EXPRESSWALA ACADEMY

Maths 1A: Addition of Vectors

Question Paper, Set 05

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Find the unit vector in the direction opposite to $\bar{a} = -11\bar{i} - 2\bar{j} + 10\bar{k}$. [1 mark]

2. Find the vectors of magnitude 30 that are parallel to $\bar{a} = -10\bar{i} - 11\bar{j} - 2\bar{k}$. [2 marks]

3. The position vectors of A and B are $\bar{i} - 4\bar{j} + 4\bar{k}$ and $-4\bar{i} + 5\bar{j} - 2\bar{k}$. Find $\overline{AB}$. [1 mark]

4. If $(\bar{a}, \bar{b}) = 145^\circ$, find $(-\bar{a}, -3\bar{b})$. [1 mark]

5. Find the magnitude of $\overline{OP}$, where $P = (5, 4, 2)$. [1 mark]

6. A vector of length 34 has direction cosines $(-\frac{8}{17}, \frac{15}{17}, 0)$. Find the vector. [2 marks]

7. Find p if the vectors $-6\bar{i} + 9\bar{j} + p\bar{k}$ and $-2\bar{i} + 3\bar{j} + 5\bar{k}$ are collinear. [2 marks]

8. Show that the points $A(-2, -3, 5)$, $B(-2, -5, 6)$ and $C(-2, -9, 8)$ are collinear. [3 marks]

9. If $\bar{a} = -2\bar{i} + 3\bar{j} + 2\bar{k}$, $\bar{b} = -2\bar{i} + 3\bar{j} + 3\bar{k}$ and $\bar{c} = -\bar{i} + 2\bar{j}$, find scalars x, y, z such that $5\bar{i} - 6\bar{j} - 11\bar{k} = x\bar{a} + y\bar{b} + z\bar{c}$. [5 marks]

10. Find p if the points $A(0, 2, -2)$, $B(-3, 1, -1)$, $C(-3, 3, 1)$ and $D(0, p, 2)$ are coplanar. [5 marks]

11. Find the midpoint of the segment joining $A(4, 2, 6)$ and $B(-1, 4, 6)$. [1 mark]

12. Find the point that divides the segment joining $A(-2, 2, 3)$ and $B(2, 6, -5)$ internally in the ratio $3 : 1$. [2 marks]

13. In what ratio does the point $C(13, 9, 14)$ divide the segment joining $A(-5, -1, -4)$ and $B(4, 4, 5)$? [3 marks]

14. Find the incentre of the triangle whose vertices are $A(-3, 12, 0)$, $B(-3, 4, 0)$ and $C(-3, 4, 6)$. [5 marks]

15. Find the vector and cartesian equations of the line through $A(-4, -2, 3)$ parallel to $4\bar{i} + 4\bar{j} + 4\bar{k}$. Also find the point of the line for $t = 3$. [3 marks]

16. Write the vector equation of the plane through $2\bar{i} + \bar{j} + 3\bar{k}$ parallel to $-3\bar{i} + 3\bar{j} + 2\bar{k}$ and $4\bar{i} + 4\bar{j} + 2\bar{k}$, and find its point with $s = 3$, $t = 1$. [2 marks]

17. Find a vector along the internal bisector of the angle between $\bar{a} = 2\bar{i} + \bar{j} - 2\bar{k}$ and $\bar{b} = -9\bar{i} - 2\bar{j} + 6\bar{k}$. [2 marks]

18. $ABCDEF$ is a regular hexagon with centre O. Prove that $\overline{AB} + \overline{AC} + \overline{AD} + \overline{AE} + \overline{AF} = 3\,\overline{AD} = 6\,\overline{AO}$. [5 marks]

19. Find the components of $\bar{a} = -3\bar{i} - 4\bar{j} + 5\bar{k}$ when the coordinate axes are rotated about the y-axis through 45°. [3 marks]

20. In triangle ABC, the lines joining a point P to A, B, C meet BC, CA, AB at D, E, F. If $\frac{BD}{DC} = \frac{2}{3}$ and $\frac{CE}{EA} = \frac{3}{4}$, find $\frac{AF}{FB}$. [1 mark]

