EXPRESSWALA ACADEMY

Maths 1A: Addition of Vectors

Question Paper, Set 01

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Find the unit vector in the direction of $\bar{a} = 8\bar{i} + 9\bar{j} + 12\bar{k}$. [1 mark]

2. Find the vectors of magnitude 8 that are parallel to $\bar{a} = 6\bar{i} + 3\bar{j} - 2\bar{k}$. [2 marks]

3. The position vectors of A and B are $5\bar{i} + \bar{j} - 2\bar{k}$ and $2\bar{i} - 3\bar{j}$. Find $\overline{AB}$. [1 mark]

4. If $(\bar{a}, \bar{b}) = 110^\circ$, find $(3\bar{a}, -5\bar{b})$. [1 mark]

5. Find the magnitude of $\overline{OP}$, where $P = (0, 6, 1)$. [1 mark]

6. Find the direction cosines of the vector $12\bar{j} - 5\bar{k}$. [2 marks]

7. Find p if the vectors $-8\bar{i} + p\bar{j} - 12\bar{k}$ and $4\bar{i} - 6\bar{j} + 6\bar{k}$ are collinear. [2 marks]

8. Show that the points $A(0, -2, -1)$, $B(2, -5, -3)$ and $C(-6, 7, 5)$ are collinear. [3 marks]

9. Express $\bar{r} = 3\bar{i} - 3\bar{j} + \bar{k}$ as a linear combination of $\bar{a} = -\bar{i} - \bar{j} + \bar{k}$, $\bar{b} = 2\bar{i} + \bar{k}$ and $\bar{c} = 2\bar{i} + 3\bar{j} - \bar{k}$. [5 marks]

10. Show that the points $A(-3, 0, 2)$, $B(-2, 2, 1)$, $C(-1, -1, 3)$ and $D(-7, -3, 3)$ are coplanar. [5 marks]

11. Find the midpoint of the segment joining $A(6, -4, 3)$ and $B(2, 1, 2)$. [1 mark]

12. Find the point that divides the segment joining $A(3, -2, 3)$ and $B(-5, -2, 3)$ internally in the ratio $3 : 1$. [2 marks]

13. Find the points of trisection of the segment joining $A(-1, 3, -2)$ and $B(2, 12, -5)$. [3 marks]

14. Find the incentre of the triangle whose vertices are $A(-7, 1, 4)$, $B(-2, 1, 4)$ and $C(-2, 13, 4)$. [5 marks]

15. Find the vector and cartesian equations of the line through $A(2, 5, -4)$ and $B(5, 3, -8)$. [3 marks]

16. Write the vector equation of the plane through the points $A(3, -3, 3)$, $B(0, 4, 4)$ and $C(1, 1, -1)$, and find its point with $s = 2$, $t = 1$. [2 marks]

17. Find a vector along the internal bisector of the angle between $\bar{a} = 4\bar{i} - 8\bar{j} + 19\bar{k}$ and $\bar{b} = 4\bar{i} + 20\bar{j} + 5\bar{k}$. [2 marks]

18. If G is the centroid of triangle ABC, prove that $\overline{GA} + \overline{GB} + \overline{GC} = \bar{0}$. [5 marks]

19. Find the components of $\bar{a} = 2\bar{i} + 5\bar{j} + 2\bar{k}$ when the coordinate axes are rotated about the x-axis through 90°. [3 marks]

20. In triangle ABC, the lines joining a point P to A, B, C meet BC, CA, AB at D, E, F. If $\frac{BD}{DC} = 4$ and $\frac{CE}{EA} = \frac{4}{5}$, find $\frac{AF}{FB}$. [1 mark]

Key sheet

Q	Answer
1	$\frac{1}{17}(8\bar{i}+9\bar{j}+12\bar{k})$
2	$\pm(\frac{48}{7}\bar{i}+\frac{24}{7}\bar{j}-\frac{16}{7}\bar{k})$
3	$-3\bar{i}-4\bar{j}+2\bar{k}$
4	70°
5	$\sqrt{37}$
6	$(0,\frac{12}{13},-\frac{5}{13})$
7	$p=12$
8	$\overline{AC}=-3\,\overline{AB}$, so A,B,C are collinear
9	$\bar{r}=-3\bar{a}+2\bar{b}-2\bar{c}$ ($x=-3$, $y=2$, $z=-2$)
10	The determinant is 0, so A,B,C,D are coplanar
11	$(4,-\frac{3}{2},\frac{5}{2})$
12	$(-3,-2,3)$
13	$(0,6,-3)$ and $(1,9,-4)$
14	$(-4,3,4)$
15	$\bar{r}=(1-t)(2\bar{i}+5\bar{j}-4\bar{k})+t(5\bar{i}+3\bar{j}-8\bar{k})$; $\frac{x-2}{3}=\frac{y-5}{-2}=\frac{z+4}{-4}$
16	$\bar{r}=(1-s-t)(3\bar{i}-3\bar{j}+3\bar{k})+s(4\bar{j}+4\bar{k})+t(\bar{i}+\bar{j}-\bar{k})$; point $(-5,15,1)$
17	$\frac{8}{21}\bar{i}+\frac{4}{7}\bar{j}+\frac{8}{7}\bar{k}$ (or any multiple)
18	Proved (see the working)
19	$(2,2,-5)$
20	$\frac{5}{16}$

