

Functions

Question Paper, Set 10

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. If $f(x) = 6x + 1$, find the pre-image of -11 . [1 mark]
2. Let $A = \{-1, 0, 1, 2\}$, $B = \{0, 2, 4, 6, 8\}$ and $f : A \rightarrow B$ be defined by $f(x) = 4 - 2x$. [2 marks]
Write the domain, codomain and range of f .
3. Let $A = \{1, 3, 5\}$ and $f : A \rightarrow B$ be given by $f(x) = 8$ for all $x \in A$. Write f as a set of [1 mark]
ordered pairs and find its range.
4. Find the points where the line $y = -5x - 10$ cuts the x-axis and the y-axis. Is its inclination [2 marks]
less than or greater than 90° ?
5. Solve $2x^2 - 9x + 4 = 0$ using the formula $x = \frac{-b \pm \sqrt{\Delta}}{2a}$. [2 marks]
6. If α, β are the roots of $5x^2 - 2x - 3 = 0$, find $\alpha + \beta$, $\alpha\beta$ and $|\alpha - \beta|$. [3 marks]
7. Write $y = x^2 - 5x + 4$ in vertex form. Find its vertex and its minimum value. [3 marks]
8. Draw the graph of $y = -2(x + 1)^2 + 8$. (Find whether it opens upward or downward, [5 marks]
where it meets the x-axis and the y-axis, and its vertex.)
9. For $y = (x + 1)^2 - 5$, find the values of x for which (i) $y < 0$ (ii) $y \leq 0$ (iii) $y > 0$ (iv) $y \geq 0$. [5 marks]
10. If $\log_6(2x + 1) = 2$, find x . [1 mark]
11. Find the points on the graph of $y = \log_3 x$ for $x = \frac{1}{9}, \frac{1}{3}, 1, 3, 9$. [2 marks]
12. Find $\log_9 8 \cdot \log_{25} 27 \cdot \log_4 125$. [3 marks]
13. Find $|3 - 10| + |10 - 3|$. [1 mark]
14. Solve $|3x + 4| \geq 2$. [2 marks]
15. Solve $|x^2 - 3x| = 4$. [3 marks]
16. Solve $|x + 4| + |2x + 5| = 6$. [5 marks]
17. Solve $x^2 + 5|x| - 6 < 0$. [5 marks]
18. Find the domain of $f(x) = \sqrt{2x^2 - 5x + 2}$. [2 marks]

19. Find $[-2.5] - [2.5]$. [1 mark]

20. Find $x - [x]$ for $x = -0.45$. [1 mark]

Key sheet

Q	Answer
1	-2
2	Domain = $\{-1, 0, 1, 2\}$, codomain = $\{0, 2, 4, 6, 8\}$, range = $\{0, 2, 4, 6\}$
3	$f = \{(1, 8), (3, 8), (5, 8)\}$, range = $\{8\}$
4	x-axis at $(-2, 0)$, y-axis at $(0, -10)$; inclination greater than 90°
5	$x = \frac{1}{2}, 4$
6	$\alpha + \beta = \frac{2}{5}$, $\alpha\beta = -\frac{3}{5}$, $ \alpha - \beta = \frac{8}{5}$
7	$y = (x - \frac{5}{2})^2 - \frac{9}{4}$; vertex $(\frac{5}{2}, -\frac{9}{4})$; minimum value $-\frac{9}{4}$
8	Downward parabola; cuts the x-axis at $(-3, 0)$ and $(1, 0)$; cuts the y-axis at $(0, 6)$; vertex $(-1, 8)$
9	(i) $x \in (-1 - \sqrt{5}, -1 + \sqrt{5})$; (ii) $x \in [-1 - \sqrt{5}, -1 + \sqrt{5}]$; (iii) $x \in (-\infty, -1 - \sqrt{5}) \cup (-1 + \sqrt{5}, \infty)$; (iv) $x \in (-\infty, -1 - \sqrt{5}] \cup [-1 + \sqrt{5}, \infty)$
10	$\frac{35}{2}$
11	$(\frac{1}{9}, -2), (\frac{1}{3}, -1), (1, 0), (3, 1), (9, 2)$
12	$\frac{27}{8}$
13	14
14	$x \in (-\infty, -2] \cup [-\frac{2}{3}, \infty)$
15	$x \in \{-1, 4\}$
16	$x \in \{-5, -1\}$
17	$x \in (-1, 1)$
18	$(-\infty, \frac{1}{2}] \cup [2, \infty)$
19	-5
20	0.55

