

Factorization

Question Paper, Set 01

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Factorize $p^2 + 10p + 25$. [1 mark]
2. Factorize $16a^2 - 56a + 49$. [1 mark]
3. Factorize $p^2 - 4p + 4 - 9q^2$. [2 marks]
4. To factorize $5y^2 + 23y + 12$ by splitting the middle term, we need two numbers whose product is 5×12 and whose sum is 23. Find them. [1 mark]
5. To factorize $3z^2 - 19z + 6$ by splitting the middle term, we need two numbers whose product is 3×6 and whose sum is -19 . Find them. [1 mark]
6. To factorize $4z^2 - 21z - 18$ by splitting the middle term, we need two numbers whose product is $4 \times (-18)$ and whose sum is -21 . Find them. [1 mark]
7. Factorize $19m^2 + 130m - 176$. [3 marks]
8. Factorize $36 - 41m - 5m^2$. [3 marks]
9. Factorize $2x^2 - 3xy + y^2$. [2 marks]
10. Factorize $4\sqrt{2}m^2 - 14m + 3\sqrt{2}$. [5 marks]
11. Write $a^2 - \frac{3}{20}a - \frac{9}{20}$ in the form $\frac{1}{20}(\dots)$, where the bracket has whole-number coefficients (the first step in factorizing it). [1 mark]
12. Factorize $6(a - 3)^2 + (a - 3)(c - 5) - 12(c - 5)^2$. [5 marks]
13. Factorize $4p^2 + 9q^2 + 16r^2 - 12pq - 24qr + 16rp$. [2 marks]
14. Factorize $t^3 + 12t^2 + 48t + 64$. [2 marks]
15. Factorize $64p^6 - 125$. [2 marks]
16. Factorize $y^6z^6 + 216$. [3 marks]
17. Factorize $x^3 - 43x + 42$. [5 marks]
18. Factorize $p^4 + 16p^2q^2 + 256q^4$. [5 marks]

19. Factorize $(3a - b)^3 + (b - 4c)^3 + (4c - 3a)^3$. [2 marks]

20. Factorize $48x^5 - 243xz^4$. [3 marks]

Key sheet

Q	Answer
1	$(p + 5)^2$
2	$(4a - 7)^2$
3	$(p - 2 + 3q)(p - 2 - 3q)$
4	3 and 20
5	-1 and -18
6	3 and -24
7	$(19m - 22)(m + 8)$
8	$(m + 9)(4 - 5m)$
9	$(2x - y)(x - y)$
10	$(2\sqrt{2}m - 1)(2m - 3\sqrt{2})$
11	$\frac{1}{20}(20a^2 - 3a - 9)$
12	$(3a - 4c + 11)(2a + 3c - 21)$
13	$(2p - 3q + 4r)^2$
14	$(t + 4)^3$
15	$(4p^2 - 5)(16p^4 + 20p^2 + 25)$
16	$(y^2z^2 + 6)(y^4z^4 - 6y^2z^2 + 36)$
17	$(x - 1)(x - 6)(x + 7)$
18	$(p^2 + 4pq + 16q^2)(p^2 - 4pq + 16q^2)$
19	$3(3a - b)(b - 4c)(4c - 3a)$
20	$3x(4x^2 + 9z^2)(2x + 3z)(2x - 3z)$

Solutions

1. Factorize $p^2 + 10p + 25$. [1]

$$p^2 + 10p + 25 = p^2 + 5^2 + 2(p)(5)$$

$$= (p + 5)^2$$

$$= (p + 5)(p + 5)$$

$$\text{using } a^2 + b^2 + 2ab = (a + b)^2$$

$$\text{Ans: } (p + 5)^2$$

2. Factorize $16a^2 - 56a + 49$. [1]

$$16a^2 - 56a + 49 = (4a)^2 + 7^2 - 2(4a)(7)$$

$$= (4a - 7)^2$$

$$= (4a - 7)(4a - 7)$$

$$\text{using } a^2 + b^2 - 2ab = (a - b)^2$$

$$\text{Ans: } (4a - 7)^2$$

3. Factorize $p^2 - 4p + 4 - 9q^2$. [2]

$$p^2 - 4p + 4 - 9q^2 = (p - 2)^2 - (3q)^2$$

$$= (p - 2 + 3q)(p - 2 - 3q)$$

$$\text{using } a^2 - 2ab + b^2 = (a - b)^2 \text{ and } a^2 - b^2 = (a + b)(a - b)$$

$$\text{Ans: } (p - 2 + 3q)(p - 2 - 3q)$$

4. To factorize $5y^2 + 23y + 12$ by splitting the middle term, we need two numbers whose product is 5×12 and whose sum is 23. Find them. [1]

$$5 \times 12 = 60$$

$$3 \times 20 = 60 \text{ and } 3 + 20 = 23$$

So the numbers are 3 and 20.

