

Functions

Question Paper, Set 03

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Is $f = \{(1, 4), (2, 5), (3, 4)\}$ a function from $A = \{1, 2, 3\}$ to $B = \{4, 5, 6\}$? Give a reason. [1 mark]
2. Let $A = \{3, 4, 5, 6\}$, $B = \{0, 1, 2, 3, 4\}$ and $f : A \rightarrow B$ be defined by $f(x) = x - 3$. Write the domain, codomain and range of f . [2 marks]
3. If $f(x) = 2x^3 - x^2 + 4$, find $f(-2)$. [1 mark]
4. Find the points where the line $y = 8 - 4x$ cuts the x-axis and the y-axis. Is its inclination less than or greater than 90° ? [2 marks]
5. Find the discriminant of $x^2 + 8x + 16 = 0$ and write the nature of its roots. [2 marks]
6. If α, β are the roots of $x^2 - 6x + 4 = 0$, find $\alpha + \beta$, $\alpha\beta$ and $|\alpha - \beta|$. [3 marks]
7. Write $y = x^2 + 6x + 4$ in vertex form. Find its vertex and its minimum value. [3 marks]
8. Draw the graph of $y = 2(x + 2)^2 + 1$. (Find whether it opens upward or downward, where it meets the x-axis and the y-axis, and its vertex.) [5 marks]
9. For $y = (3x + 4)^2$, find the values of x for which (i) $y < 0$ (ii) $y \leq 0$ (iii) $y > 0$ (iv) $y \geq 0$. [5 marks]
10. If $\log_3 x = 4$, find x . [1 mark]
11. Find $\log_8 32$. [2 marks]
12. Find $\log_3 2 \cdot \log_{11} 3 \cdot \log_{16} 11$. [3 marks]
13. If $x = -6$ and $y = 2$, find $|xy|$ and $|x||y|$. [1 mark]
14. Solve $|3x - 1| \leq 5$. [2 marks]
15. Solve $|3x - 2| = |x + 6|$. [3 marks]
16. Solve $|2x - 5| + x = 7$. [5 marks]
17. Solve $x^2 - 6|x| + 5 \leq 0$. [5 marks]
18. Find the domain of $f(x) = \sqrt{4 - x^2}$. [2 marks]

19. Find $[x] + [-x]$ for $x = 6.4$ and for $x = 6$. [1 mark]

20. If $x \in [-4, 2] - \{0\}$, find the set of values of $\frac{1}{x}$. [1 mark]

Key sheet

Q	Answer
1	Yes
2	Domain = $\{3, 4, 5, 6\}$, codomain = $\{0, 1, 2, 3, 4\}$, range = $\{0, 1, 2, 3\}$
3	-16
4	x-axis at $(2, 0)$, y-axis at $(0, 8)$; inclination greater than 90°
5	$\Delta = 0$; roots are real and equal
6	$\alpha + \beta = 6$, $\alpha\beta = 4$, $ \alpha - \beta = 2\sqrt{5}$
7	$y = (x + 3)^2 - 5$; vertex $(-3, -5)$; minimum value -5
8	Upward parabola; does not meet the x-axis; cuts the y-axis at $(0, 9)$; vertex $(-2, 1)$
9	(i) $x \in \phi$; (ii) $x \in \{-\frac{4}{3}\}$; (iii) $x \in \mathbb{R} - \{-\frac{4}{3}\}$; (iv) $x \in \mathbb{R}$
10	81
11	$\frac{5}{3}$
12	$\frac{1}{4}$
13	$ xy = 12$, $ x y = 12$
14	$x \in [-\frac{4}{3}, 2]$
15	$x \in \{-1, 4\}$
16	$x \in \{-2, 4\}$
17	$x \in [-5, -1] \cup [1, 5]$
18	$[-2, 2]$
19	$-1, 0$
20	$(-\infty, -\frac{1}{4}] \cup [\frac{1}{2}, \infty)$

Solutions

1. Is $f = \{(1, 4), (2, 5), (3, 4)\}$ a function from $A = \{1, 2, 3\}$ to $B = \{4, 5, 6\}$? Give a reason. [1]

Each element of A has exactly one image: $1 \rightarrow 4, 2 \rightarrow 5, 3 \rightarrow 4$

Two elements having the same image is allowed

$\therefore f$ is a function

Ans: Yes

2. Let $A = \{3, 4, 5, 6\}$, $B = \{0, 1, 2, 3, 4\}$ and $f : A \rightarrow B$ be defined by $f(x) = x - 3$. Write the domain, codomain and range of f . [2]

$$f(3) = (3) - 3 = 0, f(4) = (4) - 3 = 1, f(5) = (5) - 3 = 2, f(6) = (6) - 3 = 3$$

$$\text{Domain} = A = \{3, 4, 5, 6\}$$

$$\text{Codomain} = B = \{0, 1, 2, 3, 4\}$$

$$\text{Range} = f(A) = \{0, 1, 2, 3\}$$

Ans: Domain = $\{3, 4, 5, 6\}$, **codomain** = $\{0, 1, 2, 3, 4\}$, **range** = $\{0, 1, 2, 3\}$