Solutions

1. If $f(x) = 6x + 1$, find the pre-image of -11 . [1]

Let x be the pre-image of -11 . Then $f(x) = -11$

$$\Rightarrow 6x + 1 = -11$$

$$\Rightarrow 6x = -12$$

$$\Rightarrow x = -2$$

Ans: -2

2. Let $A = \{-1, 0, 1, 2\}$, $B = \{0, 2, 4, 6, 8\}$ and $f : A \rightarrow B$ be defined by $f(x) = 4 - 2x$. Write the domain, codomain and range of f . [2]

$$f(-1) = 4 - 2(-1) = 6, f(0) = 4 - 2(0) = 4, f(1) = 4 - 2(1) = 2, f(2) = 4 - 2(2) = 0$$

$$\text{Domain} = A = \{-1, 0, 1, 2\}$$

$$\text{Codomain} = B = \{0, 2, 4, 6, 8\}$$

$$\text{Range} = f(A) = \{0, 2, 4, 6\}$$

Ans: Domain = $\{-1, 0, 1, 2\}$, codomain = $\{0, 2, 4, 6, 8\}$, range = $\{0, 2, 4, 6\}$

3. Let $A = \{1, 3, 5\}$ and $f : A \rightarrow B$ be given by $f(x) = 8$ for all $x \in A$. Write f as a set of ordered pairs and find its range. [1]

$$f(1) = f(3) = f(5) = 8$$

$$f = \{(1, 8), (3, 8), (5, 8)\}$$

Range = $\{8\}$ (only one element, so f is a constant function)

Ans: $f = \{(1, 8), (3, 8), (5, 8)\}$, range = $\{8\}$

4. Find the points where the line $y = -5x - 10$ cuts the x-axis and the y-axis. Is its inclination less than or greater than 90° ? [2]

$$\text{put } x = 0 \Rightarrow y = -10: \text{y-intercept } (0, -10)$$

$$\text{put } y = 0 \Rightarrow -5x - 10 = 0 \Rightarrow x = -2: \text{x-intercept } (-2, 0)$$

$a = -5 < 0$, so the inclination is greater than 90°

Ans: x-axis at $(-2, 0)$, y-axis at $(0, -10)$; inclination greater than 90°

5. Solve $2x^2 - 9x + 4 = 0$ using the formula $x = \frac{-b \pm \sqrt{\Delta}}{2a}$. [2]

$$a = 2, b = -9, c = 4$$

$$\Delta = b^2 - 4ac = (-9)^2 - 4(2)(4) = 49$$

$$x = \frac{9 \pm 7}{4} = \frac{1}{2}, 4$$

Ans: $x = \frac{1}{2}, 4$

6. If α, β are the roots of $5x^2 - 2x - 3 = 0$, find $\alpha + \beta$, $\alpha\beta$ and $|\alpha - \beta|$. [3]

$$a = 5, b = -2, c = -3$$

$$\alpha + \beta = -\frac{b}{a} = \frac{2}{5}$$

$$\alpha\beta = \frac{c}{a} = -\frac{3}{5}$$

$$\Delta = b^2 - 4ac = (-2)^2 - 4(5)(-3) = 64$$

$$|\alpha - \beta| = \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{64}}{5} = \frac{8}{5}$$

$$\text{Ans: } \alpha + \beta = \frac{2}{5}, \alpha\beta = -\frac{3}{5}, |\alpha - \beta| = \frac{8}{5}$$

7. Write $y = x^2 - 5x + 4$ in vertex form. Find its vertex and its minimum value. [3]

$$a = 1, b = -5, c = 4$$

$$\text{Vertex} = \left(-\frac{b}{2a}, \frac{4ac-b^2}{4a}\right) = \left(\frac{5}{2}, \frac{16-25}{4}\right) = \left(\frac{5}{2}, -\frac{9}{4}\right)$$

$$y = a(x-h)^2 + k = \left(x - \frac{5}{2}\right)^2 - \frac{9}{4}$$

$$a > 0, \text{ so the parabola opens upward: minimum value} = -\frac{9}{4} \text{ at } x = \frac{5}{2}$$

$$\text{Ans: } y = \left(x - \frac{5}{2}\right)^2 - \frac{9}{4}; \text{ vertex } \left(\frac{5}{2}, -\frac{9}{4}\right); \text{ minimum value } -\frac{9}{4}$$

