EXPRESSWALA ACADEMY

Factorization

Question Paper, Set 08

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Factorize $b^4 - 81a^4$. [2 marks]

2. Factorize $16m^2 + 72m + 81$. [1 mark]

3. Factorize $x^2 - 10x + 25 - 36y^2$. [2 marks]

4. Factorize $3t^2 + 5t + 2$. [2 marks]

5. To factorize $3a^2 - 31a + 36$ by splitting the middle term, we need two numbers whose product is 3×36 and whose sum is -31. Find them. [1 mark]

6. To factorize $10p^2 + p - 24$ by splitting the middle term, we need two numbers whose product is $10 \times (-24)$ and whose sum is 1. Find them. [1 mark]

7. Factorize $19p^2 - 125p + 150$. [3 marks]

8. Factorize $5 - 27p - 18p^2$. [3 marks]

9. Factorize $(y + 3z)^2 + 2(y + 3z) - 8$. [3 marks]

10. To factorize $10a^2 - 9ab + 2b^2$ by splitting the middle term, we need two numbers whose product is 10×2 and whose sum is -9. Find them. [1 mark]

11. Factorize $y^2 - 8\sqrt{6}y + 90$. [3 marks]

12. Write $3a^2 - \frac{2}{21}a - \frac{1}{21}$ in the form $\frac{1}{21}(\ldots)$, where the bracket has whole-number coefficients (the first step in factorizing it). [1 mark]

13. Factorize $3(m + 4)^2 + 7(m + 4)(n - 2) + 2(n - 2)^2$. [5 marks]

14. Factorize $a^2 + 16b^2 + 16c^2 - 8ab - 32bc + 8ca$. [2 marks]

15. Factorize $x^3 - 3x^2z + 3xz^2 - z^3$. [2 marks]

16. Factorize $216y^3 - 1$. [2 marks]

17. Factorize $x^3 - 28x - 48$. [5 marks]

18. Factorize $(5x - 2y)^3 + (2y - 1)^3 + (1 - 5x)^3$. [1 mark]

19. Factorize $256a^9b - ab^9$. [5 marks]

20. Factorize $(a-4)(a^2+4) - 4a^2 + 16a$. [5 marks]

Key sheet

Q	Answer
1	$(b^2+9a^2)(b+3a)(b-3a)$
2	$(4m+9)^2$
3	$(x-5+6y)(x-5-6y)$
4	$(3t+2)(t+1)$
5	-4 and -27
6	-15 and 16
7	$(19p-30)(p-5)$
8	$(3p+5)(1-6p)$
9	$(y+3z-2)(y+3z+4)$
10	-4 and -5
11	$(y-3\sqrt{6})(y-5\sqrt{6})$
12	$\frac{1}{21}(63a^2-2a-1)$
13	$(3m+n+10)(m+2n)$
14	$(a-4b+4c)^2$
15	$(x-z)^3$
16	$(6y-1)(36y^2+6y+1)$
17	$(x+2)(x+4)(x-6)$
18	$3(5x-2y)(2y-1)(1-5x)$
19	$ab(16a^4+b^4)(4a^2+b^2)(2a+b)(2a-b)$
20	$(a-4)(a-2)^2$

Solutions

1. Factorize $b^4 - 81a^4$. [2]

$b^4 - 81a^4 = (b^2)^2 - (9a^2)^2$

$= (b^2 + 9a^2)(b^2 - 9a^2)$

$= (b^2 + 9a^2)[b^2 - (3a)^2]$

$= (b^2 + 9a^2)(b + 3a)(b - 3a)$

Ans: $(b^2 + 9a^2)(b + 3a)(b - 3a)$

---

2. Factorize $16m^2 + 72m + 81$. [1]

$16m^2 + 72m + 81 = (4m)^2 + 9^2 + 2(4m)(9)$

$= (4m + 9)^2$

$= (4m + 9)(4m + 9)$

using $a^2 + b^2 + 2ab = (a + b)^2$

Ans: $(4m + 9)^2$

---

3. Factorize $x^2 - 10x + 25 - 36y^2$. [2]

$x^2 - 10x + 25 - 36y^2 = (x - 5)^2 - (6y)^2$

$= (x - 5 + 6y)(x - 5 - 6y)$

using $a^2 - 2ab + b^2 = (a - b)^2$ and $a^2 - b^2 = (a + b)(a - b)$

Ans: $(x - 5 + 6y)(x - 5 - 6y)$

---

4. Factorize $3t^2 + 5t + 2$. [2]

$3 \times 2 = 6$: $2 \times 3 = 6$ and $2 + 3 = 5$

$= 3t^2 + 2t + 3t + 2$

$= t(3t + 2) + 1(3t + 2)$

$= (3t + 2)(t + 1)$

Ans: $(3t + 2)(t + 1)$

---

5. To factorize $3a^2 - 31a + 36$ by splitting the middle term, we need two numbers whose product is 3×36 and whose sum is -31. Find them. [1]

