

Functions

Question Paper, Set 02

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. If $f(x) = 2x + 5$, find the pre-image of 17. [1 mark]
2. Let $A = \{0, 1, 2, 3\}$, $B = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and $f : A \rightarrow B$ be defined by $f(x) = 2x + 1$. [2 marks]
Write the domain, codomain and range of f .
3. If $f : \mathbb{R} \rightarrow \mathbb{R}$ is a constant function and $f(5) = -3$, find $f(-2) + f(8)$. [1 mark]
4. Find the points where the line $y = 2x + 5$ cuts the x-axis and the y-axis. Is its inclination less than or greater than 90° ? [2 marks]
5. Solve $3x^2 + 5x - 2 = 0$ using the formula $x = \frac{-b \pm \sqrt{\Delta}}{2a}$. [2 marks]
6. Write $x^2 - 8x + 15$ in the form $a(x - \alpha)(x - \beta)$, where α, β are the roots of $x^2 - 8x + 15 = 0$. [3 marks]
7. Find the range of $f(x) = -x^2 + 6x - 5$, $x \in \mathbb{R}$. [3 marks]
8. Draw the graph of $y = (3x - 2)^2$. (Find whether it opens upward or downward, where it meets the x-axis and the y-axis, and its vertex.) [5 marks]
9. For $y = (4 - x)(x + 2)$, find the values of x for which (i) $y < 0$ (ii) $y \leq 0$ (iii) $y > 0$ (iv) $y \geq 0$ [5 marks]
10. Find $\log_5 125$. [1 mark]
11. Solve $\log_5(x - 4) > 0$. [2 marks]
12. Solve $\log_2 x + \log_4 x^2 + \log_8 x^3 = 6$. [3 marks]
13. Find $|2 - 9| - |-4|$. [1 mark]
14. Solve $|x + 3| \geq 4$. [2 marks]
15. Solve $|x^2 + 4x - 1| = 4$. [3 marks]
16. Solve $|x + 1| + |3x + 1| = 6$. [5 marks]
17. Solve $|x^2 - 2x| < 3$. [5 marks]

18. Find the domain of $f(x) = \frac{1}{\sqrt{3x-5}}$. [2 marks]

19. Find $[-3.4] + [2.7]$. [1 mark]

20. Find $\{-3.6\}$ and $\{3.6\}$. [1 mark]

Key sheet

Q	Answer
1	6
2	Domain = $\{0, 1, 2, 3\}$, codomain = $\{1, 2, 3, 4, 5, 6, 7, 8\}$, range = $\{1, 3, 5, 7\}$
3	-6
4	x-axis at $(-\frac{5}{2}, 0)$, y-axis at $(0, 5)$; inclination less than 90°
5	$x = -2, \frac{1}{3}$
6	$(x - 3)(x - 5)$
7	Range = $(-\infty, 4]$
8	Upward parabola; touches the x-axis at $(\frac{2}{3}, 0)$; cuts the y-axis at $(0, 4)$; vertex $(\frac{2}{3}, 0)$
9	(i) $x \in (-\infty, -2) \cup (4, \infty)$; (ii) $x \in (-\infty, -2] \cup [4, \infty)$; (iii) $x \in (-2, 4)$; (iv) $x \in [-2, 4]$
10	3
11	$x \in (5, \infty)$
12	$x = 4$
13	3
14	$x \in (-\infty, -7] \cup [1, \infty)$
15	$x \in \{-5, -3, -1, 1\}$
16	$x \in \{-2, 1\}$
17	$x \in (-1, 3)$
18	$(\frac{5}{3}, \infty)$
19	-2
20	0.4, 0.6

Solutions

1. If $f(x) = 2x + 5$, find the pre-image of 17. [1]

Let x be the pre-image of 17. Then $f(x) = 17$

$$\Rightarrow 2x + 5 = 17$$

$$\Rightarrow 2x = 12$$

$$\Rightarrow x = 6$$

Ans: 6

2. Let $A = \{0, 1, 2, 3\}$, $B = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and $f : A \rightarrow B$ be defined by $f(x) = 2x + 1$. Write the domain, codomain and range of f . [2]

$$f(0) = 2(0) + 1 = 1, f(1) = 2(1) + 1 = 3, f(2) = 2(2) + 1 = 5, f(3) = 2(3) + 1 = 7$$

$$\text{Domain} = A = \{0, 1, 2, 3\}$$

$$\text{Codomain} = B = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

$$\text{Range} = f(A) = \{1, 3, 5, 7\}$$

Ans: Domain = $\{0, 1, 2, 3\}$, codomain = $\{1, 2, 3, 4, 5, 6, 7, 8\}$, range = $\{1, 3, 5, 7\}$