Solutions

1. Find the unit vector in the direction of $\bar{a} = 8\bar{i} + 9\bar{j} + 12\bar{k}$. [1]

$|\bar{a}| = \sqrt{64 + 81 + 144} = \sqrt{289} = 17$

$\text{Unit vector} = \frac{\bar{a}}{|\bar{a}|} = \frac{1}{17}(8\bar{i} + 9\bar{j} + 12\bar{k})$

Ans: $\frac{1}{17}(8\bar{i} + 9\bar{j} + 12\bar{k})$

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2. Find the vectors of magnitude 8 that are parallel to $\bar{a} = 6\bar{i} + 3\bar{j} - 2\bar{k}$. [2]

$|\bar{a}| = \sqrt{36 + 9 + 4} = 7$

$\text{Required vectors} = \pm\frac{8\,\bar{a}}{|\bar{a}|} = \pm\frac{8}{7}(6\bar{i} + 3\bar{j} - 2\bar{k})$

$= \pm(\frac{48}{7}\bar{i} + \frac{24}{7}\bar{j} - \frac{16}{7}\bar{k})$

Ans: $\pm(\frac{48}{7}\bar{i} + \frac{24}{7}\bar{j} - \frac{16}{7}\bar{k})$

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3. The position vectors of A and B are $5\bar{i} + \bar{j} - 2\bar{k}$ and $2\bar{i} - 3\bar{j}$. Find $\overline{AB}$. [1]

$\overline{AB} = \overline{OB} - \overline{OA}$

$= (2\bar{i} - 3\bar{j}) - (5\bar{i} + \bar{j} - 2\bar{k}) = -3\bar{i} - 4\bar{j} + 2\bar{k}$

Ans: $-3\bar{i} - 4\bar{j} + 2\bar{k}$

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4. If $(\bar{a}, \bar{b}) = 110°$, find $(3\bar{a}, -5\bar{b})$. [1]

The scalars 3 and -5 have opposite signs.

$(3\bar{a}, -5\bar{b}) = 180° - 110° = 70°$

Ans: $70°$

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5. Find the magnitude of $\overline{OP}$, where $P = (0, 6, 1)$. [1]

$\overline{OP} = 6\bar{j} + \bar{k}$

$|\overline{OP}| = \sqrt{0 + 36 + 1} = \sqrt{37}$

Ans: $\sqrt{37}$

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6. Find the direction cosines of the vector $12\bar{j} - 5\bar{k}$. [2]

$r = \sqrt{0 + 144 + 25} = 13$

$l = \frac{x}{r},\ m = \frac{y}{r},\ n = \frac{z}{r}$

$\text{Direction cosines} = (0, \frac{12}{13}, -\frac{5}{13})$

Ans: $(0, \frac{12}{13}, -\frac{5}{13})$

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7. Find p if the vectors $-8\bar{i} + p\bar{j} - 12\bar{k}$ and $4\bar{i} - 6\bar{j} + 6\bar{k}$ are collinear. [2]

Collinear vectors have proportional components.

$\frac{-12}{6} = \frac{p}{-6}$

$\Rightarrow p = 12$

Ans: $p = 12$

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8. Show that the points $A(0,-2,-1)$, $B(2,-5,-3)$ and $C(-6,7,5)$ are collinear. [3]

$\overline{AB} = \overline{OB} - \overline{OA} = 2\bar{i} - 3\bar{j} - 2\bar{k}$

$\overline{AC} = \overline{OC} - \overline{OA} = -6\bar{i} + 9\bar{j} + 6\bar{k}$

$\overline{AC} = -3\,\overline{AB}$

$\overline{AB}$ and $\overline{AC}$ are parallel and have the common point A, so A, B, C are collinear.

Ans: $\overline{AC} = -3\,\overline{AB}$, so A, B, C are collinear

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9. Express $\bar{r} = 3\bar{i} - 3\bar{j} + \bar{k}$ as a linear combination of $\bar{a} = -\bar{i} - \bar{j} + \bar{k}$, $\bar{b} = 2\bar{i} + \bar{k}$ and $\bar{c} = 2\bar{i} + 3\bar{j} - \bar{k}$. [5]

Let $3\bar{i} - 3\bar{j} + \bar{k} = x(-\bar{i} - \bar{j} + \bar{k}) + y(2\bar{i} + \bar{k}) + z(2\bar{i} + 3\bar{j} - \bar{k})$.

