EXPRESSWALA ACADEMY

Factorization

Question Paper, Set 07

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Factorize $25t^2 - 81$. [1 mark]

2. Factorize $m^2 + 18mn + 81n^2$. [1 mark]

3. Factorize $9z^2 - 30z + 25$. [1 mark]

4. Factorize $x^2 + 12x + 36 - 25y^2$. [2 marks]

5. To factorize $5z^2 + 22z + 8$ by splitting the middle term, we need two numbers whose product is 5×8 and whose sum is 22. Find them. [1 mark]

6. Factorize $3x^2 - 17x + 10$. [2 marks]

7. Factorize $5p^2 + 4p - 1$. [2 marks]

8. Factorize $9y - 2y^2 + 81$. [2 marks]

9. Factorize $(3x + 4y)^2 - 4(3x + 4y) - 21$. [3 marks]

10. To factorize $4x^2 - xz - 14z^2$ by splitting the middle term, we need two numbers whose product is $4 \times (-14)$ and whose sum is -1. Find them. [1 mark]

11. Factorize $3\sqrt{3}t^2 - 35t - 4\sqrt{3}$. [5 marks]

12. Factorize $2y^2 - 2y + \frac{1}{2}$. [3 marks]

13. Factorize $10(m + 1)^2 + 27(m + 1)(n - 1) + 18(n - 1)^2$. [5 marks]

14. Factorize $9p^2 + 9q^2 + 4r^2 - 18pq - 12qr + 12rp$. [2 marks]

15. Factorize $8t^3 + 30t(2t + 5) + 125$. [3 marks]

16. Factorize $27y^3 + 27y^2z + 9yz^2 - 26z^3$. [5 marks]

17. Factorize $m^3n^3 + 125$. [2 marks]

18. Factorize $x^3 - 67x - 126$. [5 marks]

19. Factorize $(3p - 2q)^3 + (2q - 1)^3 + (1 - 3p)^3$. [1 mark]

20. Factorize $12x^2 + (2x^2 + 5)x + 30$. [3 marks]

Key sheet

Q	Answer
1	$(5t+9)(5t-9)$
2	$(m+9n)^2$
3	$(3z-5)^2$
4	$(x+6+5y)(x+6-5y)$
5	2 and 20
6	$(3x-2)(x-5)$
7	$(5p-1)(p+1)$
8	$(2y+9)(9-y)$
9	$(3x+4y+3)(3x+4y-7)$
10	7 and -8
11	$(3\sqrt{3}t+1)(t-4\sqrt{3})$
12	$\frac{1}{2}(2y-1)^2$, i.e. $\left(y-\frac{1}{2}\right)(2y-1)$
13	$(5m+6n-1)(2m+3n-1)$
14	$(3p-3q+2r)^2$
15	$(2t+5)^3$
16	$(3y-2z)(9y^2+15yz+13z^2)$
17	$(mn+5)(m^2n^2-5mn+25)$
18	$(x+2)(x+7)(x-9)$
19	$3(3p-2q)(2q-1)(1-3p)$
20	$(x+6)(2x^2+5)$

Solutions

1. Factorize $25t^2 - 81$. [1]

$25t^2 - 81 = (5t)^2 - 9^2$

$= (5t + 9)(5t - 9)$

using $a^2 - b^2 = (a + b)(a - b)$

Ans: $(5t + 9)(5t - 9)$

---

2. Factorize $m^2 + 18mn + 81n^2$. [1]

$m^2 + 18mn + 81n^2 = m^2 + (9n)^2 + 2(m)(9n)$

$= (m + 9n)^2$

$= (m + 9n)(m + 9n)$

using $a^2 + b^2 + 2ab = (a + b)^2$

Ans: $(m + 9n)^2$

---

3. Factorize $9z^2 - 30z + 25$. [1]

$9z^2 - 30z + 25 = (3z)^2 + 5^2 - 2(3z)(5)$

$= (3z - 5)^2$

$= (3z - 5)(3z - 5)$

using $a^2 + b^2 - 2ab = (a - b)^2$

Ans: $(3z - 5)^2$

---

4. Factorize $x^2 + 12x + 36 - 25y^2$. [2]

$x^2 + 12x + 36 - 25y^2 = (x + 6)^2 - (5y)^2$

$= (x + 6 + 5y)(x + 6 - 5y)$

using $a^2 + 2ab + b^2 = (a + b)^2$ and $a^2 - b^2 = (a + b)(a - b)$

Ans: $(x + 6 + 5y)(x + 6 - 5y)$

---

5. To factorize $5z^2 + 22z + 8$ by splitting the middle term, we need two numbers whose product is 5×8 and whose sum is 22. Find them. [1]

$5 \times 8 = 40$

$2 \times 20 = 40$ and $2 + 20 = 22$

So the numbers are 2 and 20.

