

Factorization

Question Paper, Set 09

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Factorize $25 - 144t^2$. [1 mark]
2. Factorize $4m^2 - 12mn + 9n^2$. [1 mark]
3. Factorize $4p^2 - 24pq + 36q^2 - 9$. [2 marks]
4. To factorize $5b^2 + 26b + 24$ by splitting the middle term, we need two numbers whose product is 5×24 and whose sum is 26. Find them. [1 mark]
5. To factorize $15t^2 - 29t + 8$ by splitting the middle term, we need two numbers whose product is 15×8 and whose sum is -29 . Find them. [1 mark]
6. To factorize $2y^2 - y - 3$ by splitting the middle term, we need two numbers whose product is $2 \times (-3)$ and whose sum is -1 . Find them. [1 mark]
7. Factorize $11t^2 + 17t - 78$. [3 marks]
8. Factorize $3z - z^2 + 28$. [2 marks]
9. Factorize $(m + p)^2 + 3(m + p) - 28$. [3 marks]
10. Factorize $4a^2 - 23ab - 6b^2$. [2 marks]
11. Factorize $6\sqrt{5}m^2 + 13m + \sqrt{5}$. [5 marks]
12. Write $y^2 - \frac{7}{20}y - \frac{3}{20}$ in the form $\frac{1}{20}(\dots)$, where the bracket has whole-number coefficients (the first step in factorizing it). [1 mark]
13. Factorize $6(a - 5)^2 - 19(a - 5)(c - 3) + 15(c - 3)^2$. [5 marks]
14. Factorize $16a^2 + 18b^2 + 2c^2 + 24\sqrt{2}ab - 12bc - 8\sqrt{2}ca$. [3 marks]
15. Factorize $x^3 - 9x^2 + 27x - 27$. [2 marks]
16. Factorize $27 + 125p^3$. [2 marks]
17. Factorize $625a^4 + 4c^4$. [5 marks]
18. Factorize $(x - 3y)^3 + (3y - 4)^3 + (4 - x)^3$. [2 marks]

19. Factorize $16x^5y - 81xy^5$. [3 marks]

20. Factorize $(y - 1)(y^2 + 16) - 8y^2 + 8y$. [5 marks]

Key sheet

Q	Answer
1	$(5 + 12t)(5 - 12t)$
2	$(2m - 3n)^2$
3	$(2p - 6q + 3)(2p - 6q - 3)$
4	6 and 20
5	-5 and -24
6	2 and -3
7	$(t - 2)(11t + 39)$
8	$(z + 4)(7 - z)$
9	$(m + p - 4)(m + p + 7)$
10	$(4a + b)(a - 6b)$
11	$(2\sqrt{5}m + 1)(3m + \sqrt{5})$
12	$\frac{1}{20}(20y^2 - 7y - 3)$
13	$(2a - 3c - 1)(3a - 5c)$
14	$(4a + 3\sqrt{2}b - \sqrt{2}c)^2$
15	$(x - 3)^3$
16	$(5p + 3)(25p^2 - 15p + 9)$
17	$(25a^2 + 10ac + 2c^2)(25a^2 - 10ac + 2c^2)$
18	$3(x - 3y)(3y - 4)(4 - x)$
19	$xy(4x^2 + 9y^2)(2x + 3y)(2x - 3y)$
20	$(y - 1)(y - 4)^2$

Solutions

1. Factorize $25 - 144t^2$. [1]

$$\begin{aligned}25 - 144t^2 &= 5^2 - (12t)^2 \\&= (5 + 12t)(5 - 12t) \\&\text{using } a^2 - b^2 = (a + b)(a - b) \\&\text{Ans: } (5 + 12t)(5 - 12t)\end{aligned}$$

2. Factorize $4m^2 - 12mn + 9n^2$. [1]

$$\begin{aligned}4m^2 - 12mn + 9n^2 &= (2m)^2 + (3n)^2 - 2(2m)(3n) \\&= (2m - 3n)^2 \\&= (2m - 3n)(2m - 3n) \\&\text{using } a^2 + b^2 - 2ab = (a - b)^2 \\&\text{Ans: } (2m - 3n)^2\end{aligned}$$

3. Factorize $4p^2 - 24pq + 36q^2 - 9$. [2]

$$\begin{aligned}4p^2 - 24pq + 36q^2 - 9 &= (2p - 6q)^2 - 3^2 \\&= (2p - 6q + 3)(2p - 6q - 3) \\&\text{using } a^2 - 2ab + b^2 = (a - b)^2 \text{ and } a^2 - b^2 = (a + b)(a - b) \\&\text{Ans: } (2p - 6q + 3)(2p - 6q - 3)\end{aligned}$$

