

Factorization

Question Paper, Set 05

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Factorize $49 - 64n^2$. [1 mark]
2. Factorize $25y^2 + 20y + 4$. [1 mark]
3. Factorize $49y^2 - 126yz + 81z^2$. [1 mark]
4. Factorize $3t^2 + 13t + 14$. [2 marks]
5. To factorize $4y^2 - 37y + 40$ by splitting the middle term, we need two numbers whose product is 4×40 and whose sum is -37 . Find them. [1 mark]
6. Factorize $12z^2 - 5z - 25$. [2 marks]
7. Factorize $29p^2 - 75p - 36$. [3 marks]
8. Factorize $-2b + 48 - b^2$. [2 marks]
9. Factorize $(p + q)^2 + 2(p + q) - 24$. [3 marks]
10. To factorize $2x^2 + 5xy + 2y^2$ by splitting the middle term, we need two numbers whose product is 2×2 and whose sum is 5. Find them. [1 mark]
11. Factorize $6t^2 - \frac{1}{5}t - \frac{2}{15}$. [5 marks]
12. Factorize $9p^2 + 9q^2 + 4r^2 + 18pq - 12qr - 12rp$. [2 marks]
13. Factorize $8z^3 + 12z^2 + 6z + 1$. [2 marks]
14. Factorize $216x^6 - z^6$. [2 marks]
15. Factorize $125y^6 + 64$. [3 marks]
16. Factorize $x^3 - 19x + 30$. [5 marks]
17. Factorize $1024a^4 + 1$. [3 marks]
18. Factorize $(a - 3b)^3 + (3b - 5)^3 + (5 - a)^3$. [1 mark]
19. Factorize $256t^9 - t$. [5 marks]

20. Factorize $p(p - 2q)^3 + 6p^2q(p - 2q)$.

[5 marks]

Key sheet

Q	Answer
1	$(7 + 8n)(7 - 8n)$
2	$(5y + 2)^2$
3	$(7y - 9z)^2$
4	$(t + 2)(3t + 7)$
5	-5 and -32
6	$(4z + 5)(3z - 5)$
7	$(29p + 12)(p - 3)$
8	$(b + 8)(6 - b)$
9	$(p + q - 4)(p + q + 6)$
10	1 and 4
11	$\frac{1}{15}(15t + 2)(6t - 1)$, i.e. $(3t + \frac{2}{5})(2t - \frac{1}{3})$
12	$(3p + 3q - 2r)^2$
13	$(2z + 1)^3$
14	$(6x^2 - z^2)(36x^4 + 6x^2z^2 + z^4)$
15	$(5y^2 + 4)(25y^4 - 20y^2 + 16)$
16	$(x - 2)(x - 3)(x + 5)$
17	$(32a^2 + 8a + 1)(32a^2 - 8a + 1)$
18	$3(a - 3b)(3b - 5)(5 - a)$
19	$t(16t^4 + 1)(4t^2 + 1)(2t + 1)(2t - 1)$
20	$p(p - 2q)(p^2 + 2pq + 4q^2)$

Solutions

1. Factorize $49 - 64n^2$. [1]

$$\begin{aligned}49 - 64n^2 &= 7^2 - (8n)^2 \\&= (7 + 8n)(7 - 8n) \\&\text{using } a^2 - b^2 = (a + b)(a - b)\end{aligned}$$

Ans: $(7 + 8n)(7 - 8n)$

2. Factorize $25y^2 + 20y + 4$. [1]

$$\begin{aligned}25y^2 + 20y + 4 &= (5y)^2 + 2^2 + 2(5y)(2) \\&= (5y + 2)^2 \\&= (5y + 2)(5y + 2) \\&\text{using } a^2 + b^2 + 2ab = (a + b)^2\end{aligned}$$

Ans: $(5y + 2)^2$

3. Factorize $49y^2 - 126yz + 81z^2$. [1]

$$\begin{aligned}49y^2 - 126yz + 81z^2 &= (7y)^2 + (9z)^2 - 2(7y)(9z) \\&= (7y - 9z)^2 \\&= (7y - 9z)(7y - 9z) \\&\text{using } a^2 + b^2 - 2ab = (a - b)^2\end{aligned}$$

Ans: $(7y - 9z)^2$

4. Factorize $3t^2 + 13t + 14$. [2]

$$\begin{aligned}3 \times 14 &= 42: 6 \times 7 = 42 \text{ and } 6 + 7 = 13 \\&= 3t^2 + 6t + 7t + 14 \\&= 3t(t + 2) + 7(t + 2) \\&= (t + 2)(3t + 7)\end{aligned}$$

