EXPRESSWALA ACADEMY

Functions

Question Paper, Set 09

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. If $f(x) = 4x - 9$, find the image of 5 under f. [1 mark]

2. Let $A = \{0, 1, 2, 3\}$, $B = \{0, 1, 2, 4, 6\}$ and $f : A \to B$ be defined by $f(x) = x^2 - x$. Write the domain, codomain and range of f. [2 marks]

3. Write the degree of the polynomial function $f(x) = 4 - x^3 + 2x^6$. [1 mark]

4. Find the points where the line $y = 6x - 21$ cuts the x-axis and the y-axis. Is its inclination less than or greater than 90°? [2 marks]

5. Find the discriminant of $2x^2 + 3x + 4 = 0$ and write the nature of its roots. [2 marks]

6. If α, β are the roots of $x^2 + 3x - 5 = 0$, find $\alpha + \beta$, $\alpha\beta$ and $|\alpha - \beta|$. [3 marks]

7. Write $y = -2x^2 - 8x - 11$ in vertex form. Find its vertex and its maximum value. [3 marks]

8. Draw the graph of $y = (2x + 5)^2$. (Find whether it opens upward or downward, where it meets the x-axis and the y-axis, and its vertex.) [5 marks]

9. For $y = -2x^2 + 5x + 3$, find the values of x for which (i) $y < 0$ (ii) $y \le 0$ (iii) $y > 0$ (iv) $y \ge 0$. [5 marks]

10. If $\log_x 64 = 3$, find x. [1 mark]

11. Find $\log_6 9 + \log_6 8 - \log_6 2$. [2 marks]

12. Solve $\log_6 x + \log_6 (x - 5) = 2$. [3 marks]

13. If $|x| = x$, what can you say about x? [1 mark]

14. Solve $|7 - 2x| < 3$. [2 marks]

15. Solve $|x - 1| = |3x + 5|$. [3 marks]

16. Solve $|x + 1| - |x - 2| < x - \frac{1}{2}$. [5 marks]

17. Solve $\left|\frac{20x}{4x^2 + 25}\right| \le 1$. [5 marks]

18. Find the domain of $f(x) = \sqrt{x^2 - 4x + 4}$. [2 marks]

19. Find $[[-3.7]]$. [1 mark]

20. Write the domain and range of $f(x) = x^3$. [1 mark]

Key sheet

Q	Answer
1	11
2	Domain $= \{0, 1, 2, 3\}$, codomain $= \{0, 1, 2, 4, 6\}$, range $= \{0, 2, 6\}$
3	6
4	x-axis at $(\frac{7}{2}, 0)$, y-axis at $(0, -21)$; inclination less than 90°
5	$\Delta = -23$; roots are not real (imaginary)
6	$\alpha + \beta = -3,\ \alpha\beta = -5,\ \lvert\alpha - \beta\rvert = \sqrt{29}$
7	$y = -2(x + 2)^2 - 3$; vertex $(-2, -3)$; maximum value -3
8	Upward parabola; touches the x-axis at $(-\frac{5}{2}, 0)$; cuts the y-axis at $(0, 25)$; vertex $(-\frac{5}{2}, 0)$
9	(i) $x \in (-\infty, -\frac{1}{2}) \cup (3, \infty)$; (ii) $x \in (-\infty, -\frac{1}{2}] \cup [3, \infty)$; (iii) $x \in (-\frac{1}{2}, 3)$; (iv) $x \in [-\frac{1}{2}, 3]$
10	4
11	2
12	9
13	$x \geq 0$, i.e. $x \in [0, \infty)$
14	$x \in (2, 5)$
15	$x \in \{-3, -1\}$
16	$x \in (-\frac{5}{2}, \frac{1}{2}) \cup (\frac{7}{2}, \infty)$
17	$x \in \mathbb{R}$
18	$\mathbb{R}$
19	-4
20	Domain $= \mathbb{R}$, Range $= \mathbb{R}$

Solutions

1. If $f(x) = 4x - 9$, find the image of 5 under f. [1]

Image of 5 is $f(5)$

$f(5) = 4(5) - 9 = 11$

Ans: 11

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2. Let $A = \{0, 1, 2, 3\}$, $B = \{0, 1, 2, 4, 6\}$ and $f : A \to B$ be defined by $f(x) = x^2 - x$. Write the domain, codomain and range of f. [2]

$f(0) = (0)^2 - (0) = 0$, $f(1) = (1)^2 - (1) = 0$, $f(2) = (2)^2 - (2) = 2$, $f(3) = (3)^2 - (3) = 6$

Domain $= A = \{0, 1, 2, 3\}$

Codomain $= B = \{0, 1, 2, 4, 6\}$

Range $= f(A) = \{0, 2, 6\}$

Ans: Domain $= \{0, 1, 2, 3\}$**, codomain** $= \{0, 1, 2, 4, 6\}$**, range** $= \{0, 2, 6\}$

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3. Write the degree of the polynomial function $f(x) = 4 - x^3 + 2x^6$. [1]

