

Factorization

Question Paper, Set 06

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Factorize $16 - m^4p^4$. [2 marks]
2. Factorize $4t^2 + 36t + 81$. [1 mark]
3. Factorize $9m^2 - 42m + 49$. [1 mark]
4. Factorize $9a^2 - 18ab + 9b^2 - 25$. [3 marks]
5. To factorize $5m^2 + 26m + 5$ by splitting the middle term, we need two numbers whose product is 5×5 and whose sum is 26. Find them. [1 mark]
6. To factorize $15n^2 - 37n + 18$ by splitting the middle term, we need two numbers whose product is 15×18 and whose sum is -37 . Find them. [1 mark]
7. To factorize $5b^2 - 14b - 24$ by splitting the middle term, we need two numbers whose product is $5 \times (-24)$ and whose sum is -14 . Find them. [1 mark]
8. Factorize $29z^2 + 234z - 243$. [3 marks]
9. Factorize $-4t - 12t^2 + 1$. [3 marks]
10. Factorize $3x^2 - xy - 4y^2$. [2 marks]
11. Factorize $7\sqrt{5}y^2 - 135y - 20\sqrt{5}$. [5 marks]
12. Write $6t^2 + \frac{73}{35}t + \frac{6}{35}$ in the form $\frac{1}{35}(\dots)$, where the bracket has whole-number coefficients (the first step in factorizing it). [1 mark]
13. Factorize $4(p-1)^2 + 29(p-1)(q-1) + 30(q-1)^2$. [5 marks]
14. Factorize $8x^2 + 8y^2 + z^2 - 16xy + 4\sqrt{2}yz - 4\sqrt{2}zx$. [3 marks]
15. Factorize $p^3 - 3p^2 + 3p - 1$. [2 marks]
16. Factorize $a^6 - 64b^3$. [2 marks]
17. Factorize $216a^3 + c^3$. [2 marks]
18. Factorize $16m^4 + 100m^2 + 625$. [5 marks]

19. Factorize $(5p - 3q)^3 + (3q - r)^3 + (r - 5p)^3$. [2 marks]

20. Factorize $2m(m + 3n)^3 - 18m^2n(m + 3n)$. [5 marks]

Key sheet

Q	Answer
1	$(4 + m^2p^2)(2 + mp)(2 - mp)$
2	$(2t + 9)^2$
3	$(3m - 7)^2$
4	$(3a - 3b + 5)(3a - 3b - 5)$
5	1 and 25
6	-10 and -27
7	6 and -20
8	$(29z - 27)(z + 9)$
9	$(2t + 1)(1 - 6t)$
10	$(x + y)(3x - 4y)$
11	$(7\sqrt{5}y + 5)(y - 4\sqrt{5})$
12	$\frac{1}{35}(210t^2 + 73t + 6)$
13	$(4p + 5q - 9)(p + 6q - 7)$
14	$(-2\sqrt{2}x + 2\sqrt{2}y + z)^2$
15	$(p - 1)^3$
16	$(a^2 - 4b)(a^4 + 4a^2b + 16b^2)$
17	$(6a + c)(36a^2 - 6ac + c^2)$
18	$(4m^2 + 10m + 25)(4m^2 - 10m + 25)$
19	$3(5p - 3q)(3q - r)(r - 5p)$
20	$2m(m + 3n)(m^2 - 3mn + 9n^2)$

Solutions

1. Factorize $16 - m^4p^4$. [2]

$$\begin{aligned}16 - m^4p^4 &= 4^2 - (m^2p^2)^2 \\&= (4 + m^2p^2)(4 - m^2p^2) \\&= (4 + m^2p^2)[2^2 - (mp)^2] \\&= (4 + m^2p^2)(2 + mp)(2 - mp) \\ \text{Ans: } &(4 + m^2p^2)(2 + mp)(2 - mp)\end{aligned}$$

2. Factorize $4t^2 + 36t + 81$. [1]

$$\begin{aligned}4t^2 + 36t + 81 &= (2t)^2 + 9^2 + 2(2t)(9) \\&= (2t + 9)^2 \\&= (2t + 9)(2t + 9) \\ \text{using } a^2 + b^2 + 2ab &= (a + b)^2 \\ \text{Ans: } &(2t + 9)^2\end{aligned}$$

3. Factorize $9m^2 - 42m + 49$. [1]

$$\begin{aligned}9m^2 - 42m + 49 &= (3m)^2 + 7^2 - 2(3m)(7) \\&= (3m - 7)^2 \\&= (3m - 7)(3m - 7) \\ \text{using } a^2 + b^2 - 2ab &= (a - b)^2 \\ \text{Ans: } &(3m - 7)^2\end{aligned}$$

