

Factorization

Question Paper, Set 03

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Factorize $16m^4 - 625$. [2 marks]
2. Factorize $16x^2 + 40x + 25$. [1 mark]
3. Factorize $n^2 - 16n + 64$. [1 mark]
4. Factorize $9x^2 - 12x + 4 - 25y^2$. [3 marks]
5. To factorize $2t^2 - 13t + 6$ by splitting the middle term, we need two numbers whose product is 2×6 and whose sum is -13 . Find them. [1 mark]
6. To factorize $3x^2 + 8x - 3$ by splitting the middle term, we need two numbers whose product is $3 \times (-3)$ and whose sum is 8. Find them. [1 mark]
7. Factorize $29b^2 + 258b + 208$. [3 marks]
8. Factorize $20p - 3p^2 - 32$. [2 marks]
9. Factorize $(2m + n)^2 - 4(2m + n) - 5$. [3 marks]
10. To factorize $3a^2 - 20ab + 12b^2$ by splitting the middle term, we need two numbers whose product is 3×12 and whose sum is -20 . Find them. [1 mark]
11. Factorize $3\sqrt{5}t^2 + 28t - 4\sqrt{5}$. [5 marks]
12. Factorize $3t^2 + \frac{13}{30}t - \frac{1}{30}$. [5 marks]
13. Factorize $3(a - 2)^2 + 17(a - 2)(c + 3) + 10(c + 3)^2$. [5 marks]
14. Factorize $9a^2 + 16b^2 + c^2 - 24ab - 8bc + 6ca$. [2 marks]
15. Factorize $p^3 - 6p^2q + 12pq^2 - 8q^3$. [2 marks]
16. Factorize $8m^3 - 27n^3$. [2 marks]
17. Factorize $64x^3 + 1$. [2 marks]
18. Factorize $(4x - y)^3 + (y - 5)^3 + (5 - 4x)^3$. [1 mark]

19. Factorize $t^9 - 256t$. [5 marks]

20. Factorize $30a^2 + (5a^2 + 6)a + 36$. [3 marks]

Key sheet

Q	Answer
1	$(4m^2 + 25)(2m + 5)(2m - 5)$
2	$(4x + 5)^2$
3	$(n - 8)^2$
4	$(3x - 2 + 5y)(3x - 2 - 5y)$
5	-1 and -12
6	-1 and 9
7	$(29b + 26)(b + 8)$
8	$(3p - 8)(4 - p)$
9	$(2m + n + 1)(2m + n - 5)$
10	-2 and -18
11	$(3\sqrt{5}t - 2)(t + 2\sqrt{5})$
12	$\frac{1}{30}(18t - 1)(5t + 1)$, i.e. $(t + \frac{1}{5})(3t - \frac{1}{6})$
13	$(3a + 2c)(a + 5c + 13)$
14	$(3a - 4b + c)^2$
15	$(p - 2q)^3$
16	$(2m - 3n)(4m^2 + 6mn + 9n^2)$
17	$(4x + 1)(16x^2 - 4x + 1)$
18	$3(4x - y)(y - 5)(5 - 4x)$
19	$t(t^4 + 16)(t^2 + 4)(t + 2)(t - 2)$
20	$(a + 6)(5a^2 + 6)$

Solutions

1. Factorize $16m^4 - 625$. [2]

$$\begin{aligned}16m^4 - 625 &= (4m^2)^2 - 25^2 \\&= (4m^2 + 25)(4m^2 - 25) \\&= (4m^2 + 25)[(2m)^2 - 5^2] \\&= (4m^2 + 25)(2m + 5)(2m - 5) \\ \text{Ans: } &(4m^2 + 25)(2m + 5)(2m - 5)\end{aligned}$$

2. Factorize $16x^2 + 40x + 25$. [1]

$$\begin{aligned}16x^2 + 40x + 25 &= (4x)^2 + 5^2 + 2(4x)(5) \\&= (4x + 5)^2 \\&= (4x + 5)(4x + 5) \\ \text{using } a^2 + b^2 + 2ab &= (a + b)^2 \\ \text{Ans: } &(4x + 5)^2\end{aligned}$$

3. Factorize $n^2 - 16n + 64$. [1]

$$\begin{aligned}n^2 - 16n + 64 &= n^2 + 8^2 - 2(n)(8) \\&= (n - 8)^2 \\&= (n - 8)(n - 8) \\ \text{using } a^2 + b^2 - 2ab &= (a - b)^2 \\ \text{Ans: } &(n - 8)^2\end{aligned}$$