The highest power of x is in the term $2x^6$ (coefficient $2 \neq 0$)

$\therefore$ degree of $f(x)$ is 6

Ans: 6

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4. Find the points where the line $y = 6x - 21$ cuts the x-axis and the y-axis. Is its inclination less than or greater than $90°$? [2]

put $x = 0 \Rightarrow y = -21$: y-intercept $(0, -21)$

put $y = 0 \Rightarrow 6x - 21 = 0 \Rightarrow x = \frac{7}{2}$: x-intercept $(\frac{7}{2}, 0)$

$a = 6 > 0$, so the inclination is less than $90°$

Ans: x-axis at $(\frac{7}{2}, 0)$**, y-axis at** $(0, -21)$**; inclination less than** $90°$

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5. Find the discriminant of $2x^2 + 3x + 4 = 0$ and write the nature of its roots. [2]

$a = 2, b = 3, c = 4$

$\Delta = b^2 - 4ac = 3^2 - 4(2)(4) = -23$

$\Delta < 0 \Rightarrow$ roots are not real (imaginary)

Ans: $\Delta = -23$**; roots are not real (imaginary)**

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6. If α, β are the roots of $x^2 + 3x - 5 = 0$, find $\alpha + \beta$, $\alpha\beta$ and $|\alpha - \beta|$. [3]

$a = 1, b = 3, c = -5$

$\alpha + \beta = -\frac{b}{a} = -3$

$\alpha\beta = \frac{c}{a} = -5$

$\Delta = b^2 - 4ac = 3^2 - 4(1)(-5) = 29$

$|\alpha - \beta| = \frac{\sqrt{\Delta}}{|a|} = \sqrt{29} = \sqrt{29}$

Ans: $\alpha + \beta = -3,\ \alpha\beta = -5,\ |\alpha - \beta| = \sqrt{29}$

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7. Write $y = -2x^2 - 8x - 11$ in vertex form. Find its vertex and its maximum value. [3]

$a = -2, b = -8, c = -11$

$\text{Vertex} = \left(-\frac{b}{2a}, \frac{4ac-b^2}{4a}\right) = \left(-2, \frac{88-64}{-8}\right) = (-2, -3)$

$y = a(x-h)^2 + k = -2(x+2)^2 - 3$

$a < 0$, so the parabola opens downward: maximum value $= -3$ at $x = -2$

Ans: $y = -2(x+2)^2 - 3$; **vertex** $(-2, -3)$; **maximum value** -3

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8. Draw the graph of $y = (2x+5)^2$. (Find whether it opens upward or downward, where it meets the x-axis and the y-axis, and its vertex.) [5]

$y = (2x+5)^2 = 4x^2 + 20x + 25$

$a = 4 > 0$ (upward parabola)

$\Delta = b^2 - 4ac = 20^2 - 4(4)(25) = 0$

$\Delta = 0$: touches the x-axis at $(-\frac{5}{2}, 0)$

put $x = 0 \Rightarrow y = 25$: cuts the y-axis at $(0, 25)$

$\text{Vertex} = \left(-\frac{b}{2a}, \frac{4ac-b^2}{4a}\right) = (-\frac{5}{2}, 0)$

Plot these points and draw the upward parabola through them (minimum value 0 at $x = -\frac{5}{2}$)

Ans: Upward parabola; touches the x-axis at $(-\frac{5}{2}, 0)$; **cuts the y-axis at** $(0, 25)$; **vertex** $(-\frac{5}{2}, 0)$

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9. For $y = -2x^2 + 5x + 3$, find the values of x for which (i) $y < 0$ (ii) $y \leq 0$ (iii) $y > 0$ (iv) $y \geq 0$. [5]

$a = -2 < 0$ (downward parabola)

$\Delta = b^2 - 4ac = 5^2 - 4(-2)(3) = 49$

$y = 0 \Rightarrow x = \frac{-5 \pm 7}{-4} = -\frac{1}{2}, 3$

Sign: $-$ outside $-\frac{1}{2}$ and 3, $+$ between them

(i) $y < 0 \Rightarrow x \in (-\infty, -\frac{1}{2}) \cup (3, \infty)$

(ii) $y \leq 0 \Rightarrow x \in (-\infty, -\frac{1}{2}] \cup [3, \infty)$

(iii) $y > 0 \Rightarrow x \in (-\frac{1}{2}, 3)$

(iv) $y \geq 0 \Rightarrow x \in [-\frac{1}{2}, 3]$

Ans: (i) $x \in (-\infty, -\frac{1}{2}) \cup (3, \infty)$; **(ii)** $x \in (-\infty, -\frac{1}{2}] \cup [3, \infty)$; **(iii)** $x \in (-\frac{1}{2}, 3)$; **(iv)** $x \in [-\frac{1}{2}, 3]$

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10. If $\log_x 64 = 3$, find x. [1]

$\log_x 64 = 3 \Leftrightarrow x^3 = 64$

$x^3 = 4^3 \Rightarrow x = 4$

Ans: 4

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11. Find $\log_6 9 + \log_6 8 - \log_6 2$. [2]

