

Factorization

Question Paper, Set 10

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Factorize $64 - 121p^2$. [1 mark]
2. Factorize $25t^2 + 60t + 36$. [1 mark]
3. Factorize $9y^2 - 12y + 4$. [1 mark]
4. Factorize $4x^2 + 12xy + 9y^2 - 4$. [3 marks]
5. To factorize $9t^2 + 15t + 4$ by splitting the middle term, we need two numbers whose product is 9×4 and whose sum is 15. Find them. [1 mark]
6. Factorize $5y^2 - 31y + 6$. [2 marks]
7. Factorize $5m^2 + 39m - 8$. [2 marks]
8. Factorize $19t^2 - 47t - 30$. [3 marks]
9. Factorize $9x - x^2 - 20$. [2 marks]
10. To factorize $4p^2 - 8pq - 5q^2$ by splitting the middle term, we need two numbers whose product is $4 \times (-5)$ and whose sum is -8 . Find them. [1 mark]
11. Factorize $9\sqrt{5}y^2 - 15y - 10\sqrt{5}$. [5 marks]
12. Factorize $2x^2 + \frac{31}{21}x + \frac{2}{21}$. [5 marks]
13. Factorize $9x^2 + 16y^2 + 9z^2 - 24xy - 24yz + 18zx$. [2 marks]
14. Factorize $x^3 + 9x(x + 3) + 27$. [3 marks]
15. Factorize $64m^6 - 125n^3$. [2 marks]
16. Factorize $125 + 8a^3$. [2 marks]
17. Factorize $x^3 - 73x - 72$. [5 marks]
18. Factorize $81x^4 + 9x^2 + 1$. [3 marks]
19. Factorize $(4m - 3n)^3 + (3n - 1)^3 + (1 - 4m)^3$. [1 mark]

20. Factorize $256p^9 - pq^8$.

[5 marks]

Key sheet

Q	Answer
1	$(8 + 11p)(8 - 11p)$
2	$(5t + 6)^2$
3	$(3y - 2)^2$
4	$(2x + 3y + 2)(2x + 3y - 2)$
5	3 and 12
6	$(5y - 1)(y - 6)$
7	$(5m - 1)(m + 8)$
8	$(19t + 10)(t - 3)$
9	$(x - 4)(5 - x)$
10	2 and -10
11	$(3\sqrt{5}y + 5)(3y - 2\sqrt{5})$
12	$\frac{1}{21}(14x + 1)(3x + 2)$, i.e. $(x + \frac{2}{3})(2x + \frac{1}{7})$
13	$(3x - 4y + 3z)^2$
14	$(x + 3)^3$
15	$(4m^2 - 5n)(16m^4 + 20m^2n + 25n^2)$
16	$(2a + 5)(4a^2 - 10a + 25)$
17	$(x + 1)(x + 8)(x - 9)$
18	$(9x^2 + 3x + 1)(9x^2 - 3x + 1)$
19	$3(4m - 3n)(3n - 1)(1 - 4m)$
20	$p(16p^4 + q^4)(4p^2 + q^2)(2p + q)(2p - q)$

Solutions

1. Factorize $64 - 121p^2$. [1]

$$\begin{aligned}64 - 121p^2 &= 8^2 - (11p)^2 \\&= (8 + 11p)(8 - 11p) \\&\text{using } a^2 - b^2 = (a + b)(a - b)\end{aligned}$$

Ans: $(8 + 11p)(8 - 11p)$

2. Factorize $25t^2 + 60t + 36$. [1]

$$\begin{aligned}25t^2 + 60t + 36 &= (5t)^2 + 6^2 + 2(5t)(6) \\&= (5t + 6)^2 \\&= (5t + 6)(5t + 6) \\&\text{using } a^2 + b^2 + 2ab = (a + b)^2\end{aligned}$$

Ans: $(5t + 6)^2$

3. Factorize $9y^2 - 12y + 4$. [1]

$$\begin{aligned}9y^2 - 12y + 4 &= (3y)^2 + 2^2 - 2(3y)(2) \\&= (3y - 2)^2 \\&= (3y - 2)(3y - 2) \\&\text{using } a^2 + b^2 - 2ab = (a - b)^2\end{aligned}$$

Ans: $(3y - 2)^2$

4. Factorize $4x^2 + 12xy + 9y^2 - 4$. [3]

$$\begin{aligned}4x^2 + 12xy + 9y^2 - 4 &= (2x + 3y)^2 - 2^2 \\&= (2x + 3y + 2)(2x + 3y - 2) \\&\text{using } a^2 + 2ab + b^2 = (a + b)^2 \text{ and } a^2 - b^2 = (a + b)(a - b)\end{aligned}$$

Ans: $(2x + 3y + 2)(2x + 3y - 2)$

5. To factorize $9t^2 + 15t + 4$ by splitting the middle term, we need two numbers whose product is 9×4 and whose sum is 15. Find them. [1]

$$9 \times 4 = 36$$

$$3 \times 12 = 36 \text{ and } 3 + 12 = 15$$

So the numbers are 3 and 12.

