnitude 22 that are parallel to $\bar{a} = -6\bar{i} + 6\bar{j} - 7\bar{k}$. [2]

$|\bar{a}| = \sqrt{36 + 36 + 49} = 11$

$\text{Required vectors} = \pm\frac{22\,\bar{a}}{|\bar{a}|} = \pm\frac{22}{11}(-6\bar{i} + 6\bar{j} - 7\bar{k})$

$= \pm(-12\bar{i} + 12\bar{j} - 14\bar{k})$

Ans: $\pm(-12\bar{i} + 12\bar{j} - 14\bar{k})$

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3. The position vectors of L and M are $\bar{i} + 3\bar{j} + 3\bar{k}$ and $\bar{i} - 2\bar{j} + 5\bar{k}$. Find $\overline{LM}$. [1]

$\overline{LM} = \overline{OM} - \overline{OL}$

$= (\bar{i} - 2\bar{j} + 5\bar{k}) - (\bar{i} + 3\bar{j} + 3\bar{k}) = -5\bar{j} + 2\bar{k}$

Ans: $-5\bar{j} + 2\bar{k}$

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4. If $(\bar{a}, \bar{b}) = 140^\circ$, find $(-2\bar{a}, 4\bar{b})$. [1]

The scalars -2 and 4 have opposite signs.

$(-2\bar{a}, 4\bar{b}) = 180^\circ - 140^\circ = 40^\circ$

Ans: 40°

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5. Find the magnitude of $\overline{OP}$, where $P = (0, -2, -4)$. [1]

$\overline{OP} = -2\bar{j} - 4\bar{k}$

$|\overline{OP}| = \sqrt{0 + 4 + 16} = 2\sqrt{5}$

Ans: $2\sqrt{5}$

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6. Find the direction cosines of the vector $6\bar{j} + 8\bar{k}$. [2]

$r = \sqrt{0 + 36 + 64} = 10$

$l = \frac{x}{r},\ m = \frac{y}{r},\ n = \frac{z}{r}$

$\text{Direction cosines} = (0, \frac{3}{5}, \frac{4}{5})$

Ans: $(0, \frac{3}{5}, \frac{4}{5})$

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7. Find p if the vectors $-10\bar{i} + 8\bar{j} + p\bar{k}$ and $-5\bar{i} + 4\bar{j} + 3\bar{k}$ are collinear. [2]

Collinear vectors have proportional components.

$\frac{-10}{-5} = \frac{p}{3}$

$\Rightarrow p = 6$

Ans: $p = 6$

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8. Show that the points $A(1,-2,-2)$, $B(3,1,-3)$ and $C(7,7,-5)$ are collinear. [3]

$\overline{AB} = \overline{OB} - \overline{OA} = 2\bar{i} + 3\bar{j} - \bar{k}$

$\overline{AC} = \overline{OC} - \overline{OA} = 6\bar{i} + 9\bar{j} - 3\bar{k}$

$\overline{AC} = 3\,\overline{AB}$

$\overline{AB}$ and $\overline{AC}$ are parallel and have the common point A, so A, B, C are collinear.

Ans: $\overline{AC} = 3\,\overline{AB}$, so A, B, C are collinear

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9. Express $\bar{r} = 14\bar{i} + \bar{j} - 11\bar{k}$ as a linear combination of $\bar{a} = -2\bar{i} + \bar{j} + 2\bar{k}$, $\bar{b} = 2\bar{i} - 2\bar{k}$ and $\bar{c} = -2\bar{i} - \bar{j} + \bar{k}$. [5]

Let $14\bar{i} + \bar{j} - 11\bar{k} = x(-2\bar{i} + \bar{j} + 2\bar{k}) + y(2\bar{i} - 2\bar{k}) + z(-2\bar{i} - \bar{j} + \bar{k})$.

Comparing the coefficients of $\bar{i}, \bar{j}, \bar{k}$:

$-2x + 2y - 2z = 14$

$x - z = 1$

$2x - 2y + z = -11$

Solving by Cramer's rule:

$$\Delta = \begin{vmatrix} -2 & 2 & -2 \\ 1 & 0 & -1 \\ 2 & -2 & 1 \end{vmatrix} = 2$$

$$\Delta_1 = \begin{vmatrix} 14 & 2 & -2 \\ 1 & 0 & -1 \\ -11 & -2 & 1 \end{vmatrix} = -4, \quad x = \frac{\Delta_1}{\Delta} = -2$$

$$\Delta_2 = \begin{vmatrix} -2 & 14 & -2 \\ 1 & 1 & -1 \\ 2 & -11 & 1 \end{vmatrix} = 4, \quad y = \frac{\Delta_2}{\Delta} = 2$$

$$\Delta_3 = \begin{vmatrix} -2 & 2 & 14 \\ 1 & 0 & 1 \\ 2 & -2 & -11 \end{vmatrix} = -6, \quad z = \frac{\Delta_3}{\Delta} = -3$$