Key sheet

Q	Answer
1	$-\frac{1}{15}(-11\bar{i}-2\bar{j}+10\bar{k})$
2	$\pm(-20\bar{i}-22\bar{j}-4\bar{k})$
3	$-5\bar{i}+9\bar{j}-6\bar{k}$
4	145°
5	$3\sqrt{5}$
6	$-16\bar{i}+30\bar{j}$
7	$p=15$
8	$\overline{AC}=3\,\overline{AB}$, so A, B, C are collinear
9	$\bar{r}=-\bar{a}-3\bar{b}+3\bar{c}$ $(x=-1,\ y=-3,\ z=3)$
10	$p=6$
11	$(\frac{3}{2}, 3, 6)$
12	$(1, 5, -3)$
13	Externally in the ratio $2:1$
14	$(-3, 6, 2)$
15	$\bar{r}=(-4\bar{i}-2\bar{j}+3\bar{k})+t(4\bar{i}+4\bar{j}+4\bar{k})$; $\frac{x+4}{4}=\frac{y+2}{4}=\frac{z-3}{4}$; point $(8, 10, 15)$
16	$\bar{r}=(2\bar{i}+\bar{j}+3\bar{k})+s(-3\bar{i}+3\bar{j}+2\bar{k})+t(4\bar{i}+4\bar{j}+2\bar{k})$; point $(-3, 14, 11)$
17	$-\frac{5}{33}\bar{i}+\frac{5}{33}\bar{j}-\frac{4}{33}\bar{k}$ (or any multiple)
18	Proved (see the working)
19	$(-4\sqrt{2}, -4, \sqrt{2})$
20	2

Solutions

1. Find the unit vector in the direction opposite to $\bar{a} = -11\bar{i} - 2\bar{j} + 10\bar{k}$. [1]

$|\bar{a}| = \sqrt{121 + 4 + 100} = \sqrt{225} = 15$

$\text{Unit vector opposite to } \bar{a} = -\frac{\bar{a}}{|\bar{a}|} = -\frac{1}{15}(-11\bar{i} - 2\bar{j} + 10\bar{k})$

Ans: $-\frac{1}{15}(-11\bar{i} - 2\bar{j} + 10\bar{k})$

---

2. Find the vectors of magnitude 30 that are parallel to $\bar{a} = -10\bar{i} - 11\bar{j} - 2\bar{k}$. [2]

$|\bar{a}| = \sqrt{100 + 121 + 4} = 15$

$\text{Required vectors} = \pm\frac{30\,\bar{a}}{|\bar{a}|} = \pm\frac{30}{15}(-10\bar{i} - 11\bar{j} - 2\bar{k})$

$= \pm(-20\bar{i} - 22\bar{j} - 4\bar{k})$

Ans: $\pm(-20\bar{i} - 22\bar{j} - 4\bar{k})$

---

3. The position vectors of A and B are $\bar{i} - 4\bar{j} + 4\bar{k}$ and $-4\bar{i} + 5\bar{j} - 2\bar{k}$. Find $\overline{AB}$. [1]

$\overline{AB} = \overline{OB} - \overline{OA}$

$= (-4\bar{i} + 5\bar{j} - 2\bar{k}) - (\bar{i} - 4\bar{j} + 4\bar{k}) = -5\bar{i} + 9\bar{j} - 6\bar{k}$

Ans: $-5\bar{i} + 9\bar{j} - 6\bar{k}$

---

4. If $(\bar{a}, \bar{b}) = 145^\circ$, find $(-\bar{a}, -3\bar{b})$. [1]

The scalars -1 and -3 have the same sign.