$\log_6 9 + \log_6 8 - \log_6 2 = \log_6 \frac{9 \times 8}{2}$

$= \log_6 36 = \log_6 6^2 = 2$

Ans: 2

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12. Solve $\log_6 x + \log_6(x-5) = 2$. [3]

$\log_6 x(x-5) = 2 \Rightarrow x(x-5) = 36$

$x^2 - 5x - 36 = 0 \Rightarrow (x-9)(x+4) = 0$

$x = 9$ (or) $x = -4$

For $x = -4$, $\log_6(-4)$ is not defined

$\therefore x = 9$

Ans: 9

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13. If $|x| = x$, what can you say about x? [1]

$|x| = x$ for $x \geq 0$ and $|x| = -x \neq x$ for $x < 0$

$\therefore x \geq 0,$ i.e. $x \in [0, \infty)$

Ans: $x \geq 0$, **i.e.** $x \in [0, \infty)$

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14. Solve $|7 - 2x| < 3$. [2]

$-3 < 7 - 2x < 3$

$\Rightarrow -3 - 7 < -2x < 3 - 7$

$\Rightarrow 2 < x < 5$ (dividing by a negative number turns the signs)

$\Rightarrow x \in (2, 5)$

Ans: $x \in (2, 5)$

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15. Solve $|x - 1| = |3x + 5|$. [3]

$|x| = |y| \Rightarrow x = \pm y$

$x - 1 = \pm(3x + 5)$

$x - 1 = 3x + 5 \Rightarrow 2x + 6 = 0 \Rightarrow x = -3$

$x - 1 = -3x - 5 \Rightarrow 4x + 4 = 0 \Rightarrow x = -1$

$\therefore x \in \{-3, -1\}$

Ans: $x \in \{-3, -1\}$

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16. Solve $|x + 1| - |x - 2| < x - \frac{1}{2}$. [5]

Critical points: $x + 1 = 0 \Rightarrow x = -1$, $x - 2 = 0 \Rightarrow x = 2$

Case (i) Suppose $x < -1$ $(x + 1 < 0, x - 2 < 0)$: $(-x - 1) - (2 - x) < x - \frac{1}{2} \Rightarrow -x - \frac{5}{2} < 0 \Rightarrow x \in (-\frac{5}{2}, \infty)$: common values $(-\frac{5}{2}, -1)$

Case (ii) Suppose $-1 \leq x \leq 2$ $(x + 1 \geq 0, x - 2 \leq 0)$: $(x + 1) - (2 - x) < x - \frac{1}{2} \Rightarrow x - \frac{1}{2} < 0 \Rightarrow x \in (-\infty, \frac{1}{2})$: common values $[-1, \frac{1}{2})$

Case (iii) Suppose $x > 2$ $(x + 1 > 0, x - 2 > 0)$: $(x + 1) - (x - 2) < x - \frac{1}{2} \Rightarrow \frac{7}{2} - x < 0 \Rightarrow x \in (\frac{7}{2}, \infty)$: common values $(\frac{7}{2}, \infty)$

Ans: $x \in (-\frac{5}{2}, \frac{1}{2}) \cup (\frac{7}{2}, \infty)$

Ans: $x \in (-\frac{5}{2}, \frac{1}{2}) \cup (\frac{7}{2}, \infty)$

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17. Solve $\left|\frac{20x}{4x^2+25}\right| \leq 1$. [5]

$\frac{20|x|}{4x^2+25} \leq 1$ (the denominator is positive)

(i) $x < 0$: $-20x \leq 4x^2 + 25 \Rightarrow 4x^2 + 20x + 25 \geq 0 \Rightarrow (2x+5)^2 \geq 0$: true for all $x < 0$

(ii) $x \geq 0$: $20x \leq 4x^2 + 25 \Rightarrow 4x^2 - 20x + 25 \geq 0 \Rightarrow (2x-5)^2 \geq 0$: true for all $x \geq 0$

From (i), (ii): $x \in (-\infty, 0) \cup [0, \infty) = \mathbb{R}$

Ans: $x \in \mathbb{R}$

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18. Find the domain of $f(x) = \sqrt{x^2 - 4x + 4}$. [2]

$\sqrt{x^2 - 4x + 4}$ is defined when $x^2 - 4x + 4 \geq 0$

$\Rightarrow (x-2)^2 \geq 0$: a perfect square

Domain $= \mathbb{R}$

Ans: $\mathbb{R}$

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19. Find $[[-3.7]]$. [1]

$[-3.7] = -4$

$[[-3.7]] = [-4] = -4$ (property $[[x]] = [x]$)

Ans: -4

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20. Write the domain and range of $f(x) = x^3$. [1]

x^3 is defined for all real x: Domain $= \mathbb{R}$

It takes every real value (negative for $x < 0$): Range $= \mathbb{R}$

Ans: Domain $= \mathbb{R}$**, Range** $= \mathbb{R}$