$\therefore \bar{r} = -2\bar{a} + 2\bar{b} - 3\bar{c}$

Ans: $\bar{r} = -2\bar{a} + 2\bar{b} - 3\bar{c}$ ($x = -2$, $y = 2$, $z = -3$)

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10. Show that the points $A(2,4,-3)$, $B(4,2,-2)$, $C(5,1,-4)$ and $D(3,3,-5)$ are coplanar. [5]

$\overline{AB} = 2\bar{i} - 2\bar{j} + \bar{k}$, $\overline{AC} = 3\bar{i} - 3\bar{j} - \bar{k}$, $\overline{AD} = \bar{i} - \bar{j} - 2\bar{k}$

The four points are coplanar when these three vectors are coplanar (linearly dependent).

$$\begin{vmatrix} 2 & -2 & 1 \\ 3 & -3 & -1 \\ 1 & -1 & -2 \end{vmatrix} = 2(5) - (-2)(-5) + 1(0)$$

$= 0$

The determinant is zero, so $\overline{AB}, \overline{AC}, \overline{AD}$ are coplanar and A, B, C, D are coplanar.

Ans: The determinant is 0, so A, B, C, D are coplanar

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11. Find the midpoint of the segment joining $A(-1,5,1)$ and $B(-4,-4,1)$. [1]

$\text{Midpoint} = \frac{\bar{a}+\bar{b}}{2} = \frac{(-5,1,2)}{2}$

$= (-\frac{5}{2}, \frac{1}{2}, 1)$

Ans: $(-\frac{5}{2}, \frac{1}{2}, 1)$

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12. Find the point that divides the segment joining $A(-4, 2, -3)$ and $B(-4, 2, 6)$ internally in the ratio $1 : 2$. [2]

$\bar{r} = \frac{m\bar{b}+n\bar{a}}{m+n}$
$= \frac{1(-4,2,6)+2(-4,2,-3)}{1+2}$
$= \frac{(-12,6,0)}{3} = (-4, 2, 0)$

Ans: $(-4, 2, 0)$

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13. Find the points of trisection of the segment joining $A(0, 4, -3)$ and $B(6, 4, 3)$. [3]

The points of trisection divide AB internally in the ratios $1 : 2$ and $2 : 1$.
$\frac{1(6,4,3)+2(0,4,-3)}{3} = \frac{(6,12,-3)}{3} = (2, 4, -1)$
$\frac{2(6,4,3)+1(0,4,-3)}{3} = \frac{(12,12,3)}{3} = (4, 4, 1)$

Ans: $(2, 4, -1)$ **and** $(4, 4, 1)$

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14. Find the incentre of the triangle whose vertices are $A(6, 4, 2)$, $B(-2, 4, 2)$ and $C(-2, -11, 2)$. [5]

$a = BC = \sqrt{0 + 225 + 0} = 15$
$b = CA = \sqrt{64 + 225 + 0} = 17$
$c = AB = \sqrt{64 + 0 + 0} = 8$
Incentre $I = \frac{a\bar{a}+b\bar{b}+c\bar{c}}{a+b+c}$
$= \frac{15(6,4,2)+17(-2,4,2)+8(-2,-11,2)}{40}$
$= \frac{(40,40,80)}{40} = (1, 1, 2)$

Ans: $(1, 1, 2)$

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15. Find the vector and cartesian equations of the line through $A(1, 2, 0)$ and $B(0, 4, -1)$. [3]

$\bar{r} = (1-t)\bar{a} + t\bar{b} = (1-t)(\bar{i} + 2\bar{j}) + t(4\bar{j} - \bar{k})$
$= (1-t)\bar{i} + (2+2t)\bar{j} + (-t)\bar{k}$
Direction $\overline{AB} = (-1, 2, -1)$
$\frac{x-1}{-1} = \frac{y-2}{2} = \frac{z}{-1}$

Ans: $\bar{r} = (1-t)(\bar{i} + 2\bar{j}) + t(4\bar{j} - \bar{k})$; $\frac{x-1}{-1} = \frac{y-2}{2} = \frac{z}{-1}$