$(-\bar{a}, -3\bar{b}) = (\bar{a}, \bar{b}) = 145^\circ$

Ans: 145°

---

5. Find the magnitude of $\overline{OP}$, where $P = (5, 4, 2)$. [1]

$\overline{OP} = 5\bar{i} + 4\bar{j} + 2\bar{k}$

$|\overline{OP}| = \sqrt{25 + 16 + 4} = 3\sqrt{5}$

Ans: $3\sqrt{5}$

---

6. A vector of length 34 has direction cosines $(-\frac{8}{17}, \frac{15}{17}, 0)$. Find the vector. [2]

$x = lr,\ y = mr,\ z = nr \text{ with } r = 34$

$x = -16,\ y = 30,\ z = 0$

$\text{Vector} = -16\bar{i} + 30\bar{j}$

Ans: $-16\bar{i} + 30\bar{j}$

---

7. Find p if the vectors $-6\bar{i} + 9\bar{j} + p\bar{k}$ and $-2\bar{i} + 3\bar{j} + 5\bar{k}$ are collinear. [2]

Collinear vectors have proportional components.

$\frac{-6}{-2} = \frac{p}{5}$

$\Rightarrow p = 15$

Ans: $p = 15$

---

8. Show that the points $A(-2,-3,5)$, $B(-2,-5,6)$ and $C(-2,-9,8)$ are collinear. [3]

$\overline{AB} = \overline{OB} - \overline{OA} = -2\bar{j} + \bar{k}$

$\overline{AC} = \overline{OC} - \overline{OA} = -6\bar{j} + 3\bar{k}$

$\overline{AC} = 3\,\overline{AB}$

$\overline{AB}$ and $\overline{AC}$ are parallel and have the common point A, so A, B, C are collinear.

Ans: $\overline{AC} = 3\,\overline{AB}$**, so** A, B, C **are collinear**

---

9. If $\bar{a} = -2\bar{i} + 3\bar{j} + 2\bar{k}$, $\bar{b} = -2\bar{i} + 3\bar{j} + 3\bar{k}$ and $\bar{c} = -\bar{i} + 2\bar{j}$, find scalars x, y, z such that $5\bar{i} - 6\bar{j} - 11\bar{k} = x\bar{a} + y\bar{b} + z\bar{c}$. [5]

Let $5\bar{i} - 6\bar{j} - 11\bar{k} = x(-2\bar{i} + 3\bar{j} + 2\bar{k}) + y(-2\bar{i} + 3\bar{j} + 3\bar{k}) + z(-\bar{i} + 2\bar{j})$.

Comparing the coefficients of $\bar{i}, \bar{j}, \bar{k}$:

$-2x - 2y - z = 5$

$3x + 3y + 2z = -6$

$2x + 3y = -11$

Solving by Cramer's rule:

$$\Delta = \begin{vmatrix} -2 & -2 & -1 \\ 3 & 3 & 2 \\ 2 & 3 & 0 \end{vmatrix} = 1$$

$$\Delta_1 = \begin{vmatrix} 5 & -2 & -1 \\ -6 & 3 & 2 \\ -11 & 3 & 0 \end{vmatrix} = -1, \quad x = \frac{\Delta_1}{\Delta} = -1$$

$$\Delta_2 = \begin{vmatrix} -2 & 5 & -1 \\ 3 & -6 & 2 \\ 2 & -11 & 0 \end{vmatrix} = -3, \quad y = \frac{\Delta_2}{\Delta} = -3$$

$$\Delta_3 = \begin{vmatrix} -2 & -2 & 5 \\ 3 & 3 & -6 \\ 2 & 3 & -11 \end{vmatrix} = 3, \quad z = \frac{\Delta_3}{\Delta} = 3$$

$\therefore \bar{r} = -\bar{a} - 3\bar{b} + 3\bar{c}$

Ans: $\bar{r} = -\bar{a} - 3\bar{b} + 3\bar{c}$ **(**$x = -1$**,** $y = -3$**,** $z = 3$**)**

---

10. Find p if the points $A(0,2,-2)$, $B(-3,1,-1)$, $C(-3,3,1)$ and $D(0,p,2)$ are coplanar. [5]

$\overline{AB} = (-3,-1,1)$, $\overline{AC} = (-3,1,3)$, $\overline{AD} = (0, p-2, 4)$