4. Factorize $9x^2 - 12x + 4 - 25y^2$. [3]

$$\begin{aligned}9x^2 - 12x + 4 - 25y^2 &= (3x - 2)^2 - (5y)^2 \\&= (3x - 2 + 5y)(3x - 2 - 5y) \\ \text{using } a^2 - 2ab + b^2 &= (a - b)^2 \text{ and } a^2 - b^2 = (a + b)(a - b) \\ \text{Ans: } &(3x - 2 + 5y)(3x - 2 - 5y)\end{aligned}$$

5. To factorize $2t^2 - 13t + 6$ by splitting the middle term, we need two numbers whose product is 2×6 and whose sum is -13 . Find them. [1]

$$\begin{aligned}2 \times 6 &= 12 \\ (-1) \times (-12) &= 12 \text{ and } -1 - 12 = -13 \\ \text{So the numbers are } &-1 \text{ and } -12. \\ \text{Ans: } &-1 \text{ and } -12\end{aligned}$$

6. To factorize $3x^2 + 8x - 3$ by splitting the middle term, we need two numbers whose product is $3 \times (-3)$ and whose sum is 8. Find them. [1]

$$\begin{aligned}3 \times (-3) &= -9 \\ (-1) \times 9 &= -9 \text{ and } -1 + 9 = 8 \\ \text{So the numbers are } &-1 \text{ and } 9. \\ \text{Ans: } &-1 \text{ and } 9\end{aligned}$$

7. Factorize $29b^2 + 258b + 208$. [3]

$$29 \times 208 = 6032$$

$$6032 = 2 \times 2 \times 2 \times 2 \times 13 \times 29 = 26 \times 232, \text{ and } 26 + 232 = 258$$

$$= 29b^2 + 26b + 232b + 208$$

$$= b(29b + 26) + 8(29b + 26)$$

$$= (29b + 26)(b + 8)$$

$$\text{Ans: } (29b + 26)(b + 8)$$

8. Factorize $20p - 3p^2 - 32$. [2]

$$\text{Writing the terms in order: } -3p^2 + 20p - 32$$

$$(-3) \times (-32) = 96; 8 \times 12 = 96 \text{ and } 8 + 12 = 20$$

$$= -3p^2 + 8p + 12p - 32$$

$$= -p(3p - 8) + 4(3p - 8)$$

$$= (3p - 8)(4 - p)$$

$$\text{Ans: } (3p - 8)(4 - p)$$

9. Factorize $(2m + n)^2 - 4(2m + n) - 5$. [3]

$$\text{Let } 2m + n = p$$

$$(2m + n)^2 - 4(2m + n) - 5 = p^2 - 4p - 5$$

$$1 \times (-5) = -5; 1 \times (-5) = -5 \text{ and } 1 - 5 = -4$$

$$= p^2 + p - 5p - 5$$

$$= p(p + 1) - 5(p + 1)$$

$$= (p + 1)(p - 5)$$

$$= [(2m + n) + 1][(2m + n) - 5]$$

$$= (2m + n + 1)(2m + n - 5)$$

$$\text{Ans: } (2m + n + 1)(2m + n - 5)$$

10. To factorize $3a^2 - 20ab + 12b^2$ by splitting the middle term, we need two numbers whose product is 3×12 and whose sum is -20 . Find them. [1]

$$3 \times 12 = 36$$

$$(-2) \times (-18) = 36 \text{ and } -2 - 18 = -20$$

So the numbers are -2 and -18 .

$$\text{Ans: } -2 \text{ and } -18$$

11. Factorize $3\sqrt{5}t^2 + 28t - 4\sqrt{5}$. [5]

$$3\sqrt{5} \times (-4\sqrt{5}) = 3 \times (-4) \times 5 = -60; (-2) \times 30 = -60 \text{ and } -2 + 30 = 28$$

$$= 3\sqrt{5}t^2 - 2t + 30t - 4\sqrt{5}$$

$$= t(3\sqrt{5}t - 2) + 2\sqrt{5}(3\sqrt{5}t - 2)$$

$$= (3\sqrt{5}t - 2)(t + 2\sqrt{5})$$

$$\text{Ans: } (3\sqrt{5}t - 2)(t + 2\sqrt{5})$$

12. Factorize $3t^2 + \frac{13}{30}t - \frac{1}{30}$. [5]

$$3t^2 + \frac{13}{30}t - \frac{1}{30} = \frac{90t^2 + 13t - 1}{30}$$

$$= \frac{1}{30}(90t^2 + 13t - 1)$$

$$90 \times (-1) = -90: (-5) \times 18 = -90 \text{ and } -5 + 18 = 13$$

$$= \frac{1}{30}[90t^2 - 5t + 18t - 1]$$

$$= \frac{1}{30}[5t(18t - 1) + 1(18t - 1)]$$

$$= \frac{1}{30}(18t - 1)(5t + 1)$$

$$= \left(t + \frac{1}{5}\right) \left(3t - \frac{1}{6}\right)$$

$$\text{Ans: } \frac{1}{30}(18t - 1)(5t + 1), \text{ i.e. } \left(t + \frac{1}{5}\right) \left(3t - \frac{1}{6}\right)$$

