

Functions

Question Paper, Set 05

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. If $f(x) = 3x - 4$, find the pre-image of 11. [1 mark]
2. Let $A = \{-1, 0, 1, 2\}$, $B = \{1, 2, 3, 4, 5\}$ and $f : A \rightarrow B$ be defined by $f(x) = x^2 + 1$. Write the domain, codomain and range of f . [2 marks]
3. Write the degree of the polynomial function $f(x) = 6 - 5x + 9x^4$. [1 mark]
4. Find the points where the line $y = -3x - 9$ cuts the x-axis and the y-axis. Is its inclination less than or greater than 90° ? [2 marks]
5. Find the discriminant of $3x^2 + 2x + 5 = 0$ and write the nature of its roots. [2 marks]
6. If α, β are the roots of $3x^2 + 7x - 6 = 0$, find $\alpha + \beta$, $\alpha\beta$ and $|\alpha - \beta|$. [3 marks]
7. Write $y = -x^2 + 4x + 1$ in vertex form. Find its vertex and its maximum value. [3 marks]
8. Draw the graph of $y = x^2 + 4x + 6$. (Find whether it opens upward or downward, where it meets the x-axis and the y-axis, and its vertex.) [5 marks]
9. For $y = (x - 2)^2 - 3$, find the values of x for which (i) $y < 0$ (ii) $y \leq 0$ (iii) $y > 0$ (iv) $y \geq 0$. [5 marks]
10. Write $\log_2 64 = 6$ in exponential form. [1 mark]
11. Find $\log_3 5 \cdot \log_5 81$. [2 marks]
12. Find $(2.5)^{\log_{2.5} 8 + 1}$. [3 marks]
13. Find $\left| \frac{-15}{3} \right|$ and $\left| \frac{-15}{3} \right|$. [1 mark]
14. Solve $|5 - x| \geq 3$. [2 marks]
15. Solve $|2x - 1| = |x^2 + 2x - 4|$. [3 marks]
16. Solve $|x + 1| - |x - 3| < x - 3$. [5 marks]
17. Solve $|x^2 + 6x| < 16$. [5 marks]
18. Find the domain of $f(x) = \sqrt{x^2 + x - 6}$. [2 marks]

19. If $2 < [x] \leq 5$, find the values of x . [1 mark]

20. Write the domain and range of $f(x) = \frac{1}{x^2}$. [1 mark]

Key sheet

Q	Answer
1	5
2	Domain = $\{-1, 0, 1, 2\}$, codomain = $\{1, 2, 3, 4, 5\}$, range = $\{1, 2, 5\}$
3	4
4	x-axis at $(-3, 0)$, y-axis at $(0, -9)$; inclination greater than 90°
5	$\Delta = -56$; roots are not real (imaginary)
6	$\alpha + \beta = -\frac{7}{3}$, $\alpha\beta = -2$, $ \alpha - \beta = \frac{11}{3}$
7	$y = -(x - 2)^2 + 5$; vertex $(2, 5)$; maximum value 5
8	Upward parabola; does not meet the x-axis; cuts the y-axis at $(0, 6)$; vertex $(-2, 2)$
9	(i) $x \in (2 - \sqrt{3}, 2 + \sqrt{3})$; (ii) $x \in [2 - \sqrt{3}, 2 + \sqrt{3}]$; (iii) $x \in (-\infty, 2 - \sqrt{3}) \cup (2 + \sqrt{3}, \infty)$; (iv) $x \in (-\infty, 2 - \sqrt{3}] \cup [2 + \sqrt{3}, \infty)$
10	$2^6 = 64$
11	4
12	20
13	5, 5
14	$x \in (-\infty, 2] \cup [8, \infty)$
15	$x \in \{-5, -\sqrt{3}, 1, \sqrt{3}\}$
16	$x \in (7, \infty)$
17	$x \in (-8, 2)$
18	$(-\infty, -3] \cup [2, \infty)$
19	$x \in [3, 6)$
20	Domain = $\mathbb{R} - \{0\}$, Range = $(0, \infty)$