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16. Write the vector equation of the plane through the points $A(3, -1, 3)$, $B(0, 2, 2)$ and $C(0, 1, -1)$, and find its point with $s = 3$, $t = 1$. [2]

$\bar{r} = (1-s-t)\bar{a} + s\bar{b} + t\bar{c}$
$\bar{r} = (1-s-t)(3\bar{i} - \bar{j} + 3\bar{k}) + s(2\bar{j} + 2\bar{k}) + t(\bar{j} - \bar{k})$
$s = 3,\ t = 1:\ \bar{r} = -3(3, -1, 3) + 3(0, 2, 2) + 1(0, 1, -1) = (-9, 10, -4)$

Ans: $\bar{r} = (1-s-t)(3\bar{i} - \bar{j} + 3\bar{k}) + s(2\bar{j} + 2\bar{k}) + t(\bar{j} - \bar{k})$; **point** $(-9, 10, -4)$

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17. Find a vector along the internal bisector of the angle between $\bar{a} = -2\bar{i} - \bar{j} - 2\bar{k}$ and $\bar{b} = -16\bar{i} + 12\bar{j} - 15\bar{k}$. [2]

$|\bar{a}| = \sqrt{4 + 1 + 4} = 3, \quad |\bar{b}| = \sqrt{256 + 144 + 225} = 25$
$\frac{\bar{a}}{|\bar{a}|} + \frac{\bar{b}}{|\bar{b}|} = \frac{1}{3}(-2\bar{i} - \bar{j} - 2\bar{k}) + \frac{1}{25}(-16\bar{i} + 12\bar{j} - 15\bar{k})$
$= -\frac{98}{75}\bar{i} + \frac{11}{75}\bar{j} - \frac{19}{15}\bar{k}$
Any non-zero multiple of this vector is also along the internal bisector.

Ans: $-\frac{98}{75}\bar{i} + \frac{11}{75}\bar{j} - \frac{19}{15}\bar{k}$ **(or any multiple)**

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18. Using vectors, prove that the line joining the midpoints of two sides of a triangle is parallel to the third side and half of it. [5]

Let $\bar{a}, \bar{b}, \bar{c}$ be the position vectors of A, B, C, and D, E the midpoints of AB, AC.
$\overline{OD} = \frac{\bar{a}+\bar{b}}{2}$, $\overline{OE} = \frac{\bar{a}+\bar{c}}{2}$
$\overline{DE} = \frac{\bar{a}+\bar{c}}{2} - \frac{\bar{a}+\bar{b}}{2} = \frac{\bar{c}-\bar{b}}{2} = \frac{1}{2}\overline{BC}$
So $\overline{DE}$ is a positive multiple of $\overline{BC}$: DE is parallel to BC.
$|\overline{DE}| = \frac{1}{2}|\overline{BC}|$

Ans: Proved (see the working)

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19. Find the components of $\bar{a} = \bar{j} + 5\bar{k}$ when the coordinate axes are rotated about the x-axis through 30°. [3]

About the x-axis through α the new components are $(a_1,\ a_2\cos\alpha + a_3\sin\alpha,\ -a_2\sin\alpha + a_3\cos\alpha)$.
$\cos 30^\circ = \frac{\sqrt{3}}{2}, \quad \sin 30^\circ = \frac{1}{2}$
Substituting $a_1 = 0$, $a_2 = 1$, $a_3 = 5$:
$(0,\ 1\cdot\frac{\sqrt{3}}{2} + 5\cdot\frac{1}{2},\ -1\cdot\frac{1}{2} + 5\cdot\frac{\sqrt{3}}{2})$
$= (0, \frac{\sqrt{3}}{2} + \frac{5}{2}, -\frac{1}{2} + \frac{5\sqrt{3}}{2})$

Ans: $(0, \frac{\sqrt{3}}{2} + \frac{5}{2}, -\frac{1}{2} + \frac{5\sqrt{3}}{2})$

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20. In triangle ABC, the lines joining a point P to A, B, C meet BC, CA, AB at D, E, F. If $\frac{BD}{DC} = \frac{5}{2}$ and $\frac{CE}{EA} = \frac{2}{5}$, find $\frac{AF}{FB}$. [1]

By Ceva's theorem $\frac{BD}{DC}\cdot\frac{CE}{EA}\cdot\frac{AF}{FB} = 1$
$\frac{AF}{FB} = \frac{1}{\frac{5}{2}\times\frac{2}{5}} = 1$

Ans: 1