$3 \times 36 = 108$

$(-4) \times (-27) = 108$ and $-4 - 27 = -31$

So the numbers are -4 and -27.

Ans: -4 **and** -27

---

6. To factorize $10p^2 + p - 24$ by splitting the middle term, we need two numbers whose product is $10 \times (-24)$ and whose sum is 1. Find them. [1]

$10 \times (-24) = -240$

$(-15) \times 16 = -240$ and $-15 + 16 = 1$

So the numbers are -15 and 16.

Ans: -15 **and** 16

---

7. Factorize $19p^2 - 125p + 150$. [3]

$19 \times 150 = 2850$

$2850 = 2 \times 3 \times 5 \times 5 \times 19 = 30 \times 95$, and $-30 - 95 = -125$

$= 19p^2 - 30p - 95p + 150$

$= p(19p - 30) - 5(19p - 30)$

$= (19p - 30)(p - 5)$

Ans: $(19p - 30)(p - 5)$

---

8. Factorize $5 - 27p - 18p^2$. [3]

Writing the terms in order: $-18p^2 - 27p + 5$

$(-18) \times 5 = -90$: $(-30) \times 3 = -90$ and $-30 + 3 = -27$

$= -18p^2 - 30p + 3p + 5$

$= -6p(3p + 5) + 1(3p + 5)$

$= (3p + 5)(1 - 6p)$

Ans: $(3p + 5)(1 - 6p)$

---

9. Factorize $(y + 3z)^2 + 2(y + 3z) - 8$. [3]

Let $y + 3z = p$

$(y + 3z)^2 + 2(y + 3z) - 8 = p^2 + 2p - 8$

$1 \times (-8) = -8$: $(-2) \times 4 = -8$ and $-2 + 4 = 2$

$= p^2 - 2p + 4p - 8$

$= p(p - 2) + 4(p - 2)$

$= (p - 2)(p + 4)$

$= [(y + 3z) - 2][(y + 3z) + 4]$

$= (y + 3z - 2)(y + 3z + 4)$

Ans: $(y + 3z - 2)(y + 3z + 4)$

---

10. To factorize $10a^2 - 9ab + 2b^2$ by splitting the middle term, we need two numbers whose product is 10×2 and whose sum is -9. Find them. [1]

$10 \times 2 = 20$

$(-4) \times (-5) = 20$ and $-4 - 5 = -9$

So the numbers are -4 and -5.

Ans: -4 **and** -5

---

11. Factorize $y^2 - 8\sqrt{6}y + 90$. [3]

Product $= 90$, sum $= -8\sqrt{6}$: $(-3\sqrt{6}) \times (-5\sqrt{6}) = 90$ and $-3\sqrt{6} + (-5\sqrt{6}) = -8\sqrt{6}$

$= y^2 - 3\sqrt{6}y - 5\sqrt{6}y + 90$

$= y(y - 3\sqrt{6}) - 5\sqrt{6}(y - 3\sqrt{6})$

$= (y - 3\sqrt{6})(y - 5\sqrt{6})$

Ans: $(y - 3\sqrt{6})(y - 5\sqrt{6})$

---

12. Write $3a^2 - \frac{2}{21}a - \frac{1}{21}$ in the form $\frac{1}{21}(\ldots)$, where the bracket has whole-number coefficients (the first step in factorizing it). [1]