3. If $f(x) = 2x^3 - x^2 + 4$, find $f(-2)$. [1]

$$f(-2) = 2(-2)^3 - (-2)^2 + 4$$

$$= -16$$

Ans: -16

4. Find the points where the line $y = 8 - 4x$ cuts the x-axis and the y-axis. Is its inclination less than or greater than 90° ? [2]

$$\text{put } x = 0 \Rightarrow y = 8: \text{y-intercept } (0, 8)$$

$$\text{put } y = 0 \Rightarrow 8 - 4x = 0 \Rightarrow x = 2: \text{x-intercept } (2, 0)$$

$$a = -4 < 0, \text{ so the inclination is greater than } 90^\circ$$

Ans: x-axis at (2, 0), y-axis at (0, 8); inclination greater than 90°

5. Find the discriminant of $x^2 + 8x + 16 = 0$ and write the nature of its roots. [2]

$$a = 1, b = 8, c = 16$$

$$\Delta = b^2 - 4ac = 8^2 - 4(1)(16) = 0$$

$$\Delta = 0 \Rightarrow \text{roots are real and equal}$$

Ans: $\Delta = 0$; roots are real and equal

6. If α, β are the roots of $x^2 - 6x + 4 = 0$, find $\alpha + \beta$, $\alpha\beta$ and $|\alpha - \beta|$. [3]

$$a = 1, b = -6, c = 4$$

$$\alpha + \beta = -\frac{b}{a} = 6$$

$$\alpha\beta = \frac{c}{a} = 4$$

$$\Delta = b^2 - 4ac = (-6)^2 - 4(1)(4) = 20$$

$$|\alpha - \beta| = \frac{\sqrt{\Delta}}{|a|} = \sqrt{20} = 2\sqrt{5}$$

Ans: $\alpha + \beta = 6$, $\alpha\beta = 4$, $|\alpha - \beta| = 2\sqrt{5}$

7. Write $y = x^2 + 6x + 4$ in vertex form. Find its vertex and its minimum value. [3]

$$a = 1, b = 6, c = 4$$

$$\text{Vertex} = \left(-\frac{b}{2a}, \frac{4ac-b^2}{4a}\right) = \left(-3, \frac{16-36}{4}\right) = (-3, -5)$$

$$y = a(x-h)^2 + k = (x+3)^2 - 5$$

$a > 0$, so the parabola opens upward: minimum value = -5 at $x = -3$

Ans: $y = (x+3)^2 - 5$; vertex $(-3, -5)$; minimum value -5

8. Draw the graph of $y = 2(x+2)^2 + 1$. (Find whether it opens upward or downward, where it meets the x-axis and the y-axis, and its vertex.) [5]

$$y = 2(x+2)^2 + 1 = 2x^2 + 8x + 9$$

$a = 2 > 0$ (upward parabola)

$$\Delta = b^2 - 4ac = 8^2 - 4(2)(9) = -8$$

$\Delta < 0$: the graph does not meet the x-axis

put $x = 0 \Rightarrow y = 9$: cuts the y-axis at $(0, 9)$

$$\text{Vertex} = \left(-\frac{b}{2a}, \frac{4ac-b^2}{4a}\right) = (-2, 1)$$

Plot these points and draw the upward parabola through them (minimum value 1 at $x = -2$)

Ans: Upward parabola; does not meet the x-axis; cuts the y-axis at $(0, 9)$; vertex $(-2, 1)$

9. For $y = (3x+4)^2$, find the values of x for which (i) $y < 0$ (ii) $y \leq 0$ (iii) $y > 0$ (iv) $y \geq 0$. [5]

$$y = (3x+4)^2 = 9x^2 + 24x + 16$$

$a = 9 > 0$ (upward parabola)

$$\Delta = b^2 - 4ac = 24^2 - 4(9)(16) = 0$$

$$\Delta = 0 \Rightarrow y = 9\left(x + \frac{4}{3}\right)^2$$

$y > 0$ for all $x \neq -\frac{4}{3}$, and $y = 0$ at $x = -\frac{4}{3}$

(i) $y < 0 \Rightarrow x \in \phi$

(ii) $y \leq 0 \Rightarrow x \in \left\{-\frac{4}{3}\right\}$

(iii) $y > 0 \Rightarrow x \in \mathbb{R} - \left\{-\frac{4}{3}\right\}$

(iv) $y \geq 0 \Rightarrow x \in \mathbb{R}$

Ans: (i) $x \in \phi$; (ii) $x \in \left\{-\frac{4}{3}\right\}$; (iii) $x \in \mathbb{R} - \left\{-\frac{4}{3}\right\}$; (iv) $x \in \mathbb{R}$