Ans: 3 and 12

6. Factorize $5y^2 - 31y + 6$. [2]

$$\begin{aligned}5 \times 6 = 30: (-1) \times (-30) &= 30 \text{ and } -1 - 30 = -31 \\&= 5y^2 - y - 30y + 6 \\&= y(5y - 1) - 6(5y - 1) \\&= (5y - 1)(y - 6)\end{aligned}$$

Ans: $(5y - 1)(y - 6)$

7. Factorize $5m^2 + 39m - 8$. [2]

$$\begin{aligned}5 \times (-8) &= -40: (-1) \times 40 = -40 \text{ and } -1 + 40 = 39 \\&= 5m^2 - m + 40m - 8 \\&= m(5m - 1) + 8(5m - 1) \\&= (5m - 1)(m + 8)\end{aligned}$$

Ans: $(5m - 1)(m + 8)$

8. Factorize $19t^2 - 47t - 30$. [3]

$$\begin{aligned}19 \times (-30) &= -570 \\570 &= 2 \times 3 \times 5 \times 19 = 10 \times 57, \text{ and } 10 - 57 = -47 \\&= 19t^2 + 10t - 57t - 30 \\&= t(19t + 10) - 3(19t + 10) \\&= (19t + 10)(t - 3)\end{aligned}$$

Ans: $(19t + 10)(t - 3)$

9. Factorize $9x - x^2 - 20$. [2]

$$\begin{aligned}\text{Writing the terms in order: } &-x^2 + 9x - 20 \\(-1) \times (-20) &= 20: 4 \times 5 = 20 \text{ and } 4 + 5 = 9 \\&= -x^2 + 4x + 5x - 20 \\&= -x(x - 4) + 5(x - 4) \\&= (x - 4)(5 - x)\end{aligned}$$

Ans: $(x - 4)(5 - x)$

10. To factorize $4p^2 - 8pq - 5q^2$ by splitting the middle term, we need two numbers whose product is $4 \times (-5)$ and whose sum is -8 . Find them. [1]

$$\begin{aligned}4 \times (-5) &= -20 \\2 \times (-10) &= -20 \text{ and } 2 - 10 = -8 \\&\text{So the numbers are } 2 \text{ and } -10.\end{aligned}$$

Ans: 2 and -10

11. Factorize $9\sqrt{5}y^2 - 15y - 10\sqrt{5}$. [5]

$$\begin{aligned}9\sqrt{5} \times (-10\sqrt{5}) &= 9 \times (-10) \times 5 = -450: 15 \times (-30) = -450 \text{ and } 15 - 30 = -15 \\&= 9\sqrt{5}y^2 + 15y - 30y - 10\sqrt{5} \\&= 3y(3\sqrt{5}y + 5) - 2\sqrt{5}(3\sqrt{5}y + 5) \\&= (3\sqrt{5}y + 5)(3y - 2\sqrt{5})\end{aligned}$$

Ans: $(3\sqrt{5}y + 5)(3y - 2\sqrt{5})$

12. Factorize $2x^2 + \frac{31}{21}x + \frac{2}{21}$. [5]

$$2x^2 + \frac{31}{21}x + \frac{2}{21} = \frac{42x^2 + 31x + 2}{21}$$

$$= \frac{1}{21}(42x^2 + 31x + 2)$$

$$42 \times 2 = 84; 3 \times 28 = 84 \text{ and } 3 + 28 = 31$$

$$= \frac{1}{21}[42x^2 + 3x + 28x + 2]$$

$$= \frac{1}{21}[3x(14x + 1) + 2(14x + 1)]$$

$$= \frac{1}{21}(14x + 1)(3x + 2)$$

$$= \left(x + \frac{2}{3}\right) \left(2x + \frac{1}{7}\right)$$

$$\text{Ans: } \frac{1}{21}(14x + 1)(3x + 2), \text{ i.e. } \left(x + \frac{2}{3}\right) \left(2x + \frac{1}{7}\right)$$