4. Factorize $9a^2 - 18ab + 9b^2 - 25$. [3]

$$\begin{aligned}9a^2 - 18ab + 9b^2 - 25 &= (3a - 3b)^2 - 5^2 \\&= (3a - 3b + 5)(3a - 3b - 5) \\ \text{using } a^2 - 2ab + b^2 &= (a - b)^2 \text{ and } a^2 - b^2 = (a + b)(a - b) \\ \text{Ans: } &(3a - 3b + 5)(3a - 3b - 5)\end{aligned}$$

5. To factorize $5m^2 + 26m + 5$ by splitting the middle term, we need two numbers whose product is 5×5 and whose sum is 26. Find them. [1]

$$\begin{aligned}5 \times 5 &= 25 \\ 1 \times 25 &= 25 \text{ and } 1 + 25 = 26 \\ \text{So the numbers are } &1 \text{ and } 25. \\ \text{Ans: } &1 \text{ and } 25\end{aligned}$$

6. To factorize $15n^2 - 37n + 18$ by splitting the middle term, we need two numbers whose product is 15×18 and whose sum is -37 . Find them. [1]

$$\begin{aligned}15 \times 18 &= 270 \\ (-10) \times (-27) &= 270 \text{ and } -10 - 27 = -37 \\ \text{So the numbers are } &-10 \text{ and } -27. \\ \text{Ans: } &-10 \text{ and } -27\end{aligned}$$

7. To factorize $5b^2 - 14b - 24$ by splitting the middle term, we need two numbers whose product is $5 \times (-24)$ and whose sum is -14 . Find them. [1]

$$5 \times (-24) = -120$$

$$6 \times (-20) = -120 \text{ and } 6 - 20 = -14$$

So the numbers are 6 and -20 .

Ans: 6 and -20

8. Factorize $29z^2 + 234z - 243$. [3]

$$29 \times (-243) = -7047$$

$$7047 = 3 \times 3 \times 3 \times 3 \times 3 \times 29 = 27 \times 261, \text{ and } -27 + 261 = 234$$

$$= 29z^2 - 27z + 261z - 243$$

$$= z(29z - 27) + 9(29z - 27)$$

$$= (29z - 27)(z + 9)$$

Ans: $(29z - 27)(z + 9)$

9. Factorize $-4t - 12t^2 + 1$. [3]

Writing the terms in order: $-12t^2 - 4t + 1$

$$(-12) \times 1 = -12; (-6) \times 2 = -12 \text{ and } -6 + 2 = -4$$

$$= -12t^2 - 6t + 2t + 1$$

$$= -6t(2t + 1) + 1(2t + 1)$$

$$= (2t + 1)(1 - 6t)$$

Ans: $(2t + 1)(1 - 6t)$

10. Factorize $3x^2 - xy - 4y^2$. [2]

$$3 \times (-4) = -12; 3 \times (-4) = -12 \text{ and } 3 - 4 = -1$$

$$= 3x^2 + 3xy - 4xy - 4y^2$$

$$= 3x(x + y) - 4y(x + y)$$

$$= (x + y)(3x - 4y)$$

Ans: $(x + y)(3x - 4y)$

11. Factorize $7\sqrt{5}y^2 - 135y - 20\sqrt{5}$. [5]

$$7\sqrt{5} \times (-20\sqrt{5}) = 7 \times (-20) \times 5 = -700; 5 \times (-140) = -700 \text{ and } 5 - 140 = -135$$

$$= 7\sqrt{5}y^2 + 5y - 140y - 20\sqrt{5}$$

$$= y(7\sqrt{5}y + 5) - 4\sqrt{5}(7\sqrt{5}y + 5)$$

$$= (7\sqrt{5}y + 5)(y - 4\sqrt{5})$$

Ans: $(7\sqrt{5}y + 5)(y - 4\sqrt{5})$

12. Write $6t^2 + \frac{73}{35}t + \frac{6}{35}$ in the form $\frac{1}{35}(\dots)$, where the bracket has whole-number coefficients (the first step in factorizing it). [1]