10. If $\log_3 x = 4$, find x . [1]

$$\log_3 x = 4 \Leftrightarrow x = 3^4$$

$$x = 81$$

Ans: 81

11. Find $\log_8 32$. [2]

$$\log_8 32 = \log_{2^3} 2^5$$

$$\log_{b^n} a^m = \frac{m}{n} \log_b a$$

$$= \frac{5}{3} \log_2 2 = \frac{5}{3}$$

Ans: $\frac{5}{3}$

12. Find $\log_3 2 \cdot \log_{11} 3 \cdot \log_{16} 11$. [3]

$$\begin{aligned}\log_b a \cdot \log_c b \cdot \log_d c &= \log_d a \\ &= \log_{16} 2 = \log_{2^4} 2^1 = \frac{1}{4}\end{aligned}$$

Ans: $\frac{1}{4}$

13. If $x = -6$ and $y = 2$, find $|xy|$ and $|x||y|$. [1]

$$\begin{aligned}|xy| &= |-12| = 12 \\ |x||y| &= 6 \times 2 = 12 \\ (\text{so } |xy| &= |x||y|)\end{aligned}$$

Ans: $|xy| = 12, |x||y| = 12$

14. Solve $|3x - 1| \leq 5$. [2]

$$\begin{aligned}-5 &\leq 3x - 1 \leq 5 \\ \Rightarrow -5 + 1 &\leq 3x \leq 5 + 1 \\ \Rightarrow -\frac{4}{3} &\leq x \leq 2 \\ \Rightarrow x &\in \left[-\frac{4}{3}, 2\right]\end{aligned}$$

Ans: $x \in \left[-\frac{4}{3}, 2\right]$

15. Solve $|3x - 2| = |x + 6|$. [3]

$$\begin{aligned}|x| &= |y| \Rightarrow x = \pm y \\ 3x - 2 &= \pm(x + 6) \\ 3x - 2 &= x + 6 \Rightarrow 2x - 8 = 0 \Rightarrow x = 4 \\ 3x - 2 &= -x - 6 \Rightarrow 4x + 4 = 0 \Rightarrow x = -1 \\ \therefore x &\in \{-1, 4\}\end{aligned}$$

Ans: $x \in \{-1, 4\}$

16. Solve $|2x - 5| + x = 7$. [5]

Critical points: $2x - 5 = 0 \Rightarrow x = \frac{5}{2}$

Case (i) Suppose $x < \frac{5}{2}$ ($2x - 5 < 0$): $(5 - 2x) + x = 7 \Rightarrow 5 - x = 7 \Rightarrow x = -2$, which is in this case

Case (ii) Suppose $x \geq \frac{5}{2}$ ($2x - 5 \geq 0$): $(2x - 5) + x = 7 \Rightarrow 3x - 5 = 7 \Rightarrow x = 4$, which is in this case

Ans: $x \in \{-2, 4\}$

Ans: $x \in \{-2, 4\}$

17. Solve $x^2 - 6|x| + 5 \leq 0$. [5]

$x = 0$ is the critical point

Case (i) Suppose $x < 0$: $|x| = -x$, $x^2 + 6x + 5 \leq 0 \Rightarrow (x + 1)(x + 5) \leq 0 \Rightarrow x \in [-5, -1]$; common values $[-5, -1]$

Case (ii) Suppose $x \geq 0$: $|x| = x$, $x^2 - 6x + 5 \leq 0 \Rightarrow (x - 5)(x - 1) \leq 0 \Rightarrow x \in [1, 5]$; common values $[1, 5]$

From (i), (ii): $x \in [-5, -1] \cup [1, 5]$

Ans: $x \in [-5, -1] \cup [1, 5]$

18. Find the domain of $f(x) = \sqrt{4 - x^2}$. [2]

$\sqrt{4 - x^2}$ is defined when $4 - x^2 \geq 0$

$$\Rightarrow x^2 - 4 \leq 0$$

$$\Rightarrow (x - 2)(x + 2) \leq 0$$

$$\text{Domain} = [-2, 2]$$

$$\text{Ans: } [-2, 2]$$

19. Find $[x] + [-x]$ for $x = 6.4$ and for $x = 6$. [1]

$$x = 6.4 \notin \mathbb{Z}: [6.4] + [-6.4] = 6 + (-7) = -1$$

$$x = 6 \in \mathbb{Z}: [6] + [-6] = 6 - 6 = 0$$

$$\text{Ans: } -1, 0$$

20. If $x \in [-4, 2] - \{0\}$, find the set of values of $\frac{1}{x}$. [1]

$$\text{For } -4 \leq x < 0: \frac{1}{x} \leq -\frac{1}{4}$$

$$\text{For } 0 < x \leq 2: \frac{1}{x} \geq \frac{1}{2}$$

$$\frac{1}{x} \in (-\infty, -\frac{1}{4}] \cup [\frac{1}{2}, \infty)$$

$$\text{Ans: } (-\infty, -\frac{1}{4}] \cup [\frac{1}{2}, \infty)$$