Solutions

1. If $f(x) = 3x - 4$, find the pre-image of 11. [1]

Let x be the pre-image of 11. Then $f(x) = 11$

$$\Rightarrow 3x - 4 = 11$$

$$\Rightarrow 3x = 15$$

$$\Rightarrow x = 5$$

Ans: 5

2. Let $A = \{-1, 0, 1, 2\}$, $B = \{1, 2, 3, 4, 5\}$ and $f : A \rightarrow B$ be defined by $f(x) = x^2 + 1$. Write the domain, codomain and range of f . [2]

$$f(-1) = (-1)^2 + 1 = 2, f(0) = (0)^2 + 1 = 1, f(1) = (1)^2 + 1 = 2, f(2) = (2)^2 + 1 = 5$$

$$\text{Domain} = A = \{-1, 0, 1, 2\}$$

$$\text{Codomain} = B = \{1, 2, 3, 4, 5\}$$

$$\text{Range} = f(A) = \{1, 2, 5\}$$

Ans: Domain = $\{-1, 0, 1, 2\}$, codomain = $\{1, 2, 3, 4, 5\}$, range = $\{1, 2, 5\}$

3. Write the degree of the polynomial function $f(x) = 6 - 5x + 9x^4$. [1]

The highest power of x is in the term $9x^4$ (coefficient $9 \neq 0$)

$\therefore$ degree of $f(x)$ is 4

Ans: 4

4. Find the points where the line $y = -3x - 9$ cuts the x-axis and the y-axis. Is its inclination less than or greater than 90° ? [2]

$$\text{put } x = 0 \Rightarrow y = -9: \text{y-intercept } (0, -9)$$

$$\text{put } y = 0 \Rightarrow -3x - 9 = 0 \Rightarrow x = -3: \text{x-intercept } (-3, 0)$$

$$a = -3 < 0, \text{ so the inclination is greater than } 90^\circ$$

Ans: x-axis at $(-3, 0)$, y-axis at $(0, -9)$; inclination greater than 90°

5. Find the discriminant of $3x^2 + 2x + 5 = 0$ and write the nature of its roots. [2]

$$a = 3, b = 2, c = 5$$

$$\Delta = b^2 - 4ac = 2^2 - 4(3)(5) = -56$$

$$\Delta < 0 \Rightarrow \text{roots are not real (imaginary)}$$

Ans: $\Delta = -56$; roots are not real (imaginary)

6. If α, β are the roots of $3x^2 + 7x - 6 = 0$, find $\alpha + \beta$, $\alpha\beta$ and $|\alpha - \beta|$. [3]

$$a = 3, b = 7, c = -6$$

$$\alpha + \beta = -\frac{b}{a} = -\frac{7}{3}$$

$$\alpha\beta = \frac{c}{a} = -2$$

$$\Delta = b^2 - 4ac = 7^2 - 4(3)(-6) = 121$$

$$|\alpha - \beta| = \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{121}}{3} = \frac{11}{3}$$

Ans: $\alpha + \beta = -\frac{7}{3}$, $\alpha\beta = -2$, $|\alpha - \beta| = \frac{11}{3}$

7. Write $y = -x^2 + 4x + 1$ in vertex form. Find its vertex and its maximum value. [3]

$$a = -1, b = 4, c = 1$$

$$\text{Vertex} = \left(-\frac{b}{2a}, \frac{4ac-b^2}{4a}\right) = \left(2, \frac{-4-16}{-4}\right) = (2, 5)$$

$$y = a(x-h)^2 + k = -(x-2)^2 + 5$$

$a < 0$, so the parabola opens downward: maximum value = 5 at $x = 2$

Ans: $y = -(x-2)^2 + 5$; vertex (2, 5); maximum value 5

8. Draw the graph of $y = x^2 + 4x + 6$. (Find whether it opens upward or downward, where it meets the x-axis and the y-axis, and its vertex.) [5]