$$6t^2 + \frac{73}{35}t + \frac{6}{35} = \frac{210t^2 + 73t + 6}{35}$$

$$= \frac{1}{35}(210t^2 + 73t + 6)$$

Ans: $\frac{1}{35}(210t^2 + 73t + 6)$

13. Factorize $4(p-1)^2 + 29(p-1)(q-1) + 30(q-1)^2$. [5]

Let $p-1 = x$ and $q-1 = y$

$$4(p-1)^2 + 29(p-1)(q-1) + 30(q-1)^2 = 4x^2 + 29xy + 30y^2$$

$$4 \times 30 = 120: 5 \times 24 = 120 \text{ and } 5 + 24 = 29$$

$$= 4x^2 + 5xy + 24xy + 30y^2$$

$$= x(4x + 5y) + 6y(4x + 5y)$$

$$= (4x + 5y)(x + 6y)$$

$$= [4(p-1) + 5(q-1)][(p-1) + 6(q-1)]$$

$$= (4p - 4 + 5q - 5)(p - 1 + 6q - 6)$$

$$= (4p + 5q - 9)(p + 6q - 7)$$

$$\text{Ans: } (4p + 5q - 9)(p + 6q - 7)$$

14. Factorize $8x^2 + 8y^2 + z^2 - 16xy + 4\sqrt{2}yz - 4\sqrt{2}zx$. [3]

$$8x^2 + 8y^2 + z^2 - 16xy + 4\sqrt{2}yz - 4\sqrt{2}zx = (2\sqrt{2}x)^2 + (2\sqrt{2}y)^2 + z^2 - 2(2\sqrt{2}x)(2\sqrt{2}y) + 2(2\sqrt{2}y)(z) - 2(z)(2\sqrt{2}x)$$

$$= (-2\sqrt{2}x + 2\sqrt{2}y + z)^2$$

$$\text{using } a^2 + b^2 + c^2 - 2ab + 2bc - 2ca = (-a + b + c)^2$$

$$\text{Ans: } (-2\sqrt{2}x + 2\sqrt{2}y + z)^2$$

15. Factorize $p^3 - 3p^2 + 3p - 1$. [2]

$$p^3 - 3p^2 + 3p - 1 = p^3 - 1^3 - 3(p)^2(1) + 3(p)(1)^2$$

$$= (p - 1)^3$$

$$\text{using } (a - b)^3 = a^3 - b^3 - 3ab(a - b)$$

$$\text{Ans: } (p - 1)^3$$

16. Factorize $a^6 - 64b^3$. [2]

$$a^6 - 64b^3 = (a^2)^3 - (4b)^3$$

$$= (a^2 - 4b)[(a^2)^2 + (a^2)(4b) + (4b)^2]$$

$$= (a^2 - 4b)(a^4 + 4a^2b + 16b^2)$$

$$\text{using } a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$\text{Ans: } (a^2 - 4b)(a^4 + 4a^2b + 16b^2)$$

17. Factorize $216a^3 + c^3$. [2]

$$216a^3 + c^3 = (6a)^3 + c^3$$

$$= (6a + c)[(6a)^2 - (6a)(c) + c^2]$$

$$= (6a + c)(36a^2 - 6ac + c^2)$$

$$\text{using } a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$\text{Ans: } (6a + c)(36a^2 - 6ac + c^2)$$

18. Factorize $16m^4 + 100m^2 + 625$. [5]

$$\begin{aligned}16m^4 + 100m^2 + 625 &= (4m^2)^2 + 2(4m^2)(25) + 25^2 - 100m^2 \\&= (4m^2 + 25)^2 - (10m)^2 \\&= (4m^2 + 25 + 10m)(4m^2 + 25 - 10m) \\&= (4m^2 + 10m + 25)(4m^2 - 10m + 25)\end{aligned}$$

Ans: $(4m^2 + 10m + 25)(4m^2 - 10m + 25)$

19. Factorize $(5p - 3q)^3 + (3q - r)^3 + (r - 5p)^3$. [2]

Here $(5p - 3q) + (3q - r) + (r - 5p) = 0$

If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

$$\therefore (5p - 3q)^3 + (3q - r)^3 + (r - 5p)^3 = 3(5p - 3q)(3q - r)(r - 5p)$$

Ans: $3(5p - 3q)(3q - r)(r - 5p)$

20. Factorize $2m(m + 3n)^3 - 18m^2n(m + 3n)$. [5]

$$\begin{aligned}2m(m + 3n)^3 - 18m^2n(m + 3n) &= 2m(m + 3n)[(m + 3n)^2 - 9mn] \\&= 2m(m + 3n)[m^2 + 6mn + 9n^2 - 9mn] \\&= 2m(m + 3n)(m^2 - 3mn + 9n^2)\end{aligned}$$

Ans: $2m(m + 3n)(m^2 - 3mn + 9n^2)$