$3a^2 - \frac{2}{21}a - \frac{1}{21} = \frac{63a^2 - 2a - 1}{21}$

$= \frac{1}{21}(63a^2 - 2a - 1)$

Ans: $\frac{1}{21}(63a^2 - 2a - 1)$

---

13. Factorize $3(m+4)^2 + 7(m+4)(n-2) + 2(n-2)^2$. [5]

Let $m + 4 = x$ and $n - 2 = y$

$3(m+4)^2 + 7(m+4)(n-2) + 2(n-2)^2 = 3x^2 + 7xy + 2y^2$

$3 \times 2 = 6$: $1 \times 6 = 6$ and $1 + 6 = 7$

$= 3x^2 + xy + 6xy + 2y^2$

$= x(3x + y) + 2y(3x + y)$

$= (3x + y)(x + 2y)$

$= [3(m+4) + (n-2)][(m+4) + 2(n-2)]$

$= (3m + 12 + n - 2)(m + 4 + 2n - 4)$

$= (3m + n + 10)(m + 2n)$

Ans: $(3m + n + 10)(m + 2n)$

---

14. Factorize $a^2 + 16b^2 + 16c^2 - 8ab - 32bc + 8ca$. [2]

$a^2 + 16b^2 + 16c^2 - 8ab - 32bc + 8ca = a^2 + (4b)^2 + (4c)^2 - 2(a)(4b) - 2(4b)(4c) + 2(4c)(a)$

$= (a - 4b + 4c)^2$

using $a^2 + b^2 + c^2 - 2ab - 2bc + 2ca = (a - b + c)^2$

Ans: $(a - 4b + 4c)^2$

---

15. Factorize $x^3 - 3x^2z + 3xz^2 - z^3$. [2]

$x^3 - 3x^2z + 3xz^2 - z^3 = x^3 - z^3 - 3(x)^2(z) + 3(x)(z)^2$

$= (x - z)^3$

using $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

Ans: $(x - z)^3$

---

16. Factorize $216y^3 - 1$. [2]

$216y^3 - 1 = (6y)^3 - 1^3$

$= (6y - 1)[(6y)^2 + (6y)(1) + 1^2]$

$= (6y - 1)(36y^2 + 6y + 1)$

using $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Ans: $(6y - 1)(36y^2 + 6y + 1)$

---

17. Factorize $x^3 - 28x - 48$. [5]

$x^3 - 28x - 48 = x^3 - 4x - 24x - 48$

$= x(x^2 - 4) - 24(x + 2)$

$= x(x + 2)(x - 2) - 24(x + 2)$

$= (x + 2)[x(x - 2) - 24]$

$= (x + 2)(x^2 - 2x - 24)$

Now $x^2 - 2x - 24$: $4 \times (-6) = -24$ and $4 - 6 = -2$

$x^2 - 2x - 24 = x^2 + 4x - 6x - 24$

$= x(x + 4) - 6(x + 4)$

$= (x + 4)(x - 6)$

So $x^3 - 28x - 48 = (x + 2)(x + 4)(x - 6)$

Ans: $(x + 2)(x + 4)(x - 6)$

---

18. Factorize $(5x - 2y)^3 + (2y - 1)^3 + (1 - 5x)^3$. [1]

Here $(5x - 2y) + (2y - 1) + (1 - 5x) = 0$

If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

$\therefore (5x - 2y)^3 + (2y - 1)^3 + (1 - 5x)^3 = 3(5x - 2y)(2y - 1)(1 - 5x)$

Ans: $3(5x - 2y)(2y - 1)(1 - 5x)$

---

19. Factorize $256a^9b - ab^9$. [5]

$256a^9b - ab^9 = ab(256a^8 - b^8)$

$= ab[(16a^4)^2 - (b^4)^2]$

$= ab(16a^4 + b^4)(16a^4 - b^4)$

$= ab(16a^4 + b^4)[(4a^2)^2 - (b^2)^2]$

$= ab(16a^4 + b^4)(4a^2 + b^2)(4a^2 - b^2)$

$= ab(16a^4 + b^4)(4a^2 + b^2)[(2a)^2 - b^2]$

$= ab(16a^4 + b^4)(4a^2 + b^2)(2a + b)(2a - b)$

Ans: $ab(16a^4 + b^4)(4a^2 + b^2)(2a + b)(2a - b)$

---

20. Factorize $(a - 4)(a^2 + 4) - 4a^2 + 16a$. [5]

$(a - 4)(a^2 + 4) - 4a^2 + 16a = (a - 4)(a^2 + 4) - 4a(a - 4)$

$= (a - 4)(a^2 + 4 - 4a)$

$= (a - 4)(a^2 - 4a + 4)$

$= (a - 4)(a - 2)^2$

Ans: $(a - 4)(a - 2)^2$