13. Factorize $3(a - 2)^2 + 17(a - 2)(c + 3) + 10(c + 3)^2$. [5]

$$\text{Let } a - 2 = x \text{ and } c + 3 = y$$

$$3(a - 2)^2 + 17(a - 2)(c + 3) + 10(c + 3)^2 = 3x^2 + 17xy + 10y^2$$

$$3 \times 10 = 30: 2 \times 15 = 30 \text{ and } 2 + 15 = 17$$

$$= 3x^2 + 2xy + 15xy + 10y^2$$

$$= x(3x + 2y) + 5y(3x + 2y)$$

$$= (3x + 2y)(x + 5y)$$

$$= [3(a - 2) + 2(c + 3)][(a - 2) + 5(c + 3)]$$

$$= (3a - 6 + 2c + 6)(a - 2 + 5c + 15)$$

$$= (3a + 2c)(a + 5c + 13)$$

$$\text{Ans: } (3a + 2c)(a + 5c + 13)$$

14. Factorize $9a^2 + 16b^2 + c^2 - 24ab - 8bc + 6ca$. [2]

$$9a^2 + 16b^2 + c^2 - 24ab - 8bc + 6ca = (3a)^2 + (4b)^2 + c^2 - 2(3a)(4b) - 2(4b)(c) + 2(c)(3a)$$

$$= (3a - 4b + c)^2$$

$$\text{using } a^2 + b^2 + c^2 - 2ab - 2bc + 2ca = (a - b + c)^2$$

$$\text{Ans: } (3a - 4b + c)^2$$

15. Factorize $p^3 - 6p^2q + 12pq^2 - 8q^3$. [2]

$$p^3 - 6p^2q + 12pq^2 - 8q^3 = p^3 - (2q)^3 - 3(p)^2(2q) + 3(p)(2q)^2$$

$$= (p - 2q)^3$$

$$\text{using } (a - b)^3 = a^3 - b^3 - 3ab(a - b)$$

$$\text{Ans: } (p - 2q)^3$$

16. Factorize $8m^3 - 27n^3$. [2]

$$8m^3 - 27n^3 = (2m)^3 - (3n)^3$$

$$= (2m - 3n)[(2m)^2 + (2m)(3n) + (3n)^2]$$

$$= (2m - 3n)(4m^2 + 6mn + 9n^2)$$

$$\text{using } a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$\text{Ans: } (2m - 3n)(4m^2 + 6mn + 9n^2)$$

17. Factorize $64x^3 + 1$. [2]

$$\begin{aligned} 64x^3 + 1 &= (4x)^3 + 1^3 \\ &= (4x + 1)[(4x)^2 - (4x)(1) + 1^2] \\ &= (4x + 1)(16x^2 - 4x + 1) \\ \text{using } a^3 + b^3 &= (a + b)(a^2 - ab + b^2) \end{aligned}$$

Ans: $(4x + 1)(16x^2 - 4x + 1)$

18. Factorize $(4x - y)^3 + (y - 5)^3 + (5 - 4x)^3$. [1]

Here $(4x - y) + (y - 5) + (5 - 4x) = 0$

If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

$\therefore (4x - y)^3 + (y - 5)^3 + (5 - 4x)^3 = 3(4x - y)(y - 5)(5 - 4x)$

Ans: $3(4x - y)(y - 5)(5 - 4x)$

19. Factorize $t^9 - 256t$. [5]

$$\begin{aligned} t^9 - 256t &= t(t^8 - 256) \\ &= t[(t^4)^2 - 16^2] \\ &= t(t^4 + 16)(t^4 - 16) \\ &= t(t^4 + 16)[(t^2)^2 - 4^2] \\ &= t(t^4 + 16)(t^2 + 4)(t^2 - 4) \\ &= t(t^4 + 16)(t^2 + 4)[t^2 - 2^2] \\ &= t(t^4 + 16)(t^2 + 4)(t + 2)(t - 2) \end{aligned}$$

Ans: $t(t^4 + 16)(t^2 + 4)(t + 2)(t - 2)$

20. Factorize $30a^2 + (5a^2 + 6)a + 36$. [3]

$$\begin{aligned} 30a^2 + (5a^2 + 6)a + 36 &= 30a^2 + 5a^3 + 6a + 36 \\ &= 5a^3 + 30a^2 + 6a + 36 \\ &= 5a^2(a + 6) + 6(a + 6) \\ &= (a + 6)(5a^2 + 6) \end{aligned}$$

Ans: $(a + 6)(5a^2 + 6)$