Comparing the coefficients of $\bar{i}, \bar{j}, \bar{k}$:

$-x + 2y + 2z = 3$

$-x + 3z = -3$

$x + y - z = 1$

Solving by Cramer's rule:

$$\Delta = \begin{vmatrix} -1 & 2 & 2 \\ -1 & 0 & 3 \\ 1 & 1 & -1 \end{vmatrix} = 5$$

$$\Delta_1 = \begin{vmatrix} 3 & 2 & 2 \\ -3 & 0 & 3 \\ 1 & 1 & -1 \end{vmatrix} = -15, \quad x = \frac{\Delta_1}{\Delta} = -3$$

$$\Delta_2 = \begin{vmatrix} -1 & 3 & 2 \\ -1 & -3 & 3 \\ 1 & 1 & -1 \end{vmatrix} = 10, \quad y = \frac{\Delta_2}{\Delta} = 2$$

$$\Delta_3 = \begin{vmatrix} -1 & 2 & 3 \\ -1 & 0 & -3 \\ 1 & 1 & 1 \end{vmatrix} = -10, \quad z = \frac{\Delta_3}{\Delta} = -2$$

$\therefore \bar{r} = -3\bar{a} + 2\bar{b} - 2\bar{c}$

Ans: $\bar{r} = -3\bar{a} + 2\bar{b} - 2\bar{c}$ ($x = -3$, $y = 2$, $z = -2$)

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10. Show that the points $A(-3,0,2)$, $B(-2,2,1)$, $C(-1,-1,3)$ and $D(-7,-3,3)$ are coplanar. [5]

$\overline{AB} = \bar{i} + 2\bar{j} - \bar{k}$, $\overline{AC} = 2\bar{i} - \bar{j} + \bar{k}$, $\overline{AD} = -4\bar{i} - 3\bar{j} + \bar{k}$

The four points are coplanar when these three vectors are coplanar (linearly dependent).

$$\begin{vmatrix} 1 & 2 & -1 \\ 2 & -1 & 1 \\ -4 & -3 & 1 \end{vmatrix} = 1(2) - 2(6) + (-1)(-10)$$

$= 0$

The determinant is zero, so $\overline{AB}, \overline{AC}, \overline{AD}$ are coplanar and A, B, C, D are coplanar.

Ans: The determinant is 0, so A, B, C, D are coplanar

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11. Find the midpoint of the segment joining $A(6,-4,3)$ and $B(2,1,2)$. [1]

$\text{Midpoint} = \frac{\bar{a}+\bar{b}}{2} = \frac{(8,-3,5)}{2}$

$= (4, -\frac{3}{2}, \frac{5}{2})$

Ans: $(4, -\frac{3}{2}, \frac{5}{2})$

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12. Find the point that divides the segment joining $A(3,-2,3)$ and $B(-5,-2,3)$ internally in the ratio $3:1$. [2]

$\bar{r} = \frac{m\bar{b}+n\bar{a}}{m+n}$
$= \frac{3(-5,-2,3)+1(3,-2,3)}{3+1}$
$= \frac{(-12,-8,12)}{4} = (-3,-2,3)$

Ans: $(-3,-2,3)$

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13. Find the points of trisection of the segment joining $A(-1,3,-2)$ and $B(2,12,-5)$. [3]

The points of trisection divide AB internally in the ratios $1:2$ and $2:1$.
$\frac{1(2,12,-5)+2(-1,3,-2)}{3} = \frac{(0,18,-9)}{3} = (0,6,-3)$
$\frac{2(2,12,-5)+1(-1,3,-2)}{3} = \frac{(3,27,-12)}{3} = (1,9,-4)$

Ans: $(0,6,-3)$ **and** $(1,9,-4)$

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14. Find the incentre of the triangle whose vertices are $A(-7,1,4)$, $B(-2,1,4)$ and $C(-2,13,4)$. [5]

$a = BC = \sqrt{0+144+0} = 12$
$b = CA = \sqrt{25+144+0} = 13$
$c = AB = \sqrt{25+0+0} = 5$
Incentre $I = \frac{a\bar{a}+b\bar{b}+c\bar{c}}{a+b+c}$
$= \frac{12(-7,1,4)+13(-2,1,4)+5(-2,13,4)}{30}$
$= \frac{(-120,90,120)}{30} = (-4,3,4)$

Ans: $(-4,3,4)$

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15. Find the vector and cartesian equations of the line through $A(2,5,-4)$ and $B(5,3,-8)$. [3]