$$a = 1 > 0 \text{ (upward parabola)}$$

$$\Delta = b^2 - 4ac = 4^2 - 4(1)(6) = -8$$

$$a > 0, \Delta < 0: y > 0 \text{ for all } x \in \mathbb{R}$$

$\Delta < 0$: the graph does not meet the x-axis

put $x = 0 \Rightarrow y = 6$: cuts the y-axis at (0, 6)

$$\text{Vertex} = \left(-\frac{b}{2a}, \frac{4ac-b^2}{4a}\right) = (-2, 2)$$

Plot these points and draw the upward parabola through them (minimum value 2 at $x = -2$)

Ans: Upward parabola; does not meet the x-axis; cuts the y-axis at (0, 6); vertex (-2, 2)

9. For $y = (x-2)^2 - 3$, find the values of x for which (i) $y < 0$ (ii) $y \leq 0$ (iii) $y > 0$ (iv) $y \geq 0$. [5]

$$y = (x-2)^2 - 3 = x^2 - 4x + 1$$

$$a = 1 > 0 \text{ (upward parabola)}$$

$$\Delta = b^2 - 4ac = (-4)^2 - 4(1)(1) = 12$$

$$y = 0 \Rightarrow x = \frac{4 \pm \sqrt{12}}{2} = 2 \pm \sqrt{3}$$

Sign: + outside $2 - \sqrt{3}$ and $2 + \sqrt{3}$, - between them

$$\text{(i) } y < 0 \Rightarrow x \in (2 - \sqrt{3}, 2 + \sqrt{3})$$

$$\text{(ii) } y \leq 0 \Rightarrow x \in [2 - \sqrt{3}, 2 + \sqrt{3}]$$

$$\text{(iii) } y > 0 \Rightarrow x \in (-\infty, 2 - \sqrt{3}) \cup (2 + \sqrt{3}, \infty)$$

$$\text{(iv) } y \geq 0 \Rightarrow x \in (-\infty, 2 - \sqrt{3}] \cup [2 + \sqrt{3}, \infty)$$

Ans: (i) $x \in (2 - \sqrt{3}, 2 + \sqrt{3})$; (ii) $x \in [2 - \sqrt{3}, 2 + \sqrt{3}]$; (iii) $x \in (-\infty, 2 - \sqrt{3}) \cup (2 + \sqrt{3}, \infty)$; (iv) $x \in (-\infty, 2 - \sqrt{3}] \cup [2 + \sqrt{3}, \infty)$

10. Write $\log_2 64 = 6$ in exponential form. [1]

$$x = \log_a N \Leftrightarrow a^x = N$$

$$\log_2 64 = 6 \Leftrightarrow 2^6 = 64$$

Ans: $2^6 = 64$

11. Find $\log_3 5 \cdot \log_5 81$. [2]

$$\log_b a \cdot \log_c b = \log_c a$$

$$\log_5 81 \cdot \log_3 5 = \log_3 81$$

$$= \log_3 3^4 = 4$$

Ans: 4

12. Find $(2.5)^{\log_{2.5} 8 + 1}$. [3]

$$= (2.5)^{\log_{2.5} 8} \cdot (2.5)^1$$

$$a^{\log_a N} = N, a^{m+n} = a^m \cdot a^n$$

$$= 8 \times 2.5 = 20$$

Ans: 20

13. Find $\frac{|-15|}{|3|}$ and $|\frac{-15}{3}|$. [1]

$$\frac{|-15|}{|3|} = \frac{15}{3} = 5$$

$$|\frac{-15}{3}| = |-5| = 5$$

$$(\text{property } |\frac{x}{y}| = \frac{|x|}{|y|})$$

Ans: 5, 5

14. Solve $|5 - x| \geq 3$. [2]

$$5 - x \leq -3 \text{ (or) } 5 - x \geq 3$$

$$\Rightarrow x \leq 2 \text{ (or) } x \geq 8 \text{ (dividing by a negative number turns the signs)}$$

