

Factorization

Question Paper, Set 02

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Factorize $64t^2 - 9$. [1 mark]
2. Factorize $x^2 + 10xz + 25z^2$. [1 mark]
3. Factorize $25a^2 - 60ac + 36c^2$. [1 mark]
4. Factorize $4x^2 - 4x + 1 - 4y^2$. [2 marks]
5. Factorize $4x^2 + 37x + 40$. [2 marks]
6. Factorize $6t^2 - 35t + 36$. [2 marks]
7. Factorize $5b^2 - 27b - 18$. [2 marks]
8. Factorize $15x - 18 - 2x^2$. [2 marks]
9. To factorize $12x^2 + 19xz + 5z^2$ by splitting the middle term, we need two numbers whose product is 12×5 and whose sum is 19. Find them. [1 mark]
10. Factorize $a^2 - 2\sqrt{6}a - 18$. [3 marks]
11. Write $4t^2 + \frac{7}{5}t + \frac{1}{10}$ in the form $\frac{1}{10}(\dots)$, where the bracket has whole-number coefficients (the first step in factorizing it). [1 mark]
12. Factorize $x^2 + 2y^2 + 32z^2 - 2\sqrt{2}xy + 16yz - 8\sqrt{2}zx$. [3 marks]
13. Factorize $a^3 + 15a(a + 5) + 125$. [3 marks]
14. Factorize $27t^3 + 27t^2 + 9t - 7$. [5 marks]
15. Factorize $64 + 27a^3$. [2 marks]
16. Factorize $x^3 - 31x - 30$. [5 marks]
17. Factorize $x^4 + 4x^2z^2 + 16z^4$. [3 marks]
18. Factorize $(5a - 4b)^3 + (4b - 3c)^3 + (3c - 5a)^3$. [1 mark]
19. Factorize $2m^9 - 2mn^8$. [5 marks]

20. Factorize $(t + 4)(t^2 + 25) - 10t^2 - 40t$.

[5 marks]

Key sheet

Q	Answer
1	$(8t + 3)(8t - 3)$
2	$(x + 5z)^2$
3	$(5a - 6c)^2$
4	$(2x - 1 + 2y)(2x - 1 - 2y)$
5	$(4x + 5)(x + 8)$
6	$(3t - 4)(2t - 9)$
7	$(5b + 3)(b - 6)$
8	$(2x - 3)(6 - x)$
9	4 and 15
10	$(a + \sqrt{6})(a - 3\sqrt{6})$
11	$\frac{1}{10}(40t^2 + 14t + 1)$
12	$(-x + \sqrt{2}y + 4\sqrt{2}z)^2$
13	$(a + 5)^3$
14	$(3t - 1)(9t^2 + 12t + 7)$
15	$(3a + 4)(9a^2 - 12a + 16)$
16	$(x + 1)(x + 5)(x - 6)$
17	$(x^2 + 2xz + 4z^2)(x^2 - 2xz + 4z^2)$
18	$3(5a - 4b)(4b - 3c)(3c - 5a)$
19	$2m(m^4 + n^4)(m^2 + n^2)(m + n)(m - n)$
20	$(t + 4)(t - 5)^2$

Solutions

1. Factorize $64t^2 - 9$. [1]

$$64t^2 - 9 = (8t)^2 - 3^2$$

$$= (8t + 3)(8t - 3)$$

$$\text{using } a^2 - b^2 = (a + b)(a - b)$$

$$\text{Ans: } (8t + 3)(8t - 3)$$

2. Factorize $x^2 + 10xz + 25z^2$. [1]

$$x^2 + 10xz + 25z^2 = x^2 + (5z)^2 + 2(x)(5z)$$

$$= (x + 5z)^2$$

$$= (x + 5z)(x + 5z)$$

$$\text{using } a^2 + b^2 + 2ab = (a + b)^2$$

$$\text{Ans: } (x + 5z)^2$$

3. Factorize $25a^2 - 60ac + 36c^2$. [1]

$$25a^2 - 60ac + 36c^2 = (5a)^2 + (6c)^2 - 2(5a)(6c)$$

$$= (5a - 6c)^2$$

$$= (5a - 6c)(5a - 6c)$$

$$\text{using } a^2 + b^2 - 2ab = (a - b)^2$$

$$\text{Ans: } (5a - 6c)^2$$

4. Factorize $4x^2 - 4x + 1 - 4y^2$. [2]

$$4x^2 - 4x + 1 - 4y^2 = (2x - 1)^2 - (2y)^2$$

$$= (2x - 1 + 2y)(2x - 1 - 2y)$$

$$\text{using } a^2 - 2ab + b^2 = (a - b)^2 \text{ and } a^2 - b^2 = (a + b)(a - b)$$

