EXPRESSWALA ACADEMY

Maths 1A: Addition of Vectors

Question Paper, Set 02

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Find the unit vector in the direction opposite to $\bar{a} = -12\bar{i} + 9\bar{j} - 8\bar{k}$. [1 mark]

2. If $\bar{a} = -\bar{i} - 2\bar{j} - 2\bar{k}$ and $\bar{b} = 10\bar{i} + 4\bar{j} + 8\bar{k}$, find the vector of magnitude 22 in the direction of $\bar{a} + \bar{b}$. [2 marks]

3. If $\overrightarrow{OP} = 5\bar{i} + 5\bar{j} + 5\bar{k}$ and $\overrightarrow{OQ} = -3\bar{i} + 3\bar{j} - \bar{k}$, find $\overrightarrow{PQ}$. [1 mark]

4. If $(\bar{a}, \bar{b}) = 40^\circ$, find $(-5\bar{a}, -5\bar{b})$. [1 mark]

5. Find the distance $|\overrightarrow{AB}|$ between $A(-2, -1, -4)$ and $B(-4, -2, -4)$. [1 mark]

6. A vector of length 26 has direction cosines $(0, \frac{5}{13}, \frac{12}{13})$. Find the vector. [2 marks]

7. Find p if the vectors $p\bar{i} - 4\bar{j} + 12\bar{k}$ and $-5\bar{i} - 2\bar{j} + 6\bar{k}$ are collinear. [2 marks]

8. Find p so that the points $A(-2, 0, 5)$, $B(-3, 1, 8)$ and $C(-1, p, 2)$ are collinear. [3 marks]

9. If $\bar{a} = -2\bar{i} - 2\bar{j}$, $\bar{b} = \bar{i} + 2\bar{j} + 2\bar{k}$ and $\bar{c} = -2\bar{i} - 2\bar{j} - \bar{k}$, find scalars x, y, z such that $-3\bar{i} - 4\bar{j} - 5\bar{k} = x\bar{a} + y\bar{b} + z\bar{c}$. [5 marks]

10. Find p if the points $A(4, 0, -2)$, $B(5, -3, -3)$, $C(3, 2, -3)$ and $D(p, -8, -3)$ are coplanar. [5 marks]

11. The midpoint of AB is $(\frac{5}{2}, \frac{11}{2}, \frac{5}{2})$ and $A = (0, 6, 5)$. Find B. [1 mark]

12. Find the point that divides the segment joining $A(5, 4, 0)$ and $B(-1, -5, 0)$ externally in the ratio $2 : 1$. [2 marks]

13. In what ratio does the point $C(14, 7, 6)$ divide the segment joining $A(-4, -5, -2)$ and $B(5, 1, 2)$? [3 marks]

14. Find the incentre of the triangle whose vertices are $A(-3, 3, -3)$, $B(1, 3, 0)$ and $C(5, 3, -3)$. [5 marks]

15. Find the vector and cartesian equations of the line through $A(-4, 2, 5)$ parallel to $3\bar{i} + 3\bar{j} - 4\bar{k}$. Also find the point of the line for $t = -1$. [3 marks]

16. Write the vector equation of the plane through $4\bar{i} + \bar{j} - 3\bar{k}$ parallel to $3\bar{i} - \bar{j} - \bar{k}$ and $3\bar{i} - 3\bar{j}$, and find its point with $s = 3$, $t = 1$. [2 marks]

17. Find a vector along the external bisector of the angle between $\bar{a} = 2\bar{i} - 2\bar{j} - \bar{k}$ and $\bar{b} = -12\bar{i} - 12\bar{j} - \bar{k}$. [2 marks]

18. In triangle ABC, D, E, F are the midpoints of BC, CA, AB. Prove that $\overline{AD} + \overline{BE} + \overline{CF} = \bar{0}$. [5 marks]

19. Find the components of $\bar{a} = 5\bar{i} - 2\bar{j} - 3\bar{k}$ when the coordinate axes are rotated about the y-axis through 60°. [3 marks]