13. Factorize $9x^2 + 16y^2 + 9z^2 - 24xy - 24yz + 18zx$. [2]

$$9x^2 + 16y^2 + 9z^2 - 24xy - 24yz + 18zx = (3x)^2 + (4y)^2 + (3z)^2 - 2(3x)(4y) - 2(4y)(3z) + 2(3z)(3x)$$

$$= (3x - 4y + 3z)^2$$

$$\text{using } a^2 + b^2 + c^2 - 2ab - 2bc + 2ca = (a - b + c)^2$$

$$\text{Ans: } (3x - 4y + 3z)^2$$

14. Factorize $x^3 + 9x(x + 3) + 27$. [3]

$$x^3 + 9x(x + 3) + 27 = x^3 + 9x^2 + 27x + 27$$

$$= x^3 + 3^3 + 3(x)^2(3) + 3(x)(3)^2$$

$$= (x + 3)^3$$

$$\text{using } (a + b)^3 = a^3 + b^3 + 3ab(a + b)$$

$$\text{Ans: } (x + 3)^3$$

15. Factorize $64m^6 - 125n^3$. [2]

$$64m^6 - 125n^3 = (4m^2)^3 - (5n)^3$$

$$= (4m^2 - 5n)[(4m^2)^2 + (4m^2)(5n) + (5n)^2]$$

$$= (4m^2 - 5n)(16m^4 + 20m^2n + 25n^2)$$

$$\text{using } a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$\text{Ans: } (4m^2 - 5n)(16m^4 + 20m^2n + 25n^2)$$

16. Factorize $125 + 8a^3$. [2]

$$125 + 8a^3 = 5^3 + (2a)^3$$

$$= (5 + 2a)[5^2 - (5)(2a) + (2a)^2]$$

$$= (2a + 5)(4a^2 - 10a + 25)$$

$$\text{using } a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$\text{Ans: } (2a + 5)(4a^2 - 10a + 25)$$

17. Factorize $x^3 - 73x - 72$. [5]

$$x^3 - 73x - 72 = x^3 - x - 72x - 72$$

$$= x(x^2 - 1) - 72(x + 1)$$

$$= x(x + 1)(x - 1) - 72(x + 1)$$

$$= (x + 1)[x(x - 1) - 72]$$

$$= (x + 1)(x^2 - x - 72)$$

$$\text{Now } x^2 - x - 72: 8 \times (-9) = -72 \text{ and } 8 - 9 = -1$$

$$x^2 - x - 72 = x^2 + 8x - 9x - 72$$

$$= x(x + 8) - 9(x + 8)$$

$$= (x + 8)(x - 9)$$

$$\text{So } x^3 - 73x - 72 = (x + 1)(x + 8)(x - 9)$$

$$\text{Ans: } (x + 1)(x + 8)(x - 9)$$

18. Factorize $81x^4 + 9x^2 + 1$. [3]

$$81x^4 + 9x^2 + 1 = (9x^2)^2 + 2(9x^2)(1) + 1^2 - 9x^2$$

$$= (9x^2 + 1)^2 - (3x)^2$$

$$= (9x^2 + 1 + 3x)(9x^2 + 1 - 3x)$$

$$= (9x^2 + 3x + 1)(9x^2 - 3x + 1)$$

$$\text{Ans: } (9x^2 + 3x + 1)(9x^2 - 3x + 1)$$

19. Factorize $(4m - 3n)^3 + (3n - 1)^3 + (1 - 4m)^3$. [1]

$$\text{Here } (4m - 3n) + (3n - 1) + (1 - 4m) = 0$$

$$\text{If } a + b + c = 0, \text{ then } a^3 + b^3 + c^3 = 3abc$$

$$\therefore (4m - 3n)^3 + (3n - 1)^3 + (1 - 4m)^3 = 3(4m - 3n)(3n - 1)(1 - 4m)$$

$$\text{Ans: } 3(4m - 3n)(3n - 1)(1 - 4m)$$

20. Factorize $256p^9 - pq^8$. [5]

$$256p^9 - pq^8 = p(256p^8 - q^8)$$

$$= p[(16p^4)^2 - (q^4)^2]$$

$$= p(16p^4 + q^4)(16p^4 - q^4)$$

$$= p(16p^4 + q^4)[(4p^2)^2 - (q^2)^2]$$

$$= p(16p^4 + q^4)(4p^2 + q^2)(4p^2 - q^2)$$

$$= p(16p^4 + q^4)(4p^2 + q^2)[(2p)^2 - q^2]$$

$$= p(16p^4 + q^4)(4p^2 + q^2)(2p + q)(2p - q)$$

$$\text{Ans: } p(16p^4 + q^4)(4p^2 + q^2)(2p + q)(2p - q)$$