8. Draw the graph of $y = -2(x+1)^2 + 8$. (Find whether it opens upward or downward, where it meets the x-axis and the y-axis, and its vertex.) [5]

$$y = -2(x+1)^2 + 8 = -2x^2 - 4x + 6$$

$$a = -2 < 0 \text{ (downward parabola)}$$

$$\Delta = b^2 - 4ac = (-4)^2 - 4(-2)(6) = 64$$

$$y = 0 \Rightarrow x = \frac{4 \pm 8}{-4} = -3, 1: \text{ cuts the x-axis at } (-3, 0), (1, 0)$$

$$\text{put } x = 0 \Rightarrow y = 6: \text{ cuts the y-axis at } (0, 6)$$

$$\text{Vertex} = \left(-\frac{b}{2a}, \frac{4ac-b^2}{4a}\right) = (-1, 8)$$

Plot these points and draw the downward parabola through them (maximum value 8 at $x = -1$)

Ans: Downward parabola; cuts the x-axis at $(-3, 0)$ and $(1, 0)$; cuts the y-axis at $(0, 6)$; vertex $(-1, 8)$

9. For $y = (x+1)^2 - 5$, find the values of x for which (i) $y < 0$ (ii) $y \leq 0$ (iii) $y > 0$ (iv) $y \geq 0$. [5]

$$y = (x+1)^2 - 5 = x^2 + 2x - 4$$

$$a = 1 > 0 \text{ (upward parabola)}$$

$$\Delta = b^2 - 4ac = 2^2 - 4(1)(-4) = 20$$

$$y = 0 \Rightarrow x = \frac{-2 \pm \sqrt{20}}{2} = -1 \pm \sqrt{5}$$

Sign: + outside $-1 - \sqrt{5}$ and $-1 + \sqrt{5}$, - between them

$$(i) y < 0 \Rightarrow x \in (-1 - \sqrt{5}, -1 + \sqrt{5})$$

$$(ii) y \leq 0 \Rightarrow x \in [-1 - \sqrt{5}, -1 + \sqrt{5}]$$

$$(iii) y > 0 \Rightarrow x \in (-\infty, -1 - \sqrt{5}) \cup (-1 + \sqrt{5}, \infty)$$

$$(iv) y \geq 0 \Rightarrow x \in (-\infty, -1 - \sqrt{5}] \cup [-1 + \sqrt{5}, \infty)$$

Ans: (i) $x \in (-1 - \sqrt{5}, -1 + \sqrt{5})$; (ii) $x \in [-1 - \sqrt{5}, -1 + \sqrt{5}]$; (iii) $x \in (-\infty, -1 - \sqrt{5}) \cup (-1 + \sqrt{5}, \infty)$; (iv) $x \in (-\infty, -1 - \sqrt{5}] \cup [-1 + \sqrt{5}, \infty)$

10. If $\log_6(2x + 1) = 2$, find x . [1]

$$2x + 1 = 6^2 = 36$$

$$2x = 35 \Rightarrow x = \frac{35}{2}$$

$$\text{Ans: } \frac{35}{2}$$

11. Find the points on the graph of $y = \log_3 x$ for $x = \frac{1}{9}, \frac{1}{3}, 1, 3, 9$. [2]

$$\log_3 \frac{1}{9} = \log_3 3^{-2} = -2, \log_3 \frac{1}{3} = -1$$

$$\log_3 1 = 0, \log_3 3 = 1, \log_3 9 = 2$$

Points: $(\frac{1}{9}, -2), (\frac{1}{3}, -1), (1, 0), (3, 1), (9, 2)$ (an increasing curve, base $3 > 1$)

$$\text{Ans: } (\frac{1}{9}, -2), (\frac{1}{3}, -1), (1, 0), (3, 1), (9, 2)$$

12. Find $\log_9 8 \cdot \log_{25} 27 \cdot \log_4 125$. [3]

$$\begin{aligned} \log_{3^2} 2^3 \cdot \log_{5^2} 3^3 \cdot \log_{2^2} 5^3 &= \frac{3}{2} \log_3 2 \cdot \frac{3}{2} \log_5 3 \cdot \frac{3}{2} \log_2 5 \\ &= \frac{27}{8} \log_3 2 \cdot \log_5 3 \cdot \log_2 5 = \frac{27}{8} \log_5 2 \cdot \log_2 5 = \frac{27}{8} \end{aligned}$$