Ans: $(t + 2)(3t + 7)$

5. To factorize $4y^2 - 37y + 40$ by splitting the middle term, we need two numbers whose product is 4×40 and whose sum is -37 . Find them. [1]

$$\begin{aligned}4 \times 40 &= 160 \\(-5) \times (-32) &= 160 \text{ and } -5 - 32 = -37 \\&\text{So the numbers are } -5 \text{ and } -32.\end{aligned}$$

Ans: -5 and -32

6. Factorize $12z^2 - 5z - 25$. [2]

$$\begin{aligned}12 \times (-25) &= -300: 15 \times (-20) = -300 \text{ and } 15 - 20 = -5 \\&= 12z^2 + 15z - 20z - 25 \\&= 3z(4z + 5) - 5(4z + 5) \\&= (4z + 5)(3z - 5)\end{aligned}$$

Ans: $(4z + 5)(3z - 5)$

7. Factorize $29p^2 - 75p - 36$. [3]

$$29 \times (-36) = -1044$$

$$1044 = 2 \times 2 \times 3 \times 3 \times 29 = 12 \times 87, \text{ and } 12 - 87 = -75$$

$$= 29p^2 + 12p - 87p - 36$$

$$= p(29p + 12) - 3(29p + 12)$$

$$= (29p + 12)(p - 3)$$

$$\text{Ans: } (29p + 12)(p - 3)$$

8. Factorize $-2b + 48 - b^2$. [2]

Writing the terms in order: $-b^2 - 2b + 48$

$$(-1) \times 48 = -48: (-8) \times 6 = -48 \text{ and } -8 + 6 = -2$$

$$= -b^2 - 8b + 6b + 48$$

$$= -b(b + 8) + 6(b + 8)$$

$$= (b + 8)(6 - b)$$

$$\text{Ans: } (b + 8)(6 - b)$$

9. Factorize $(p + q)^2 + 2(p + q) - 24$. [3]

Let $p + q = t$

$$(p + q)^2 + 2(p + q) - 24 = t^2 + 2t - 24$$

$$1 \times (-24) = -24: (-4) \times 6 = -24 \text{ and } -4 + 6 = 2$$

$$= t^2 - 4t + 6t - 24$$

$$= t(t - 4) + 6(t - 4)$$

$$= (t - 4)(t + 6)$$

$$= [(p + q) - 4][(p + q) + 6]$$

$$= (p + q - 4)(p + q + 6)$$

$$\text{Ans: } (p + q - 4)(p + q + 6)$$

10. To factorize $2x^2 + 5xy + 2y^2$ by splitting the middle term, we need two numbers whose product is 2×2 and whose sum is 5. Find them. [1]

$$2 \times 2 = 4$$

$$1 \times 4 = 4 \text{ and } 1 + 4 = 5$$

So the numbers are 1 and 4.

$$\text{Ans: } 1 \text{ and } 4$$

11. Factorize $6t^2 - \frac{1}{5}t - \frac{2}{15}$. [5]

$$6t^2 - \frac{1}{5}t - \frac{2}{15} = \frac{90t^2 - 3t - 2}{15}$$

$$= \frac{1}{15}(90t^2 - 3t - 2)$$

$$90 \times (-2) = -180: 12 \times (-15) = -180 \text{ and } 12 - 15 = -3$$

$$= \frac{1}{15}[90t^2 + 12t - 15t - 2]$$

$$= \frac{1}{15}[6t(15t + 2) - 1(15t + 2)]$$

$$= \frac{1}{15}(15t + 2)(6t - 1)$$

$$= \left(3t + \frac{2}{5}\right)\left(2t - \frac{1}{3}\right)$$

$$\text{Ans: } \frac{1}{15}(15t + 2)(6t - 1), \text{ i.e. } \left(3t + \frac{2}{5}\right)\left(2t - \frac{1}{3}\right)$$

12. Factorize $9p^2 + 9q^2 + 4r^2 + 18pq - 12qr - 12rp$. [2]

$$\begin{aligned} 9p^2 + 9q^2 + 4r^2 + 18pq - 12qr - 12rp &= (3p)^2 + (3q)^2 + (2r)^2 + 2(3p)(3q) - 2(3q)(2r) - 2(2r)(3p) \\ &= (3p + 3q - 2r)^2 \\ \text{using } a^2 + b^2 + c^2 + 2ab - 2bc - 2ca &= (a + b - c)^2 \end{aligned}$$