$$\text{Ans: } (2x - 1 + 2y)(2x - 1 - 2y)$$

5. Factorize $4x^2 + 37x + 40$. [2]

$$4 \times 40 = 160: 5 \times 32 = 160 \text{ and } 5 + 32 = 37$$

$$= 4x^2 + 5x + 32x + 40$$

$$= x(4x + 5) + 8(4x + 5)$$

$$= (4x + 5)(x + 8)$$

$$\text{Ans: } (4x + 5)(x + 8)$$

6. Factorize $6t^2 - 35t + 36$. [2]

$$6 \times 36 = 216: (-8) \times (-27) = 216 \text{ and } -8 - 27 = -35$$

$$= 6t^2 - 8t - 27t + 36$$

$$= 2t(3t - 4) - 9(3t - 4)$$

$$= (3t - 4)(2t - 9)$$

$$\text{Ans: } (3t - 4)(2t - 9)$$

7. Factorize $5b^2 - 27b - 18$. [2]

$$\begin{aligned}5 \times (-18) &= -90: 3 \times (-30) = -90 \text{ and } 3 - 30 = -27 \\&= 5b^2 + 3b - 30b - 18 \\&= b(5b + 3) - 6(5b + 3) \\&= (5b + 3)(b - 6)\end{aligned}$$

Ans: $(5b + 3)(b - 6)$

8. Factorize $15x - 18 - 2x^2$. [2]

$$\begin{aligned}\text{Writing the terms in order: } &-2x^2 + 15x - 18 \\(-2) \times (-18) &= 36: 3 \times 12 = 36 \text{ and } 3 + 12 = 15 \\&= -2x^2 + 3x + 12x - 18 \\&= -x(2x - 3) + 6(2x - 3) \\&= (2x - 3)(6 - x)\end{aligned}$$

Ans: $(2x - 3)(6 - x)$

9. To factorize $12x^2 + 19xz + 5z^2$ by splitting the middle term, we need two numbers whose product is 12×5 and whose sum is 19. Find them. [1]

$$\begin{aligned}12 \times 5 &= 60 \\4 \times 15 &= 60 \text{ and } 4 + 15 = 19 \\ \text{So the numbers are } &4 \text{ and } 15.\end{aligned}$$

Ans: 4 and 15

10. Factorize $a^2 - 2\sqrt{6}a - 18$. [3]

$$\begin{aligned}\text{Product} &= -18, \text{ sum} = -2\sqrt{6}: \sqrt{6} \times (-3\sqrt{6}) = -18 \text{ and } \sqrt{6} + (-3\sqrt{6}) = -2\sqrt{6} \\&= a^2 + \sqrt{6}a - 3\sqrt{6}a - 18 \\&= a(a + \sqrt{6}) - 3\sqrt{6}(a + \sqrt{6}) \\&= (a + \sqrt{6})(a - 3\sqrt{6})\end{aligned}$$

Ans: $(a + \sqrt{6})(a - 3\sqrt{6})$

11. Write $4t^2 + \frac{7}{5}t + \frac{1}{10}$ in the form $\frac{1}{10}(\dots)$, where the bracket has whole-number coefficients (the first step in factorizing it). [1]

$$\begin{aligned}4t^2 + \frac{7}{5}t + \frac{1}{10} &= \frac{40t^2 + 14t + 1}{10} \\&= \frac{1}{10}(40t^2 + 14t + 1)\end{aligned}$$

Ans: $\frac{1}{10}(40t^2 + 14t + 1)$

12. Factorize $x^2 + 2y^2 + 32z^2 - 2\sqrt{2}xy + 16yz - 8\sqrt{2}zx$. [3]

$$\begin{aligned}x^2 + 2y^2 + 32z^2 - 2\sqrt{2}xy + 16yz - 8\sqrt{2}zx &= x^2 + (\sqrt{2}y)^2 + (4\sqrt{2}z)^2 - 2(x)(\sqrt{2}y) + \\&2(\sqrt{2}y)(4\sqrt{2}z) - 2(4\sqrt{2}z)(x) \\&= (-x + \sqrt{2}y + 4\sqrt{2}z)^2\end{aligned}$$

$$\text{using } a^2 + b^2 + c^2 - 2ab + 2bc - 2ca = (-a + b + c)^2$$

Ans: $(-x + \sqrt{2}y + 4\sqrt{2}z)^2$

13. Factorize $a^3 + 15a(a + 5) + 125$. [3]

$$\begin{aligned}a^3 + 15a(a + 5) + 125 &= a^3 + 15a^2 + 75a + 125 \\&= a^3 + 5^3 + 3(a)^2(5) + 3(a)(5)^2 \\&= (a + 5)^3\end{aligned}$$