$$\text{Ans: } \frac{27}{8}$$

13. Find $|3 - 10| + |10 - 3|$. [1]

$$|3 - 10| = |-7| = 7, |10 - 3| = 7$$

(property $|-x| = |x|$)

$$7 + 7 = 14$$

$$\text{Ans: } 14$$

14. Solve $|3x + 4| \geq 2$. [2]

$$3x + 4 \leq -2 \text{ (or) } 3x + 4 \geq 2$$

$$\Rightarrow x \leq -2 \text{ (or) } x \geq -\frac{2}{3}$$

$$\Rightarrow x \in (-\infty, -2] \cup [-\frac{2}{3}, \infty)$$

$$\text{Ans: } x \in (-\infty, -2] \cup [-\frac{2}{3}, \infty)$$

15. Solve $|x^2 - 3x| = 4$. [3]

$$|x| = k \Rightarrow x = \pm k$$

$$x^2 - 3x = \pm 4$$

$$x^2 - 3x = 4 \Rightarrow x^2 - 3x - 4 = 0 \Rightarrow x = \frac{3 \pm 5}{2} = -1, 4$$

$$x^2 - 3x = -4 \Rightarrow x^2 - 3x + 4 = 0, \Delta = -7 < 0: \text{ no real } x$$

$$\therefore x \in \{-1, 4\}$$

$$\text{Ans: } x \in \{-1, 4\}$$

16. Solve $|x + 4| + |2x + 5| = 6$. [5]

Critical points: $x + 4 = 0 \Rightarrow x = -4$, $2x + 5 = 0 \Rightarrow x = -\frac{5}{2}$

Case (i) Suppose $x < -4$ ($x + 4 < 0$, $2x + 5 < 0$): $(-x - 4) + (-2x - 5) = 6 \Rightarrow -3x - 9 = 6 \Rightarrow x = -5$, which is in this case

Case (ii) Suppose $-4 \leq x \leq -\frac{5}{2}$ ($x + 4 \geq 0$, $2x + 5 \leq 0$): $(x + 4) + (-2x - 5) = 6 \Rightarrow -x - 1 = 6 \Rightarrow x = -7$, not in this case: no solution

Case (iii) Suppose $x > -\frac{5}{2}$ ($x + 4 > 0$, $2x + 5 > 0$): $(x + 4) + (2x + 5) = 6 \Rightarrow 3x + 9 = 6 \Rightarrow x = -1$, which is in this case

Ans: $x \in \{-5, -1\}$

Ans: $x \in \{-5, -1\}$

17. Solve $x^2 + 5|x| - 6 < 0$. [5]

$x = 0$ is the critical point

Case (i) Suppose $x < 0$: $|x| = -x$, $x^2 - 5x - 6 < 0 \Rightarrow (x - 6)(x + 1) < 0 \Rightarrow x \in (-1, 6)$; common values $(-1, 0)$

Case (ii) Suppose $x \geq 0$: $|x| = x$, $x^2 + 5x - 6 < 0 \Rightarrow (x - 1)(x + 6) < 0 \Rightarrow x \in (-6, 1)$; common values $[0, 1)$

From (i), (ii): $x \in (-1, 1)$

Ans: $x \in (-1, 1)$

18. Find the domain of $f(x) = \sqrt{2x^2 - 5x + 2}$. [2]

$\sqrt{2x^2 - 5x + 2}$ is defined when $2x^2 - 5x + 2 \geq 0$

$\Rightarrow (x - 2)(2x - 1) \geq 0$

Domain = $(-\infty, \frac{1}{2}] \cup [2, \infty)$

Ans: $(-\infty, \frac{1}{2}] \cup [2, \infty)$

19. Find $[-2.5] - [2.5]$. [1]

$[-2.5] = -3$, $[2.5] = 2$

$-3 - 2 = -5$

Ans: -5

20. Find $x - [x]$ for $x = -0.45$. [1]

$[-0.45] = -1$

$x - [x] = -0.45 - (-1) = 0.55$

Ans: 0.55