Ans: $(3p + 3q - 2r)^2$

13. Factorize $8z^3 + 12z^2 + 6z + 1$. [2]

$$\begin{aligned} 8z^3 + 12z^2 + 6z + 1 &= (2z)^3 + 1^3 + 3(2z)^2(1) + 3(2z)(1)^2 \\ &= (2z + 1)^3 \\ \text{using } (a + b)^3 &= a^3 + b^3 + 3ab(a + b) \end{aligned}$$

Ans: $(2z + 1)^3$

14. Factorize $216x^6 - z^6$. [2]

$$\begin{aligned} 216x^6 - z^6 &= (6x^2)^3 - (z^2)^3 \\ &= (6x^2 - z^2)[(6x^2)^2 + (6x^2)(z^2) + (z^2)^2] \\ &= (6x^2 - z^2)(36x^4 + 6x^2z^2 + z^4) \\ \text{using } a^3 - b^3 &= (a - b)(a^2 + ab + b^2) \end{aligned}$$

Ans: $(6x^2 - z^2)(36x^4 + 6x^2z^2 + z^4)$

15. Factorize $125y^6 + 64$. [3]

$$\begin{aligned} 125y^6 + 64 &= (5y^2)^3 + 4^3 \\ &= (5y^2 + 4)[(5y^2)^2 - (5y^2)(4) + 4^2] \\ &= (5y^2 + 4)(25y^4 - 20y^2 + 16) \\ \text{using } a^3 + b^3 &= (a + b)(a^2 - ab + b^2) \end{aligned}$$

Ans: $(5y^2 + 4)(25y^4 - 20y^2 + 16)$

16. Factorize $x^3 - 19x + 30$. [5]

$$\begin{aligned} x^3 - 19x + 30 &= x^3 - 4x - 15x + 30 \\ &= x(x^2 - 4) - 15(x - 2) \\ &= x(x - 2)(x + 2) - 15(x - 2) \\ &= (x - 2)[x(x + 2) - 15] \\ &= (x - 2)(x^2 + 2x - 15) \end{aligned}$$

Now $x^2 + 2x - 15$: $(-3) \times 5 = -15$ and $-3 + 5 = 2$

$$x^2 + 2x - 15 = x^2 - 3x + 5x - 15$$

$$= x(x - 3) + 5(x - 3)$$

$$= (x - 3)(x + 5)$$

$$\text{So } x^3 - 19x + 30 = (x - 2)(x - 3)(x + 5)$$

Ans: $(x - 2)(x - 3)(x + 5)$

17. Factorize $1024a^4 + 1$. [3]

$$\begin{aligned}1024a^4 + 1 &= 1^2 + (32a^2)^2 + 2(1)(32a^2) - 64a^2 \\&= (32a^2 + 1)^2 - (8a)^2 \\&= (32a^2 + 1 + 8a)(32a^2 + 1 - 8a) \\&= (32a^2 + 8a + 1)(32a^2 - 8a + 1)\end{aligned}$$

Ans: $(32a^2 + 8a + 1)(32a^2 - 8a + 1)$

18. Factorize $(a - 3b)^3 + (3b - 5)^3 + (5 - a)^3$. [1]

Here $(a - 3b) + (3b - 5) + (5 - a) = 0$

If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

$\therefore (a - 3b)^3 + (3b - 5)^3 + (5 - a)^3 = 3(a - 3b)(3b - 5)(5 - a)$

Ans: $3(a - 3b)(3b - 5)(5 - a)$

19. Factorize $256t^9 - t$. [5]

$$\begin{aligned}256t^9 - t &= t(256t^8 - 1) \\&= t[(16t^4)^2 - 1^2] \\&= t(16t^4 + 1)(16t^4 - 1) \\&= t(16t^4 + 1)[(4t^2)^2 - 1^2] \\&= t(16t^4 + 1)(4t^2 + 1)(4t^2 - 1) \\&= t(16t^4 + 1)(4t^2 + 1)[(2t)^2 - 1^2] \\&= t(16t^4 + 1)(4t^2 + 1)(2t + 1)(2t - 1)\end{aligned}$$

Ans: $t(16t^4 + 1)(4t^2 + 1)(2t + 1)(2t - 1)$

20. Factorize $p(p - 2q)^3 + 6p^2q(p - 2q)$. [5]

$$\begin{aligned}p(p - 2q)^3 + 6p^2q(p - 2q) &= p(p - 2q)[(p - 2q)^2 + 6pq] \\&= p(p - 2q)[p^2 - 4pq + 4q^2 + 6pq] \\&= p(p - 2q)(p^2 + 2pq + 4q^2)\end{aligned}$$

Ans: $p(p - 2q)(p^2 + 2pq + 4q^2)$