$$\text{using } (a + b)^3 = a^3 + b^3 + 3ab(a + b)$$

$$\text{Ans: } (a + 5)^3$$

14. Factorize $27t^3 + 27t^2 + 9t - 7$. [5]

$$\begin{aligned}27t^3 + 27t^2 + 9t - 7 &= (3t)^3 + 3(3t)^2(1) + 3(3t)(1)^2 + 1^3 - 8 \\&= (3t + 1)^3 - 2^3 \\&= (3t + 1 - 2)[(3t + 1)^2 + (3t + 1)2 + 2^2] \\&= (3t - 1)[9t^2 + 6t + 1 + 6t + 2 + 4] \\&= (3t - 1)(9t^2 + 12t + 7)\end{aligned}$$

$$\text{using } a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$\text{Ans: } (3t - 1)(9t^2 + 12t + 7)$$

15. Factorize $64 + 27a^3$. [2]

$$\begin{aligned}64 + 27a^3 &= 4^3 + (3a)^3 \\&= (4 + 3a)[4^2 - (4)(3a) + (3a)^2] \\&= (3a + 4)(9a^2 - 12a + 16) \\&\text{using } a^3 + b^3 = (a + b)(a^2 - ab + b^2)\end{aligned}$$

$$\text{Ans: } (3a + 4)(9a^2 - 12a + 16)$$

16. Factorize $x^3 - 31x - 30$. [5]

$$\begin{aligned}x^3 - 31x - 30 &= x^3 - x - 30x - 30 \\&= x(x^2 - 1) - 30(x + 1) \\&= x(x + 1)(x - 1) - 30(x + 1) \\&= (x + 1)[x(x - 1) - 30] \\&= (x + 1)(x^2 - x - 30) \\&\text{Now } x^2 - x - 30: 5 \times (-6) = -30 \text{ and } 5 - 6 = -1 \\&x^2 - x - 30 = x^2 + 5x - 6x - 30 \\&= x(x + 5) - 6(x + 5) \\&= (x + 5)(x - 6)\end{aligned}$$

$$\text{So } x^3 - 31x - 30 = (x + 1)(x + 5)(x - 6)$$

$$\text{Ans: } (x + 1)(x + 5)(x - 6)$$

17. Factorize $x^4 + 4x^2z^2 + 16z^4$. [3]

$$\begin{aligned}x^4 + 4x^2z^2 + 16z^4 &= (x^2)^2 + 2(x^2)(4z^2) + (4z^2)^2 - 4x^2z^2 \\&= (x^2 + 4z^2)^2 - (2xz)^2 \\&= (x^2 + 4z^2 + 2xz)(x^2 + 4z^2 - 2xz) \\&= (x^2 + 2xz + 4z^2)(x^2 - 2xz + 4z^2)\end{aligned}$$

$$\text{Ans: } (x^2 + 2xz + 4z^2)(x^2 - 2xz + 4z^2)$$

18. Factorize $(5a - 4b)^3 + (4b - 3c)^3 + (3c - 5a)^3$. [1]

Here $(5a - 4b) + (4b - 3c) + (3c - 5a) = 0$

If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

$\therefore (5a - 4b)^3 + (4b - 3c)^3 + (3c - 5a)^3 = 3(5a - 4b)(4b - 3c)(3c - 5a)$

Ans: $3(5a - 4b)(4b - 3c)(3c - 5a)$

19. Factorize $2m^9 - 2mn^8$. [5]

$$2m^9 - 2mn^8 = 2m(m^8 - n^8)$$

$$= 2m[(m^4)^2 - (n^4)^2]$$

$$= 2m(m^4 + n^4)(m^4 - n^4)$$

$$= 2m(m^4 + n^4)[(m^2)^2 - (n^2)^2]$$

$$= 2m(m^4 + n^4)(m^2 + n^2)(m^2 - n^2)$$

$$= 2m(m^4 + n^4)(m^2 + n^2)(m + n)(m - n)$$

Ans: $2m(m^4 + n^4)(m^2 + n^2)(m + n)(m - n)$

20. Factorize $(t + 4)(t^2 + 25) - 10t^2 - 40t$. [5]

$$(t + 4)(t^2 + 25) - 10t^2 - 40t = (t + 4)(t^2 + 25) - 10t(t + 4)$$

$$= (t + 4)(t^2 + 25 - 10t)$$

$$= (t + 4)(t^2 - 10t + 25)$$

$$= (t + 4)(t - 5)^2$$

Ans: $(t + 4)(t - 5)^2$