4. To factorize $5b^2 + 26b + 24$ by splitting the middle term, we need two numbers whose product is 5×24 and whose sum is 26. Find them. [1]

$$\begin{aligned}5 \times 24 &= 120 \\6 \times 20 &= 120 \text{ and } 6 + 20 = 26 \\&\text{So the numbers are 6 and 20.} \\&\text{Ans: 6 and 20}\end{aligned}$$

5. To factorize $15t^2 - 29t + 8$ by splitting the middle term, we need two numbers whose product is 15×8 and whose sum is -29 . Find them. [1]

$$\begin{aligned}15 \times 8 &= 120 \\(-5) \times (-24) &= 120 \text{ and } -5 - 24 = -29 \\&\text{So the numbers are } -5 \text{ and } -24. \\&\text{Ans: } -5 \text{ and } -24\end{aligned}$$

6. To factorize $2y^2 - y - 3$ by splitting the middle term, we need two numbers whose product is $2 \times (-3)$ and whose sum is -1 . Find them. [1]

$$\begin{aligned}2 \times (-3) &= -6 \\2 \times (-3) &= -6 \text{ and } 2 - 3 = -1 \\&\text{So the numbers are 2 and } -3. \\&\text{Ans: 2 and } -3\end{aligned}$$

7. Factorize $11t^2 + 17t - 78$. [3]

$$11 \times (-78) = -858$$

$$858 = 2 \times 3 \times 11 \times 13 = 22 \times 39, \text{ and } -22 + 39 = 17$$

$$= 11t^2 - 22t + 39t - 78$$

$$= 11t(t - 2) + 39(t - 2)$$

$$= (t - 2)(11t + 39)$$

$$\text{Ans: } (t - 2)(11t + 39)$$

8. Factorize $3z - z^2 + 28$. [2]

Writing the terms in order: $-z^2 + 3z + 28$

$$(-1) \times 28 = -28: (-4) \times 7 = -28 \text{ and } -4 + 7 = 3$$

$$= -z^2 - 4z + 7z + 28$$

$$= -z(z + 4) + 7(z + 4)$$

$$= (z + 4)(7 - z)$$

$$\text{Ans: } (z + 4)(7 - z)$$

9. Factorize $(m + p)^2 + 3(m + p) - 28$. [3]

Let $m + p = t$

$$(m + p)^2 + 3(m + p) - 28 = t^2 + 3t - 28$$

$$1 \times (-28) = -28: (-4) \times 7 = -28 \text{ and } -4 + 7 = 3$$

$$= t^2 - 4t + 7t - 28$$

$$= t(t - 4) + 7(t - 4)$$

$$= (t - 4)(t + 7)$$

$$= [(m + p) - 4][(m + p) + 7]$$

$$= (m + p - 4)(m + p + 7)$$

$$\text{Ans: } (m + p - 4)(m + p + 7)$$

10. Factorize $4a^2 - 23ab - 6b^2$. [2]

$$4 \times (-6) = -24: 1 \times (-24) = -24 \text{ and } 1 - 24 = -23$$

$$= 4a^2 + ab - 24ab - 6b^2$$

$$= a(4a + b) - 6b(4a + b)$$

$$= (4a + b)(a - 6b)$$

$$\text{Ans: } (4a + b)(a - 6b)$$

11. Factorize $6\sqrt{5}m^2 + 13m + \sqrt{5}$. [5]

$$6\sqrt{5} \times \sqrt{5} = 6 \times 1 \times 5 = 30: 3 \times 10 = 30 \text{ and } 3 + 10 = 13$$

$$= 6\sqrt{5}m^2 + 3m + 10m + \sqrt{5}$$

$$= 3m(2\sqrt{5}m + 1) + \sqrt{5}(2\sqrt{5}m + 1)$$

$$= (2\sqrt{5}m + 1)(3m + \sqrt{5})$$

$$\text{Ans: } (2\sqrt{5}m + 1)(3m + \sqrt{5})$$

12. Write $y^2 - \frac{7}{20}y - \frac{3}{20}$ in the form $\frac{1}{20}(\dots)$, where the bracket has whole-number coefficients (the first step in factorizing it). [1]

$$\begin{aligned} y^2 - \frac{7}{20}y - \frac{3}{20} &= \frac{20y^2 - 7y - 3}{20} \\ &= \frac{1}{20}(20y^2 - 7y - 3) \end{aligned}$$