For coplanar points these vectors are linearly dependent:

$$\begin{vmatrix} -3 & -1 & 1 \\ -3 & 1 & 3 \\ 0 & p-2 & 4 \end{vmatrix} = 0$$

$\Rightarrow 6p - 36 = 0$

$\Rightarrow p = 6$

Ans: $p = 6$

---

11. Find the midpoint of the segment joining $A(4,2,6)$ and $B(-1,4,6)$. [1]

$\text{Midpoint} = \frac{\bar{a}+\bar{b}}{2} = \frac{(3,6,12)}{2}$

$= (\frac{3}{2}, 3, 6)$

Ans: $(\frac{3}{2}, 3, 6)$

---

12. Find the point that divides the segment joining $A(-2, 2, 3)$ and $B(2, 6, -5)$ internally in the ratio $3 : 1$. [2]

$\bar{r} = \frac{m\bar{b}+n\bar{a}}{m+n}$
$= \frac{3(2,6,-5)+1(-2,2,3)}{3+1}$
$= \frac{(4,20,-12)}{4} = (1, 5, -3)$

Ans: $(1, 5, -3)$

---

13. In what ratio does the point $C(13, 9, 14)$ divide the segment joining $A(-5, -1, -4)$ and $B(4, 4, 5)$? [3]

Let C divide AB in the ratio $\lambda : 1$. Then $\bar{c} = \frac{\lambda\bar{b}+\bar{a}}{\lambda+1}$.
Comparing one coordinate: $13 = \frac{4\lambda+(-5)}{\lambda+1}$
$\Rightarrow \lambda = -2$
$\lambda < 0$: C divides AB externally in the ratio $2 : 1$.

Ans: Externally in the ratio $2 : 1$

---

14. Find the incentre of the triangle whose vertices are $A(-3, 12, 0)$, $B(-3, 4, 0)$ and $C(-3, 4, 6)$. [5]

$a = BC = \sqrt{0 + 0 + 36} = 6$
$b = CA = \sqrt{0 + 64 + 36} = 10$
$c = AB = \sqrt{0 + 64 + 0} = 8$
$\text{Incentre } I = \frac{a\bar{a}+b\bar{b}+c\bar{c}}{a+b+c}$
$= \frac{6(-3,12,0)+10(-3,4,0)+8(-3,4,6)}{24}$
$= \frac{(-72,144,48)}{24} = (-3, 6, 2)$

Ans: $(-3, 6, 2)$

---

15. Find the vector and cartesian equations of the line through $A(-4, -2, 3)$ parallel to $4\bar{i} + 4\bar{j} + 4\bar{k}$. Also find the point of the line for $t = 3$. [3]

$\bar{r} = \bar{a} + t\bar{b}$
$\bar{r} = (-4\bar{i} - 2\bar{j} + 3\bar{k}) + t(4\bar{i} + 4\bar{j} + 4\bar{k})$
$\text{Cartesian: } \frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$
$\frac{x+4}{4} = \frac{y+2}{4} = \frac{z-3}{4}$
$t = 3: \ \bar{r} = (8, 10, 15)$

Ans: $\bar{r} = (-4\bar{i} - 2\bar{j} + 3\bar{k}) + t(4\bar{i} + 4\bar{j} + 4\bar{k})$; $\frac{x+4}{4} = \frac{y+2}{4} = \frac{z-3}{4}$; **point** $(8, 10, 15)$

---

16. Write the vector equation of the plane through $2\bar{i} + \bar{j} + 3\bar{k}$ parallel to $-3\bar{i} + 3\bar{j} + 2\bar{k}$ and $4\bar{i} + 4\bar{j} + 2\bar{k}$, and find its point with $s = 3$, $t = 1$. [2]

$\bar{r} = \bar{a} + s\bar{b} + t\bar{c}$
$\bar{r} = (2\bar{i} + \bar{j} + 3\bar{k}) + s(-3\bar{i} + 3\bar{j} + 2\bar{k}) + t(4\bar{i} + 4\bar{j} + 2\bar{k})$
$s = 3, \ t = 1: \ \bar{r} = (-3, 14, 11)$