20. A straight line cuts the lines BC, CA, AB of triangle ABC at D, E, F. If $\frac{BD}{DC} = -\frac{3}{4}$ and $\frac{CE}{EA} = \frac{5}{3}$ (signed ratios), find $\frac{AF}{FB}$. [1 mark]

Key sheet

Q	Answer
1	$-\frac{1}{17}(-12\bar{i}+9\bar{j}-8\bar{k})$
2	$18\bar{i}+4\bar{j}+12\bar{k}$
3	$-8\bar{i}-2\bar{j}-6\bar{k}$
4	40°
5	$\sqrt{5}$
6	$10\bar{j}+24\bar{k}$
7	$p=-10$
8	$p=-1$
9	$\bar{r}=-2\bar{a}-\bar{b}+3\bar{c}$ $(x=-2,\ y=-1,\ z=3)$
10	$p=7$
11	$(5,5,0)$
12	$(-7,-14,0)$
13	Externally in the ratio $2:1$
14	$(1,3,-\frac{5}{3})$
15	$\bar{r}=(-4\bar{i}+2\bar{j}+5\bar{k})+t(3\bar{i}+3\bar{j}-4\bar{k})$; $\frac{x+4}{3}=\frac{y-2}{3}=\frac{z-5}{-4}$; point $(-7,-1,9)$
16	$\bar{r}=(4\bar{i}+\bar{j}-3\bar{k})+s(3\bar{i}-\bar{j}-\bar{k})+t(3\bar{i}-3\bar{j})$; point $(16,-5,-6)$
17	$\frac{70}{51}\bar{i}+\frac{2}{51}\bar{j}-\frac{14}{51}\bar{k}$ (or any multiple)
18	Proved (see the working)
19	$(\frac{5}{2}+\frac{3\sqrt{3}}{2},-2,-\frac{3}{2}+\frac{5\sqrt{3}}{2})$
20	$\frac{4}{5}$

Solutions

1. Find the unit vector in the direction opposite to $\bar{a} = -12\bar{i} + 9\bar{j} - 8\bar{k}$. [1]

$|\bar{a}| = \sqrt{144 + 81 + 64} = \sqrt{289} = 17$

Unit vector opposite to $\bar{a} = -\frac{\bar{a}}{|\bar{a}|} = -\frac{1}{17}(-12\bar{i} + 9\bar{j} - 8\bar{k})$

Ans: $-\frac{1}{17}(-12\bar{i} + 9\bar{j} - 8\bar{k})$

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2. If $\bar{a} = -\bar{i} - 2\bar{j} - 2\bar{k}$ and $\bar{b} = 10\bar{i} + 4\bar{j} + 8\bar{k}$, find the vector of magnitude 22 in the direction of $\bar{a} + \bar{b}$. [2]

$\bar{a} + \bar{b} = 9\bar{i} + 2\bar{j} + 6\bar{k}$

$|\bar{a} + \bar{b}| = \sqrt{81 + 4 + 36} = 11$

Required vector $= \frac{22(\bar{a}+\bar{b})}{|\bar{a}+\bar{b}|} = \frac{22}{11}(9\bar{i} + 2\bar{j} + 6\bar{k}) = 18\bar{i} + 4\bar{j} + 12\bar{k}$

Ans: $18\bar{i} + 4\bar{j} + 12\bar{k}$

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3. If $\overline{OP} = 5\bar{i} + 5\bar{j} + 5\bar{k}$ and $\overline{OQ} = -3\bar{i} + 3\bar{j} - \bar{k}$, find $\overline{PQ}$. [1]

$\overline{PQ} = \overline{OQ} - \overline{OP}$

$= (-3\bar{i} + 3\bar{j} - \bar{k}) - (5\bar{i} + 5\bar{j} + 5\bar{k}) = -8\bar{i} - 2\bar{j} - 6\bar{k}$

Ans: $-8\bar{i} - 2\bar{j} - 6\bar{k}$

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4. If $(\bar{a}, \bar{b}) = 40^\circ$, find $(-5\bar{a}, -5\bar{b})$. [1]

The scalars -5 and -5 have the same sign.