Ans: 2 and 20

---

6. Factorize $3x^2 - 17x + 10$. [2]

$3 \times 10 = 30$: $(-2) \times (-15) = 30$ and $-2 - 15 = -17$

$= 3x^2 - 2x - 15x + 10$

$= x(3x - 2) - 5(3x - 2)$

$= (3x - 2)(x - 5)$

Ans: $(3x - 2)(x - 5)$

---

7. Factorize $5p^2 + 4p - 1$. [2]

$5 \times (-1) = -5$: $(-1) \times 5 = -5$ and $-1 + 5 = 4$

$= 5p^2 - p + 5p - 1$

$= p(5p - 1) + 1(5p - 1)$

$= (5p - 1)(p + 1)$

Ans: $(5p - 1)(p + 1)$

---

8. Factorize $9y - 2y^2 + 81$. [2]

Writing the terms in order: $-2y^2 + 9y + 81$

$(-2) \times 81 = -162$: $(-9) \times 18 = -162$ and $-9 + 18 = 9$

$= -2y^2 - 9y + 18y + 81$

$= -y(2y + 9) + 9(2y + 9)$

$= (2y + 9)(9 - y)$

Ans: $(2y + 9)(9 - y)$

---

9. Factorize $(3x + 4y)^2 - 4(3x + 4y) - 21$. [3]

Let $3x + 4y = p$

$(3x + 4y)^2 - 4(3x + 4y) - 21 = p^2 - 4p - 21$

$1 \times (-21) = -21$: $3 \times (-7) = -21$ and $3 - 7 = -4$

$= p^2 + 3p - 7p - 21$

$= p(p + 3) - 7(p + 3)$

$= (p + 3)(p - 7)$

$= [(3x + 4y) + 3][(3x + 4y) - 7]$

$= (3x + 4y + 3)(3x + 4y - 7)$

Ans: $(3x + 4y + 3)(3x + 4y - 7)$

---

10. To factorize $4x^2 - xz - 14z^2$ by splitting the middle term, we need two numbers whose product is $4 \times (-14)$ and whose sum is -1. Find them. [1]

$4 \times (-14) = -56$

$7 \times (-8) = -56$ and $7 - 8 = -1$

So the numbers are 7 and -8.

Ans: 7 **and** -8

---

11. Factorize $3\sqrt{3}t^2 - 35t - 4\sqrt{3}$. [5]

$3\sqrt{3} \times (-4\sqrt{3}) = 3 \times (-4) \times 3 = -36$: $1 \times (-36) = -36$ and $1 - 36 = -35$