Ans: $\bar{r} = (2\bar{i} + \bar{j} + 3\bar{k}) + s(-3\bar{i} + 3\bar{j} + 2\bar{k}) + t(4\bar{i} + 4\bar{j} + 2\bar{k})$; **point** $(-3, 14, 11)$

---

17. Find a vector along the internal bisector of the angle between $\bar{a} = 2\bar{i} + \bar{j} - 2\bar{k}$ and $\bar{b} = -9\bar{i} - 2\bar{j} + 6\bar{k}$. [2]

$|\bar{a}| = \sqrt{4+1+4} = 3, \quad |\bar{b}| = \sqrt{81+4+36} = 11$

$\frac{\bar{a}}{|\bar{a}|} + \frac{\bar{b}}{|\bar{b}|} = \frac{1}{3}(2\bar{i} + \bar{j} - 2\bar{k}) + \frac{1}{11}(-9\bar{i} - 2\bar{j} + 6\bar{k})$

$= -\frac{5}{33}\bar{i} + \frac{5}{33}\bar{j} - \frac{4}{33}\bar{k}$

Any non-zero multiple of this vector is also along the internal bisector.

Ans: $-\frac{5}{33}\bar{i} + \frac{5}{33}\bar{j} - \frac{4}{33}\bar{k}$ **(or any multiple)**

---

18. $ABCDEF$ is a regular hexagon with centre O. Prove that $\overline{AB} + \overline{AC} + \overline{AD} + \overline{AE} + \overline{AF} = 3\,\overline{AD} = 6\,\overline{AO}$. [5]

Let $\overline{AB} = \bar{a}$ and $\overline{BC} = \bar{b}$.

$\overline{AC} = \bar{a} + \bar{b},\ \overline{AD} = 2\bar{b},\ \overline{AE} = \overline{AD} + \overline{DE} = 2\bar{b} - \bar{a},\ \overline{AF} = \overline{AE} + \overline{EF} = \bar{b} - \bar{a}$

$\text{Sum} = \bar{a} + (\bar{a} + \bar{b}) + 2\bar{b} + (2\bar{b} - \bar{a}) + (\bar{b} - \bar{a}) = 6\bar{b}$

$6\bar{b} = 3(2\bar{b}) = 3\,\overline{AD}$

O is the midpoint of AD, so $\overline{AD} = 2\,\overline{AO}$ and $3\,\overline{AD} = 6\,\overline{AO}$.

Ans: Proved (see the working)

---

19. Find the components of $\bar{a} = -3\bar{i} - 4\bar{j} + 5\bar{k}$ when the coordinate axes are rotated about the y-axis through $45°$. [3]

About the y-axis through α the new components are $(-a_3 \sin\alpha + a_1 \cos\alpha,\ a_2,\ a_3 \cos\alpha + a_1 \sin\alpha)$.

$\cos 45° = \frac{\sqrt{2}}{2}, \quad \sin 45° = \frac{\sqrt{2}}{2}$

Substituting $a_1 = -3$, $a_2 = -4$, $a_3 = 5$:

$(-5 \cdot \frac{\sqrt{2}}{2} + (-3) \cdot \frac{\sqrt{2}}{2},\ (-4),\ 5 \cdot \frac{\sqrt{2}}{2} + (-3) \cdot \frac{\sqrt{2}}{2})$

$= (-4\sqrt{2}, -4, \sqrt{2})$

Ans: $(-4\sqrt{2}, -4, \sqrt{2})$

---

20. In triangle ABC, the lines joining a point P to A, B, C meet BC, CA, AB at D, E, F. If $\frac{BD}{DC} = \frac{2}{3}$ and $\frac{CE}{EA} = \frac{3}{4}$, find $\frac{AF}{FB}$. [1]

By Ceva's theorem $\frac{BD}{DC} \cdot \frac{CE}{EA} \cdot \frac{AF}{FB} = 1$

$\frac{AF}{FB} = \frac{1}{\frac{2}{3} \times \frac{3}{4}} = 2$

Ans: 2