$(-5\bar{a}, -5\bar{b}) = (\bar{a}, \bar{b}) = 40^\circ$

Ans: 40°

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5. Find the distance $|\overline{AB}|$ between $A(-2, -1, -4)$ and $B(-4, -2, -4)$. [1]

$\overline{AB} = -2\bar{i} - \bar{j}$

$|\overline{AB}| = \sqrt{4 + 1 + 0} = \sqrt{5}$

Ans: $\sqrt{5}$

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6. A vector of length 26 has direction cosines $(0, \frac{5}{13}, \frac{12}{13})$. Find the vector. [2]

$x = lr,\ y = mr,\ z = nr$ with $r = 26$

$x = 0,\ y = 10,\ z = 24$

Vector $= 10\bar{j} + 24\bar{k}$

Ans: $10\bar{j} + 24\bar{k}$

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7. Find p if the vectors $p\bar{i} - 4\bar{j} + 12\bar{k}$ and $-5\bar{i} - 2\bar{j} + 6\bar{k}$ are collinear. [2]

Collinear vectors have proportional components.

$\frac{-4}{-2} = \frac{p}{-5}$

$\Rightarrow p = -10$

Ans: $p = -10$

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8. Find p so that the points $A(-2, 0, 5)$, $B(-3, 1, 8)$ and $C(-1, p, 2)$ are collinear. [3]

$\overline{AB} = (-1, 1, 3)$

$\overline{AC} = (1, p, -3)$

For collinear points the components are proportional:

$\frac{1}{-1} = \frac{p}{1}$

$\Rightarrow p = -1$

Ans: $p = -1$

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9. If $\bar{a} = -2\bar{i} - 2\bar{j}$, $\bar{b} = \bar{i} + 2\bar{j} + 2\bar{k}$ and $\bar{c} = -2\bar{i} - 2\bar{j} - \bar{k}$, find scalars x, y, z such that $-3\bar{i} - 4\bar{j} - 5\bar{k} = x\bar{a} + y\bar{b} + z\bar{c}$. [5]

Let $-3\bar{i} - 4\bar{j} - 5\bar{k} = x(-2\bar{i} - 2\bar{j}) + y(\bar{i} + 2\bar{j} + 2\bar{k}) + z(-2\bar{i} - 2\bar{j} - \bar{k})$.

Comparing the coefficients of $\bar{i}, \bar{j}, \bar{k}$:

$-2x + y - 2z = -3$

$-2x + 2y - 2z = -4$

$2y - z = -5$

Solving by Cramer's rule:

$$\Delta = \begin{vmatrix} -2 & 1 & -2 \\ -2 & 2 & -2 \\ 0 & 2 & -1 \end{vmatrix} = 2$$

$$\Delta_1 = \begin{vmatrix} -3 & 1 & -2 \\ -4 & 2 & -2 \\ -5 & 2 & -1 \end{vmatrix} = -4, \quad x = \frac{\Delta_1}{\Delta} = -2$$

$$\Delta_2 = \begin{vmatrix} -2 & -3 & -2 \\ -2 & -4 & -2 \\ 0 & -5 & -1 \end{vmatrix} = -2, \quad y = \frac{\Delta_2}{\Delta} = -1$$

$$\Delta_3 = \begin{vmatrix} -2 & 1 & -3 \\ -2 & 2 & -4 \\ 0 & 2 & -5 \end{vmatrix} = 6, \quad z = \frac{\Delta_3}{\Delta} = 3$$

$\therefore \bar{r} = -2\bar{a} - \bar{b} + 3\bar{c}$

Ans: $\bar{r} = -2\bar{a} - \bar{b} + 3\bar{c}$ **($x = -2$, $y = -1$, $z = 3$)**

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10. Find p if the points $A(4, 0, -2)$, $B(5, -3, -3)$, $C(3, 2, -3)$ and $D(p, -8, -3)$ are coplanar. [5]

$\overline{AB} = (1, -3, -1)$, $\overline{AC} = (-1, 2, -1)$, $\overline{AD} = (p - 4, -8, -1)$

