

Factorization

Question Paper, Set 04

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Factorize $144x^2 - 169z^2$. [1 mark]
2. Factorize $25x^2 + 90x + 81$. [1 mark]
3. Factorize $49t^2 - 42t + 9$. [1 mark]
4. Factorize $x^2 + 6x + 9 - 25y^2$. [2 marks]
5. To factorize $15m^2 + 38m + 7$ by splitting the middle term, we need two numbers whose product is 15×7 and whose sum is 38. Find them. [1 mark]
6. Factorize $10a^2 - 29a + 21$. [2 marks]
7. To factorize $12b^2 - 11b - 5$ by splitting the middle term, we need two numbers whose product is $12 \times (-5)$ and whose sum is -11 . Find them. [1 mark]
8. Factorize $-20 + 13t - 2t^2$. [2 marks]
9. Factorize $(3m - 2p)^2 + (3m - 2p) - 2$. [3 marks]
10. Factorize $12x^2 - 4xz - 5z^2$. [2 marks]
11. Factorize $t^2 - 7\sqrt{7}t + 84$. [3 marks]
12. Write $2y^2 + \frac{7}{30}y - \frac{1}{30}$ in the form $\frac{1}{30}(\dots)$, where the bracket has whole-number coefficients (the first step in factorizing it). [1 mark]
13. Factorize $3(m - 4)^2 - 19(m - 4)(n - 3) + 6(n - 3)^2$. [5 marks]
14. Factorize $27a^3 + 54a^2 + 36a + 7$. [5 marks]
15. Factorize $125m^3 + 8$. [2 marks]
16. Factorize $x^3 - 52x - 96$. [5 marks]
17. Factorize $81a^4 + 64c^4$. [5 marks]
18. Factorize $(m - 5n)^3 + (5n - 4p)^3 + (4p - m)^3$. [2 marks]

19. Factorize $81ac^4 - a^5$. [3 marks]

20. Factorize $20y^2 + (4y^2 + 1)y + 5$. [3 marks]

Key sheet

Q	Answer
1	$(12x + 13z)(12x - 13z)$
2	$(5x + 9)^2$
3	$(7t - 3)^2$
4	$(x + 3 + 5y)(x + 3 - 5y)$
5	3 and 35
6	$(5a - 7)(2a - 3)$
7	4 and -15
8	$(2t - 5)(4 - t)$
9	$(3m - 2p - 1)(3m - 2p + 2)$
10	$(2x + z)(6x - 5z)$
11	$(t - 3\sqrt{7})(t - 4\sqrt{7})$
12	$\frac{1}{30}(60y^2 + 7y - 1)$
13	$(3m - n - 9)(m - 6n + 14)$
14	$(3a + 1)(9a^2 + 15a + 7)$
15	$(5m + 2)(25m^2 - 10m + 4)$
16	$(x + 2)(x + 6)(x - 8)$
17	$(9a^2 + 12ac + 8c^2)(9a^2 - 12ac + 8c^2)$
18	$3(m - 5n)(5n - 4p)(4p - m)$
19	$a(a^2 + 9c^2)(a + 3c)(3c - a)$
20	$(y + 5)(4y^2 + 1)$

Solutions

1. Factorize $144x^2 - 169z^2$. [1]

$$144x^2 - 169z^2 = (12x)^2 - (13z)^2$$

$$= (12x + 13z)(12x - 13z)$$

$$\text{using } a^2 - b^2 = (a + b)(a - b)$$

$$\text{Ans: } (12x + 13z)(12x - 13z)$$

2. Factorize $25x^2 + 90x + 81$. [1]

$$25x^2 + 90x + 81 = (5x)^2 + 9^2 + 2(5x)(9)$$

$$= (5x + 9)^2$$

$$= (5x + 9)(5x + 9)$$

$$\text{using } a^2 + b^2 + 2ab = (a + b)^2$$

$$\text{Ans: } (5x + 9)^2$$

3. Factorize $49t^2 - 42t + 9$. [1]

$$49t^2 - 42t + 9 = (7t)^2 + 3^2 - 2(7t)(3)$$

$$= (7t - 3)^2$$

$$= (7t - 3)(7t - 3)$$

$$\text{using } a^2 + b^2 - 2ab = (a - b)^2$$

$$\text{Ans: } (7t - 3)^2$$

4. Factorize $x^2 + 6x + 9 - 25y^2$. [2]

$$x^2 + 6x + 9 - 25y^2 = (x + 3)^2 - (5y)^2$$

$$= (x + 3 + 5y)(x + 3 - 5y)$$

$$\text{using } a^2 + 2ab + b^2 = (a + b)^2 \text{ and } a^2 - b^2 = (a + b)(a - b)$$

$$\text{Ans: } (x + 3 + 5y)(x + 3 - 5y)$$

5. To factorize $15m^2 + 38m + 7$ by splitting the middle term, we need two numbers whose product is 15×7 and whose sum is 38. Find them. [1]

$$15 \times 7 = 105$$

$$3 \times 35 = 105 \text{ and } 3 + 35 = 38$$

So the numbers are 3 and 35.