3. If $f : \mathbb{R} \rightarrow \mathbb{R}$ is a constant function and $f(5) = -3$, find $f(-2) + f(8)$. [1]

f is constant, so $f(x) = -3$ for every x

$$f(-2) + f(8) = -3 + (-3) = -6$$

Ans: -6

4. Find the points where the line $y = 2x + 5$ cuts the x-axis and the y-axis. Is its inclination less than or greater than 90° ? [2]

$$\text{put } x = 0 \Rightarrow y = 5: \text{y-intercept } (0, 5)$$

$$\text{put } y = 0 \Rightarrow 2x + 5 = 0 \Rightarrow x = -\frac{5}{2}: \text{x-intercept } (-\frac{5}{2}, 0)$$

$a = 2 > 0$, so the inclination is less than 90°

Ans: x-axis at $(-\frac{5}{2}, 0)$, y-axis at $(0, 5)$; inclination less than 90°

5. Solve $3x^2 + 5x - 2 = 0$ using the formula $x = \frac{-b \pm \sqrt{\Delta}}{2a}$. [2]

$$a = 3, b = 5, c = -2$$

$$\Delta = b^2 - 4ac = 5^2 - 4(3)(-2) = 49$$

$$x = \frac{-5 \pm 7}{6} = -2, \frac{1}{3}$$

Ans: $x = -2, \frac{1}{3}$

6. Write $x^2 - 8x + 15$ in the form $a(x - \alpha)(x - \beta)$, where α, β are the roots of $x^2 - 8x + 15 = 0$. [3]

$$\Delta = b^2 - 4ac = (-8)^2 - 4(1)(15) = 4$$

$$x = \frac{8 \pm 2}{2} = 3, 5$$

$$\alpha = 3, \beta = 5, a = 1$$

$$x^2 - 8x + 15 = (x - 3)(x - 5)$$

Ans: $(x - 3)(x - 5)$

7. Find the range of $f(x) = -x^2 + 6x - 5$, $x \in \mathbb{R}$. [3]

$$a = -1, b = 6, c = -5$$

$$\text{Vertex} = \left(-\frac{b}{2a}, \frac{4ac-b^2}{4a}\right) = \left(3, \frac{20-36}{-4}\right) = (3, 4)$$

$a < 0$: maximum value is 4

$$\therefore \text{Range} = (-\infty, 4]$$

$$\text{Ans: Range} = (-\infty, 4]$$

8. Draw the graph of $y = (3x - 2)^2$. (Find whether it opens upward or downward, where it meets the x-axis and the y-axis, and its vertex.) [5]

$$y = (3x - 2)^2 = 9x^2 - 12x + 4$$

$a = 9 > 0$ (upward parabola)

$$\Delta = b^2 - 4ac = (-12)^2 - 4(9)(4) = 0$$

$\Delta = 0$: touches the x-axis at $(\frac{2}{3}, 0)$

put $x = 0 \Rightarrow y = 4$: cuts the y-axis at $(0, 4)$

$$\text{Vertex} = \left(-\frac{b}{2a}, \frac{4ac-b^2}{4a}\right) = \left(\frac{2}{3}, 0\right)$$

Plot these points and draw the upward parabola through them (minimum value 0 at $x = \frac{2}{3}$)

Ans: Upward parabola; touches the x-axis at $(\frac{2}{3}, 0)$; cuts the y-axis at $(0, 4)$; vertex $(\frac{2}{3}, 0)$

9. For $y = (4 - x)(x + 2)$, find the values of x for which (i) $y < 0$ (ii) $y \leq 0$ (iii) $y > 0$ (iv) $y \geq 0$. [5]

$$y = (4 - x)(x + 2) = -x^2 + 2x + 8$$

$a = -1 < 0$ (downward parabola)

$$\Delta = b^2 - 4ac = 2^2 - 4(-1)(8) = 36$$

$$y = 0 \Rightarrow x = \frac{-2 \pm 6}{-2} = -2, 4$$

Sign: $-$ outside -2 and 4 , $+$ between them

$$(i) y < 0 \Rightarrow x \in (-\infty, -2) \cup (4, \infty)$$

$$(ii) y \leq 0 \Rightarrow x \in (-\infty, -2] \cup [4, \infty)$$

$$(iii) y > 0 \Rightarrow x \in (-2, 4)$$

$$(iv) y \geq 0 \Rightarrow x \in [-2, 4]$$

Ans: (i) $x \in (-\infty, -2) \cup (4, \infty)$; (ii) $x \in (-\infty, -2] \cup [4, \infty)$; (iii) $x \in (-2, 4)$; (iv) $x \in [-2, 4]$