For coplanar points these vectors are linearly dependent:

$$\begin{vmatrix} 1 & -3 & -1 \\ -1 & 2 & -1 \\ p-4 & -8 & -1 \end{vmatrix} = 0$$

$\Rightarrow 5p - 35 = 0$

$\Rightarrow p = 7$

Ans: $p = 7$

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11. The midpoint of AB is $(\frac{5}{2}, \frac{11}{2}, \frac{5}{2})$ and $A = (0, 6, 5)$. Find B. [1]

$\frac{\bar{a} + \bar{b}}{2} = \bar{m} \Rightarrow \bar{b} = 2\bar{m} - \bar{a}$

$= (5, 11, 5) - (0, 6, 5) = (5, 5, 0)$

Ans: $(5, 5, 0)$

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12. Find the point that divides the segment joining $A(5, 4, 0)$ and $B(-1, -5, 0)$ externally in the ratio $2 : 1$. [2]

$\bar{r} = \frac{m\bar{b} - n\bar{a}}{m-n}$
$= \frac{2(-1,-5,0) - 1(5,4,0)}{2-1}$
$= \frac{(-7,-14,0)}{1} = (-7, -14, 0)$

Ans: $(-7, -14, 0)$

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13. In what ratio does the point $C(14, 7, 6)$ divide the segment joining $A(-4, -5, -2)$ and $B(5, 1, 2)$? [3]

Let C divide AB in the ratio $\lambda : 1$. Then $\bar{c} = \frac{\lambda\bar{b} + \bar{a}}{\lambda + 1}$.
Comparing one coordinate: $14 = \frac{5\lambda + (-4)}{\lambda + 1}$
$\Rightarrow \lambda = -2$
$\lambda < 0$: C divides AB externally in the ratio $2 : 1$.

Ans: Externally in the ratio $2 : 1$

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14. Find the incentre of the triangle whose vertices are $A(-3, 3, -3)$, $B(1, 3, 0)$ and $C(5, 3, -3)$. [5]

$a = BC = \sqrt{16 + 0 + 9} = 5$
$b = CA = \sqrt{64 + 0 + 0} = 8$
$c = AB = \sqrt{16 + 0 + 9} = 5$
Incentre $I = \frac{a\bar{a} + b\bar{b} + c\bar{c}}{a + b + c}$
$= \frac{5(-3,3,-3) + 8(1,3,0) + 5(5,3,-3)}{18}$
$= \frac{(18, 54, -30)}{18} = (1, 3, -\frac{5}{3})$

Ans: $(1, 3, -\frac{5}{3})$

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15. Find the vector and cartesian equations of the line through $A(-4, 2, 5)$ parallel to $3\bar{i} + 3\bar{j} - 4\bar{k}$. Also find the point of the line for $t = -1$. [3]

$\bar{r} = \bar{a} + t\bar{b}$
$\bar{r} = (-4\bar{i} + 2\bar{j} + 5\bar{k}) + t(3\bar{i} + 3\bar{j} - 4\bar{k})$
Cartesian: $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$
$\frac{x+4}{3} = \frac{y-2}{3} = \frac{z-5}{-4}$
$t = -1 :\ \bar{r} = (-7, -1, 9)$

Ans: $\bar{r} = (-4\bar{i} + 2\bar{j} + 5\bar{k}) + t(3\bar{i} + 3\bar{j} - 4\bar{k})$; $\frac{x+4}{3} = \frac{y-2}{3} = \frac{z-5}{-4}$; **point** $(-7, -1, 9)$

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16. Write the vector equation of the plane through $4\bar{i} + \bar{j} - 3\bar{k}$ parallel to $3\bar{i} - \bar{j} - \bar{k}$ and $3\bar{i} - 3\bar{j}$, and find its point with $s = 3$, $t = 1$. [2]

$\bar{r} = \bar{a} + s\bar{b} + t\bar{c}$
$\bar{r} = (4\bar{i} + \bar{j} - 3\bar{k}) + s(3\bar{i} - \bar{j} - \bar{k}) + t(3\bar{i} - 3\bar{j})$
$s = 3,\ t = 1 :\ \bar{r} = (16, -5, -6)$