$$\text{Ans: } 3 \text{ and } 35$$

6. Factorize $10a^2 - 29a + 21$. [2]

$$10 \times 21 = 210: (-14) \times (-15) = 210 \text{ and } -14 - 15 = -29$$

$$= 10a^2 - 14a - 15a + 21$$

$$= 2a(5a - 7) - 3(5a - 7)$$

$$= (5a - 7)(2a - 3)$$

$$\text{Ans: } (5a - 7)(2a - 3)$$

7. To factorize $12b^2 - 11b - 5$ by splitting the middle term, we need two numbers whose product is $12 \times (-5)$ and whose sum is -11 . Find them. [1]

$$12 \times (-5) = -60$$

$$4 \times (-15) = -60 \text{ and } 4 - 15 = -11$$

So the numbers are 4 and -15 .

Ans: 4 and -15

8. Factorize $-20 + 13t - 2t^2$. [2]

Writing the terms in order: $-2t^2 + 13t - 20$

$$(-2) \times (-20) = 40; 5 \times 8 = 40 \text{ and } 5 + 8 = 13$$

$$= -2t^2 + 5t + 8t - 20$$

$$= -t(2t - 5) + 4(2t - 5)$$

$$= (2t - 5)(4 - t)$$

Ans: $(2t - 5)(4 - t)$

9. Factorize $(3m - 2p)^2 + (3m - 2p) - 2$. [3]

Let $3m - 2p = t$

$$(3m - 2p)^2 + (3m - 2p) - 2 = t^2 + t - 2$$

$$1 \times (-2) = -2; (-1) \times 2 = -2 \text{ and } -1 + 2 = 1$$

$$= t^2 - t + 2t - 2$$

$$= t(t - 1) + 2(t - 1)$$

$$= (t - 1)(t + 2)$$

$$= [(3m - 2p) - 1][(3m - 2p) + 2]$$

$$= (3m - 2p - 1)(3m - 2p + 2)$$

Ans: $(3m - 2p - 1)(3m - 2p + 2)$

10. Factorize $12x^2 - 4xz - 5z^2$. [2]

$$12 \times (-5) = -60; 6 \times (-10) = -60 \text{ and } 6 - 10 = -4$$

$$= 12x^2 + 6xz - 10xz - 5z^2$$

$$= 6x(2x + z) - 5z(2x + z)$$

$$= (2x + z)(6x - 5z)$$

Ans: $(2x + z)(6x - 5z)$

11. Factorize $t^2 - 7\sqrt{7}t + 84$. [3]

$$\text{Product} = 84, \text{ sum} = -7\sqrt{7}; (-3\sqrt{7}) \times (-4\sqrt{7}) = 84 \text{ and } -3\sqrt{7} + (-4\sqrt{7}) = -7\sqrt{7}$$

$$= t^2 - 3\sqrt{7}t - 4\sqrt{7}t + 84$$

$$= t(t - 3\sqrt{7}) - 4\sqrt{7}(t - 3\sqrt{7})$$

$$= (t - 3\sqrt{7})(t - 4\sqrt{7})$$

Ans: $(t - 3\sqrt{7})(t - 4\sqrt{7})$

12. Write $2y^2 + \frac{7}{30}y - \frac{1}{30}$ in the form $\frac{1}{30}(\dots)$, where the bracket has whole-number coefficients (the first step in factorizing it). [1]

$$2y^2 + \frac{7}{30}y - \frac{1}{30} = \frac{60y^2 + 7y - 1}{30}$$
$$= \frac{1}{30}(60y^2 + 7y - 1)$$