$\bar{r} = (1-t)\bar{a} + t\bar{b} = (1-t)(2\bar{i}+5\bar{j}-4\bar{k}) + t(5\bar{i}+3\bar{j}-8\bar{k})$
$= (2+3t)\bar{i} + (5-2t)\bar{j} + (-4-4t)\bar{k}$
Direction $\overline{AB} = (3,-2,-4)$
$\frac{x-2}{3} = \frac{y-5}{-2} = \frac{z+4}{-4}$

Ans: $\bar{r} = (1-t)(2\bar{i}+5\bar{j}-4\bar{k}) + t(5\bar{i}+3\bar{j}-8\bar{k})$; $\frac{x-2}{3} = \frac{y-5}{-2} = \frac{z+4}{-4}$

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16. Write the vector equation of the plane through the points $A(3,-3,3)$, $B(0,4,4)$ and $C(1,1,-1)$, and find its point with $s=2$, $t=1$. [2]

$\bar{r} = (1-s-t)\bar{a} + s\bar{b} + t\bar{c}$
$\bar{r} = (1-s-t)(3\bar{i}-3\bar{j}+3\bar{k}) + s(4\bar{j}+4\bar{k}) + t(\bar{i}+\bar{j}-\bar{k})$
$s=2,\ t=1:\ \bar{r} = -2(3,-3,3) + 2(0,4,4) + 1(1,1,-1) = (-5,15,1)$

Ans: $\bar{r} = (1-s-t)(3\bar{i}-3\bar{j}+3\bar{k}) + s(4\bar{j}+4\bar{k}) + t(\bar{i}+\bar{j}-\bar{k})$; **point** $(-5,15,1)$

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17. Find a vector along the internal bisector of the angle between $\bar{a} = 4\bar{i}-8\bar{j}+19\bar{k}$ and $\bar{b} = 4\bar{i}+20\bar{j}+5\bar{k}$. [2]

$|\bar{a}| = \sqrt{16+64+361} = 21, \quad |\bar{b}| = \sqrt{16+400+25} = 21$
$\frac{\bar{a}}{|\bar{a}|} + \frac{\bar{b}}{|\bar{b}|} = \frac{1}{21}(4\bar{i}-8\bar{j}+19\bar{k}) + \frac{1}{21}(4\bar{i}+20\bar{j}+5\bar{k})$
$= \frac{8}{21}\bar{i} + \frac{4}{7}\bar{j} + \frac{8}{7}\bar{k}$
Any non-zero multiple of this vector is also along the internal bisector.

Ans: $\frac{8}{21}\bar{i} + \frac{4}{7}\bar{j} + \frac{8}{7}\bar{k}$ **(or any multiple)**

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18. If G is the centroid of triangle ABC, prove that $\overline{GA} + \overline{GB} + \overline{GC} = \bar{0}$. [5]

Take G as the origin, and let $\bar{a}, \bar{b}, \bar{c}$ be the position vectors of A, B, C.

The centroid has position vector $\frac{\bar{a}+\bar{b}+\bar{c}}{3}$, and the centroid is the origin.

$\therefore \frac{\bar{a}+\bar{b}+\bar{c}}{3} = \bar{0} \Rightarrow \bar{a} + \bar{b} + \bar{c} = \bar{0}$

But $\overline{GA} = \bar{a}$, $\overline{GB} = \bar{b}$, $\overline{GC} = \bar{c}$.

$\therefore \overline{GA} + \overline{GB} + \overline{GC} = \bar{0}$

Ans: Proved (see the working)

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19. Find the components of $\bar{a} = 2\bar{i} + 5\bar{j} + 2\bar{k}$ when the coordinate axes are rotated about the x-axis through 90°. [3]

About the x-axis through α the new components are $(a_1,\ a_2\cos\alpha + a_3\sin\alpha,\ -a_2\sin\alpha + a_3\cos\alpha)$.

$\cos 90^\circ = 0, \quad \sin 90^\circ = 1$

Substituting $a_1 = 2$, $a_2 = 5$, $a_3 = 2$:

$(2,\ 5\cdot 0 + 2\cdot 1,\ -5\cdot 1 + 2\cdot 0)$

$= (2, 2, -5)$

Ans: $(2, 2, -5)$

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20. In triangle ABC, the lines joining a point P to A, B, C meet BC, CA, AB at D, E, F. If $\frac{BD}{DC} = 4$ and $\frac{CE}{EA} = \frac{4}{5}$, find $\frac{AF}{FB}$. [1]

By Ceva's theorem $\frac{BD}{DC} \cdot \frac{CE}{EA} \cdot \frac{AF}{FB} = 1$

$\frac{AF}{FB} = \frac{1}{4 \times \frac{4}{5}} = \frac{5}{16}$

Ans: $\frac{5}{16}$