Ans: $\bar{r} = (4\bar{i} + \bar{j} - 3\bar{k}) + s(3\bar{i} - \bar{j} - \bar{k}) + t(3\bar{i} - 3\bar{j})$; **point** $(16, -5, -6)$

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17. Find a vector along the external bisector of the angle between $\bar{a} = 2\bar{i} - 2\bar{j} - \bar{k}$ and $\bar{b} = -12\bar{i} - 12\bar{j} - \bar{k}$. [2]

$|\bar{a}| = \sqrt{4+4+1} = 3, \quad |\bar{b}| = \sqrt{144+144+1} = 17$

$\frac{\bar{a}}{|\bar{a}|} - \frac{\bar{b}}{|\bar{b}|} = \frac{1}{3}(2\bar{i} - 2\bar{j} - \bar{k}) - \frac{1}{17}(-12\bar{i} - 12\bar{j} - \bar{k})$

$= \frac{70}{51}\bar{i} + \frac{2}{51}\bar{j} - \frac{14}{51}\bar{k}$

Any non-zero multiple of this vector is also along the external bisector.

Ans: $\frac{70}{51}\bar{i} + \frac{2}{51}\bar{j} - \frac{14}{51}\bar{k}$ (or any multiple)

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18. In triangle ABC, D, E, F are the midpoints of BC, CA, AB. Prove that $\overline{AD} + \overline{BE} + \overline{CF} = \bar{0}$. [5]

Let $\bar{a}, \bar{b}, \bar{c}$ be the position vectors of A, B, C.

$\overline{OD} = \frac{\bar{b}+\bar{c}}{2}$, $\overline{OE} = \frac{\bar{c}+\bar{a}}{2}$, $\overline{OF} = \frac{\bar{a}+\bar{b}}{2}$

$\overline{AD} = \frac{\bar{b}+\bar{c}}{2} - \bar{a}$, $\overline{BE} = \frac{\bar{c}+\bar{a}}{2} - \bar{b}$, $\overline{CF} = \frac{\bar{a}+\bar{b}}{2} - \bar{c}$

$\overline{AD} + \overline{BE} + \overline{CF} = \frac{2(\bar{a}+\bar{b}+\bar{c})}{2} - (\bar{a}+\bar{b}+\bar{c})$

$= \bar{0}$

Ans: Proved (see the working)

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19. Find the components of $\bar{a} = 5\bar{i} - 2\bar{j} - 3\bar{k}$ when the coordinate axes are rotated about the y-axis through 60°. [3]

About the y-axis through α the new components are $(-a_3 \sin\alpha + a_1 \cos\alpha,\ a_2,\ a_3 \cos\alpha + a_1 \sin\alpha)$.

$\cos 60^\circ = \frac{1}{2}, \quad \sin 60^\circ = \frac{\sqrt{3}}{2}$

Substituting $a_1 = 5$, $a_2 = -2$, $a_3 = -3$:

$(-(-3)\cdot\frac{\sqrt{3}}{2} + 5\cdot\frac{1}{2},\ (-2),\ (-3)\cdot\frac{1}{2} + 5\cdot\frac{\sqrt{3}}{2})$

$= (\frac{5}{2} + \frac{3\sqrt{3}}{2}, -2, -\frac{3}{2} + \frac{5\sqrt{3}}{2})$

Ans: $(\frac{5}{2} + \frac{3\sqrt{3}}{2}, -2, -\frac{3}{2} + \frac{5\sqrt{3}}{2})$

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20. A straight line cuts the lines BC, CA, AB of triangle ABC at D, E, F. If $\frac{BD}{DC} = -\frac{3}{4}$ and $\frac{CE}{EA} = \frac{5}{3}$ (signed ratios), find $\frac{AF}{FB}$. [1]

By Menelaus' theorem $\frac{BD}{DC} \cdot \frac{CE}{EA} \cdot \frac{AF}{FB} = -1$

$\frac{AF}{FB} = \frac{-1}{(-\frac{3}{4})(\frac{5}{3})} = \frac{4}{5}$

Ans: $\frac{4}{5}$