10. Find $\log_5 125$. [1]

$$5^3 = 125$$

$$\therefore \log_5 125 = 3$$

Ans: 3

11. Solve $\log_5(x - 4) > 0$. [2]

Base $5 > 1$: $\log_a T > 0 \Leftrightarrow T > 1$

$$x - 4 > 1 \Rightarrow x > 5$$

$$x \in (5, \infty)$$

Ans: $x \in (5, \infty)$

12. Solve $\log_2 x + \log_4 x^2 + \log_8 x^3 = 6$. [3]

$$\log_2 x + \log_{2^2} x^2 + \log_{2^3} x^3 = 6$$

$$\log_2 x + \frac{2}{2} \log_2 x + \frac{3}{3} \log_2 x = 6$$

$$3 \log_2 x = 6 \Rightarrow \log_2 x = 2 \Rightarrow x = 2^2 = 4$$

Ans: $x = 4$

13. Find $|2 - 9| - |-4|$. [1]

$$|2 - 9| = |-7| = 7, |-4| = 4$$

$$7 - 4 = 3$$

Ans: 3

14. Solve $|x + 3| \geq 4$. [2]

$$x + 3 \leq -4 \text{ (or) } x + 3 \geq 4$$

$$\Rightarrow x \leq -7 \text{ (or) } x \geq 1$$

$$\Rightarrow x \in (-\infty, -7] \cup [1, \infty)$$

Ans: $x \in (-\infty, -7] \cup [1, \infty)$

15. Solve $|x^2 + 4x - 1| = 4$. [3]

$$|x| = k \Rightarrow x = \pm k$$

$$x^2 + 4x - 1 = \pm 4$$

$$x^2 + 4x - 1 = 4 \Rightarrow x^2 + 4x - 5 = 0 \Rightarrow x = \frac{-4 \pm 6}{2} = -5, 1$$

$$x^2 + 4x - 1 = -4 \Rightarrow x^2 + 4x + 3 = 0 \Rightarrow x = \frac{-4 \pm 2}{2} = -3, -1$$

$$\therefore x \in \{-5, -3, -1, 1\}$$

Ans: $x \in \{-5, -3, -1, 1\}$

16. Solve $|x + 1| + |3x + 1| = 6$. [5]

$$\text{Critical points: } x + 1 = 0 \Rightarrow x = -1, 3x + 1 = 0 \Rightarrow x = -\frac{1}{3}$$

$$\text{Case (i) Suppose } x < -1 \text{ (} x + 1 < 0, 3x + 1 < 0 \text{): } (-x - 1) + (-3x - 1) = 6 \Rightarrow -4x - 2 = 6 \Rightarrow x = -2, \text{ which is in this case}$$

$$\text{Case (ii) Suppose } -1 \leq x \leq -\frac{1}{3} \text{ (} x + 1 \geq 0, 3x + 1 \leq 0 \text{): } (x + 1) + (-3x - 1) = 6 \Rightarrow -2x = 6 \Rightarrow x = -3, \text{ not in this case: no solution}$$

$$\text{Case (iii) Suppose } x > -\frac{1}{3} \text{ (} x + 1 > 0, 3x + 1 > 0 \text{): } (x + 1) + (3x + 1) = 6 \Rightarrow 4x + 2 = 6 \Rightarrow x = 1, \text{ which is in this case}$$

$$\text{Ans: } x \in \{-2, 1\}$$

$$\text{Ans: } x \in \{-2, 1\}$$

17. Solve $|x^2 - 2x| < 3$. [5]

$$-3 < x^2 - 2x < 3$$

$$\Rightarrow x^2 - 2x + 3 > 0 \text{ and } x^2 - 2x - 3 < 0$$

$$x^2 - 2x + 3 > 0: a > 0, \Delta = -8 < 0 \Rightarrow x \in \mathbb{R}$$

$$x^2 - 2x - 3 < 0: \Delta = 16, x = \frac{2 \pm 4}{2} = -1, 3 \Rightarrow x \in (-1, 3)$$

$$\text{Common values: } x \in (-1, 3)$$

$$\text{Ans: } x \in (-1, 3)$$

18. Find the domain of $f(x) = \frac{1}{\sqrt{3x-5}}$. [2]

$\frac{1}{\sqrt{3x-5}}$ is defined when $3x - 5 > 0$

$$\Rightarrow 3x > 5$$

$$\Rightarrow x > \frac{5}{3}$$

$$\text{Domain} = \left(\frac{5}{3}, \infty\right)$$

$$\text{Ans: } \left(\frac{5}{3}, \infty\right)$$

19. Find $[-3.4] + [2.7]$. [1]

$$-4 \leq -3.4 < -3 \Rightarrow [-3.4] = -4$$

$$2 \leq 2.7 < 3 \Rightarrow [2.7] = 2$$

$$-4 + 2 = -2$$

$$\text{Ans: } -2$$

20. Find $\{-3.6\}$ and $\{3.6\}$. [1]

$$\{x\} = x - [x]$$

$$\{-3.6\} = -3.6 - [-3.6] = -3.6 - (-4) = 0.4$$

$$\{3.6\} = 3.6 - 3 = 0.6$$

$$\text{Ans: } 0.4, 0.6$$