$= 3\sqrt{3}t^2 + t - 36t - 4\sqrt{3}$

$= t(3\sqrt{3}t + 1) - 4\sqrt{3}(3\sqrt{3}t + 1)$

$= (3\sqrt{3}t + 1)(t - 4\sqrt{3})$

Ans: $(3\sqrt{3}t + 1)(t - 4\sqrt{3})$

---

12. Factorize $2y^2 - 2y + \frac{1}{2}$. [3]

$2y^2 - 2y + \frac{1}{2} = \frac{4y^2 - 4y + 1}{2}$

$= \frac{1}{2}(4y^2 - 4y + 1)$

$= \frac{1}{2}[(2y)^2 + 1^2 - 2(2y)(1)]$

$= \frac{1}{2}(2y - 1)^2$

$= \left(y - \frac{1}{2}\right)(2y - 1)$

Ans: $\frac{1}{2}(2y - 1)^2$, **i.e.** $\left(y - \frac{1}{2}\right)(2y - 1)$

---

13. Factorize $10(m + 1)^2 + 27(m + 1)(n - 1) + 18(n - 1)^2$. [5]

Let $m + 1 = x$ and $n - 1 = y$

$10(m + 1)^2 + 27(m + 1)(n - 1) + 18(n - 1)^2 = 10x^2 + 27xy + 18y^2$

$10 \times 18 = 180$: $12 \times 15 = 180$ and $12 + 15 = 27$

$= 10x^2 + 12xy + 15xy + 18y^2$

$= 2x(5x + 6y) + 3y(5x + 6y)$

$= (5x + 6y)(2x + 3y)$

$= [5(m + 1) + 6(n - 1)][2(m + 1) + 3(n - 1)]$

$= (5m + 5 + 6n - 6)(2m + 2 + 3n - 3)$

$= (5m + 6n - 1)(2m + 3n - 1)$

Ans: $(5m + 6n - 1)(2m + 3n - 1)$

---

14. Factorize $9p^2 + 9q^2 + 4r^2 - 18pq - 12qr + 12rp$. [2]

$9p^2 + 9q^2 + 4r^2 - 18pq - 12qr + 12rp = (3p)^2 + (3q)^2 + (2r)^2 - 2(3p)(3q) - 2(3q)(2r) + 2(2r)(3p)$

$= (3p - 3q + 2r)^2$

using $a^2 + b^2 + c^2 - 2ab - 2bc + 2ca = (a - b + c)^2$

Ans: $(3p - 3q + 2r)^2$

---

15. Factorize $8t^3 + 30t(2t + 5) + 125$. [3]

$8t^3 + 30t(2t + 5) + 125 = 8t^3 + 60t^2 + 150t + 125$

$= (2t)^3 + 5^3 + 3(2t)^2(5) + 3(2t)(5)^2$

$= (2t + 5)^3$

using $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

Ans: $(2t + 5)^3$

---

16. Factorize $27y^3 + 27y^2z + 9yz^2 - 26z^3$. [5]

$27y^3 + 27y^2z + 9yz^2 - 26z^3 = (3y)^3 + 3(3y)^2(z) + 3(3y)(z)^2 + z^3 - 27z^3$

$= (3y + z)^3 - (3z)^3$

$= (3y + z - 3z)[(3y + z)^2 + (3y + z)(3z) + (3z)^2]$

$= (3y - 2z)[9y^2 + 6yz + z^2 + 9yz + 3z^2 + 9z^2]$

$= (3y - 2z)(9y^2 + 15yz + 13z^2)$

using $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Ans: $(3y - 2z)(9y^2 + 15yz + 13z^2)$

---

17. Factorize $m^3n^3 + 125$. [2]

$m^3n^3 + 125 = (mn)^3 + 5^3$

$= (mn + 5)[(mn)^2 - (mn)(5) + 5^2]$

$= (mn + 5)(m^2n^2 - 5mn + 25)$

using $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Ans: $(mn + 5)(m^2n^2 - 5mn + 25)$

---

18. Factorize $x^3 - 67x - 126$. [5]

$x^3 - 67x - 126 = x^3 - 4x - 63x - 126$

$= x(x^2 - 4) - 63(x + 2)$

$= x(x + 2)(x - 2) - 63(x + 2)$

$= (x + 2)[x(x - 2) - 63]$

$= (x + 2)(x^2 - 2x - 63)$

Now $x^2 - 2x - 63$: $7 \times (-9) = -63$ and $7 - 9 = -2$

$x^2 - 2x - 63 = x^2 + 7x - 9x - 63$

$= x(x + 7) - 9(x + 7)$

$= (x + 7)(x - 9)$

So $x^3 - 67x - 126 = (x + 2)(x + 7)(x - 9)$

Ans: $(x + 2)(x + 7)(x - 9)$

---

19. Factorize $(3p - 2q)^3 + (2q - 1)^3 + (1 - 3p)^3$. [1]

Here $(3p - 2q) + (2q - 1) + (1 - 3p) = 0$

If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

$\therefore (3p - 2q)^3 + (2q - 1)^3 + (1 - 3p)^3 = 3(3p - 2q)(2q - 1)(1 - 3p)$

Ans: $3(3p - 2q)(2q - 1)(1 - 3p)$

---

20. Factorize $12x^2 + (2x^2 + 5)x + 30$. [3]

$12x^2 + (2x^2 + 5)x + 30 = 12x^2 + 2x^3 + 5x + 30$

$= 2x^3 + 12x^2 + 5x + 30$

$= 2x^2(x + 6) + 5(x + 6)$

$= (x + 6)(2x^2 + 5)$

Ans: $(x + 6)(2x^2 + 5)$