Ans: $\frac{1}{20}(20y^2 - 7y - 3)$

13. Factorize $6(a - 5)^2 - 19(a - 5)(c - 3) + 15(c - 3)^2$. [5]

Let $a - 5 = x$ and $c - 3 = y$

$$6(a - 5)^2 - 19(a - 5)(c - 3) + 15(c - 3)^2 = 6x^2 - 19xy + 15y^2$$

$$6 \times 15 = 90; (-9) \times (-10) = 90 \text{ and } -9 - 10 = -19$$

$$= 6x^2 - 9xy - 10xy + 15y^2$$

$$= 3x(2x - 3y) - 5y(2x - 3y)$$

$$= (2x - 3y)(3x - 5y)$$

$$= [2(a - 5) - 3(c - 3)][3(a - 5) - 5(c - 3)]$$

$$= (2a - 10 - 3c + 9)(3a - 15 - 5c + 15)$$

$$= (2a - 3c - 1)(3a - 5c)$$

Ans: $(2a - 3c - 1)(3a - 5c)$

14. Factorize $16a^2 + 18b^2 + 2c^2 + 24\sqrt{2}ab - 12bc - 8\sqrt{2}ca$. [3]

$$\begin{aligned} 16a^2 + 18b^2 + 2c^2 + 24\sqrt{2}ab - 12bc - 8\sqrt{2}ca &= (4a)^2 + (3\sqrt{2}b)^2 + (\sqrt{2}c)^2 + 2(4a)(3\sqrt{2}b) - \\ &2(3\sqrt{2}b)(\sqrt{2}c) - 2(\sqrt{2}c)(4a) \end{aligned}$$

$$= (4a + 3\sqrt{2}b - \sqrt{2}c)^2$$

$$\text{using } a^2 + b^2 + c^2 + 2ab - 2bc - 2ca = (a + b - c)^2$$

Ans: $(4a + 3\sqrt{2}b - \sqrt{2}c)^2$

15. Factorize $x^3 - 9x^2 + 27x - 27$. [2]

$$x^3 - 9x^2 + 27x - 27 = x^3 - 3^3 - 3(x)^2(3) + 3(x)(3)^2$$

$$= (x - 3)^3$$

$$\text{using } (a - b)^3 = a^3 - b^3 - 3ab(a - b)$$

Ans: $(x - 3)^3$

16. Factorize $27 + 125p^3$. [2]

$$27 + 125p^3 = 3^3 + (5p)^3$$

$$= (3 + 5p)[3^2 - (3)(5p) + (5p)^2]$$

$$= (5p + 3)(25p^2 - 15p + 9)$$

$$\text{using } a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Ans: $(5p + 3)(25p^2 - 15p + 9)$

17. Factorize $625a^4 + 4c^4$. [5]

$$\begin{aligned}625a^4 + 4c^4 &= (25a^2)^2 + (2c^2)^2 + 2(25a^2)(2c^2) - 100a^2c^2 \\&= (25a^2 + 2c^2)^2 - (10ac)^2 \\&= (25a^2 + 2c^2 + 10ac)(25a^2 + 2c^2 - 10ac) \\&= (25a^2 + 10ac + 2c^2)(25a^2 - 10ac + 2c^2) \\ \text{Ans: } &(25a^2 + 10ac + 2c^2)(25a^2 - 10ac + 2c^2)\end{aligned}$$

18. Factorize $(x - 3y)^3 + (3y - 4)^3 + (4 - x)^3$. [2]

$$\begin{aligned}\text{Here } &(x - 3y) + (3y - 4) + (4 - x) = 0 \\ \text{If } &a + b + c = 0, \text{ then } a^3 + b^3 + c^3 = 3abc \\ \therefore &(x - 3y)^3 + (3y - 4)^3 + (4 - x)^3 = 3(x - 3y)(3y - 4)(4 - x) \\ \text{Ans: } &3(x - 3y)(3y - 4)(4 - x)\end{aligned}$$

19. Factorize $16x^5y - 81xy^5$. [3]

$$\begin{aligned}16x^5y - 81xy^5 &= xy(16x^4 - 81y^4) \\&= xy[(4x^2)^2 - (9y^2)^2] \\&= xy(4x^2 + 9y^2)(4x^2 - 9y^2) \\&= xy(4x^2 + 9y^2)[(2x)^2 - (3y)^2] \\&= xy(4x^2 + 9y^2)(2x + 3y)(2x - 3y) \\ \text{Ans: } &xy(4x^2 + 9y^2)(2x + 3y)(2x - 3y)\end{aligned}$$

20. Factorize $(y - 1)(y^2 + 16) - 8y^2 + 8y$. [5]

$$\begin{aligned}(y - 1)(y^2 + 16) - 8y^2 + 8y &= (y - 1)(y^2 + 16) - 8y(y - 1) \\&= (y - 1)(y^2 + 16 - 8y) \\&= (y - 1)(y^2 - 8y + 16) \\&= (y - 1)(y - 4)^2 \\ \text{Ans: } &(y - 1)(y - 4)^2\end{aligned}$$