Ans: $\frac{1}{30}(60y^2 + 7y - 1)$

13. Factorize $3(m-4)^2 - 19(m-4)(n-3) + 6(n-3)^2$. [5]

Let $m-4 = x$ and $n-3 = y$

$$3(m-4)^2 - 19(m-4)(n-3) + 6(n-3)^2 = 3x^2 - 19xy + 6y^2$$

$$3 \times 6 = 18: (-1) \times (-18) = 18 \text{ and } -1 - 18 = -19$$

$$= 3x^2 - xy - 18xy + 6y^2$$

$$= x(3x - y) - 6y(3x - y)$$

$$= (3x - y)(x - 6y)$$

$$= [3(m-4) - (n-3)][(m-4) - 6(n-3)]$$

$$= (3m - 12 - n + 3)(m - 4 - 6n + 18)$$

$$= (3m - n - 9)(m - 6n + 14)$$

Ans: $(3m - n - 9)(m - 6n + 14)$

14. Factorize $27a^3 + 54a^2 + 36a + 7$. [5]

$$27a^3 + 54a^2 + 36a + 7 = (3a)^3 + 3(3a)^2(2) + 3(3a)(2)^2 + 2^3 - 1$$

$$= (3a + 2)^3 - 1^3$$

$$= (3a + 2 - 1)[(3a + 2)^2 + (3a + 2)1 + 1^2]$$

$$= (3a + 1)[9a^2 + 12a + 4 + 3a + 2 + 1]$$

$$= (3a + 1)(9a^2 + 15a + 7)$$

$$\text{using } a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Ans: $(3a + 1)(9a^2 + 15a + 7)$

15. Factorize $125m^3 + 8$. [2]

$$125m^3 + 8 = (5m)^3 + 2^3$$

$$= (5m + 2)[(5m)^2 - (5m)(2) + 2^2]$$

$$= (5m + 2)(25m^2 - 10m + 4)$$

$$\text{using } a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Ans: $(5m + 2)(25m^2 - 10m + 4)$

16. Factorize $x^3 - 52x - 96$. [5]

$$x^3 - 52x - 96 = x^3 - 4x - 48x - 96$$

$$= x(x^2 - 4) - 48(x + 2)$$

$$= x(x + 2)(x - 2) - 48(x + 2)$$

$$= (x + 2)[x(x - 2) - 48]$$

$$= (x + 2)(x^2 - 2x - 48)$$

$$\text{Now } x^2 - 2x - 48: 6 \times (-8) = -48 \text{ and } 6 - 8 = -2$$

$$x^2 - 2x - 48 = x^2 + 6x - 8x - 48$$

$$= x(x + 6) - 8(x + 6)$$

$$= (x + 6)(x - 8)$$

$$\text{So } x^3 - 52x - 96 = (x + 2)(x + 6)(x - 8)$$

$$\text{Ans: } (x + 2)(x + 6)(x - 8)$$

17. Factorize $81a^4 + 64c^4$. [5]

$$81a^4 + 64c^4 = (9a^2)^2 + (8c^2)^2 + 2(9a^2)(8c^2) - 144a^2c^2$$

$$= (9a^2 + 8c^2)^2 - (12ac)^2$$

$$= (9a^2 + 8c^2 + 12ac)(9a^2 + 8c^2 - 12ac)$$

$$= (9a^2 + 12ac + 8c^2)(9a^2 - 12ac + 8c^2)$$

$$\text{Ans: } (9a^2 + 12ac + 8c^2)(9a^2 - 12ac + 8c^2)$$

18. Factorize $(m - 5n)^3 + (5n - 4p)^3 + (4p - m)^3$. [2]

$$\text{Here } (m - 5n) + (5n - 4p) + (4p - m) = 0$$

$$\text{If } a + b + c = 0, \text{ then } a^3 + b^3 + c^3 = 3abc$$

$$\therefore (m - 5n)^3 + (5n - 4p)^3 + (4p - m)^3 = 3(m - 5n)(5n - 4p)(4p - m)$$

$$\text{Ans: } 3(m - 5n)(5n - 4p)(4p - m)$$

19. Factorize $81ac^4 - a^5$. [3]

$$81ac^4 - a^5 = a(81c^4 - a^4)$$

$$= a[(9c^2)^2 - (a^2)^2]$$

$$= a(a^2 + 9c^2)(9c^2 - a^2)$$

$$= a(a^2 + 9c^2)[(3c)^2 - a^2]$$

$$= a(a^2 + 9c^2)(a + 3c)(3c - a)$$

$$\text{Ans: } a(a^2 + 9c^2)(a + 3c)(3c - a)$$

20. Factorize $20y^2 + (4y^2 + 1)y + 5$. [3]

$$20y^2 + (4y^2 + 1)y + 5 = 20y^2 + 4y^3 + y + 5$$

$$= 4y^3 + 20y^2 + y + 5$$

$$= 4y^2(y + 5) + 1(y + 5)$$

$$= (y + 5)(4y^2 + 1)$$

$$\text{Ans: } (y + 5)(4y^2 + 1)$$