$$\Rightarrow x \in (-\infty, 2] \cup [8, \infty)$$

Ans: $x \in (-\infty, 2] \cup [8, \infty)$

15. Solve $|2x - 1| = |x^2 + 2x - 4|$. [3]

$$|x| = |y| \Rightarrow x = \pm y$$

$$2x - 1 = \pm(x^2 + 2x - 4)$$

$$2x - 1 = x^2 + 2x - 4 \Rightarrow x^2 - 3 = 0 \Rightarrow x = \pm\sqrt{3}$$

$$2x - 1 = -x^2 - 2x + 4 \Rightarrow x^2 + 4x - 5 = 0 \Rightarrow x = \frac{-4 \pm 6}{2} = -5, 1$$

$$\therefore x \in \{-5, -\sqrt{3}, 1, \sqrt{3}\}$$

Ans: $x \in \{-5, -\sqrt{3}, 1, \sqrt{3}\}$

16. Solve $|x + 1| - |x - 3| < x - 3$. [5]

$$\text{Critical points: } x + 1 = 0 \Rightarrow x = -1, x - 3 = 0 \Rightarrow x = 3$$

Case (i) Suppose $x < -1$ ($x + 1 < 0, x - 3 < 0$): $(-x - 1) - (3 - x) < x - 3 \Rightarrow -x - 1 < 0 \Rightarrow x \in (-1, \infty)$: no common value

Case (ii) Suppose $-1 \leq x \leq 3$ ($x + 1 \geq 0, x - 3 \leq 0$): $(x + 1) - (3 - x) < x - 3 \Rightarrow x + 1 < 0 \Rightarrow x \in (-\infty, -1)$: no common value

Case (iii) Suppose $x > 3$ ($x + 1 > 0, x - 3 > 0$): $(x + 1) - (x - 3) < x - 3 \Rightarrow 7 - x < 0 \Rightarrow x \in (7, \infty)$: common values $(7, \infty)$

Ans: $x \in (7, \infty)$

Ans: $x \in (7, \infty)$

17. Solve $|x^2 + 6x| < 16$. [5]

$$-16 < x^2 + 6x < 16$$

$$\Rightarrow x^2 + 6x + 16 > 0 \text{ and } x^2 + 6x - 16 < 0$$

$$x^2 + 6x + 16 > 0: a > 0, \Delta = -28 < 0 \Rightarrow x \in \mathbb{R}$$

$$x^2 + 6x - 16 < 0: \Delta = 100, x = \frac{-6 \pm 10}{2} = -8, 2 \Rightarrow x \in (-8, 2)$$

Common values: $x \in (-8, 2)$

Ans: $x \in (-8, 2)$

18. Find the domain of $f(x) = \sqrt{x^2 + x - 6}$. [2]

$$\sqrt{x^2 + x - 6} \text{ is defined when } x^2 + x - 6 \geq 0$$

$$\Rightarrow (x - 2)(x + 3) \geq 0$$

$$\text{Domain} = (-\infty, -3] \cup [2, \infty)$$

Ans: $(-\infty, -3] \cup [2, \infty)$

19. If $2 < [x] \leq 5$, find the values of x . [1]

$[x]$ is an integer: $[x] = 3, 4$ or 5

$[x] = 3$ for $3 \leq x < 4$, ..., $[x] = 5$ for $5 \leq x < 6$

$\therefore x \in [3, 6)$

Ans: $x \in [3, 6)$

20. Write the domain and range of $f(x) = \frac{1}{x^2}$. [1]

$\frac{1}{x^2}$ is not defined at $x = 0$: Domain = $\mathbb{R} - \{0\}$

$x^2 > 0$, so $\frac{1}{x^2} > 0$: Range = $(0, \infty)$

Ans: Domain = $\mathbb{R} - \{0\}$, Range = $(0, \infty)$