$$\text{Ans: 3 and 20}$$

5. To factorize $3z^2 - 19z + 6$ by splitting the middle term, we need two numbers whose product is 3×6 and whose sum is -19 . Find them. [1]

$$3 \times 6 = 18$$

$$(-1) \times (-18) = 18 \text{ and } -1 - 18 = -19$$

So the numbers are -1 and -18 .

$$\text{Ans: } -1 \text{ and } -18$$

6. To factorize $4z^2 - 21z - 18$ by splitting the middle term, we need two numbers whose product is $4 \times (-18)$ and whose sum is -21 . Find them. [1]

$$4 \times (-18) = -72$$

$$3 \times (-24) = -72 \text{ and } 3 - 24 = -21$$

So the numbers are 3 and -24 .

$$\text{Ans: 3 and } -24$$

7. Factorize $19m^2 + 130m - 176$. [3]

$$19 \times (-176) = -3344$$

$$3344 = 2 \times 2 \times 2 \times 2 \times 11 \times 19 = 22 \times 152, \text{ and } -22 + 152 = 130$$

$$= 19m^2 - 22m + 152m - 176$$

$$= m(19m - 22) + 8(19m - 22)$$

$$= (19m - 22)(m + 8)$$

$$\text{Ans: } (19m - 22)(m + 8)$$

8. Factorize $36 - 41m - 5m^2$. [3]

$$\text{Writing the terms in order: } -5m^2 - 41m + 36$$

$$(-5) \times 36 = -180; (-45) \times 4 = -180 \text{ and } -45 + 4 = -41$$

$$= -5m^2 - 45m + 4m + 36$$

$$= -5m(m + 9) + 4(m + 9)$$

$$= (m + 9)(4 - 5m)$$

$$\text{Ans: } (m + 9)(4 - 5m)$$

9. Factorize $2x^2 - 3xy + y^2$. [2]

$$2 \times 1 = 2; (-1) \times (-2) = 2 \text{ and } -1 - 2 = -3$$

$$= 2x^2 - xy - 2xy + y^2$$

$$= x(2x - y) - y(2x - y)$$

$$= (2x - y)(x - y)$$

$$\text{Ans: } (2x - y)(x - y)$$

10. Factorize $4\sqrt{2}m^2 - 14m + 3\sqrt{2}$. [5]

$$4\sqrt{2} \times 3\sqrt{2} = 4 \times 3 \times 2 = 24; (-2) \times (-12) = 24 \text{ and } -2 - 12 = -14$$

$$= 4\sqrt{2}m^2 - 2m - 12m + 3\sqrt{2}$$

$$= 2m(2\sqrt{2}m - 1) - 3\sqrt{2}(2\sqrt{2}m - 1)$$

$$= (2\sqrt{2}m - 1)(2m - 3\sqrt{2})$$

$$\text{Ans: } (2\sqrt{2}m - 1)(2m - 3\sqrt{2})$$

11. Write $a^2 - \frac{3}{20}a - \frac{9}{20}$ in the form $\frac{1}{20}(\dots)$, where the bracket has whole-number coefficients (the first step in factorizing it). [1]

$$a^2 - \frac{3}{20}a - \frac{9}{20} = \frac{20a^2 - 3a - 9}{20}$$

$$= \frac{1}{20}(20a^2 - 3a - 9)$$

$$\text{Ans: } \frac{1}{20}(20a^2 - 3a - 9)$$

12. Factorize $6(a-3)^2 + (a-3)(c-5) - 12(c-5)^2$. [5]

Let $a-3 = x$ and $c-5 = y$

$$6(a-3)^2 + (a-3)(c-5) - 12(c-5)^2 = 6x^2 + xy - 12y^2$$

$$6 \times (-12) = -72: (-8) \times 9 = -72 \text{ and } -8 + 9 = 1$$

$$= 6x^2 - 8xy + 9xy - 12y^2$$

$$= 2x(3x - 4y) + 3y(3x - 4y)$$

$$= (3x - 4y)(2x + 3y)$$

$$= [3(a-3) - 4(c-5)][2(a-3) + 3(c-5)]$$

$$= (3a - 9 - 4c + 20)(2a - 6 + 3c - 15)$$

$$= (3a - 4c + 11)(2a + 3c - 21)$$

$$\text{Ans: } (3a - 4c + 11)(2a + 3c - 21)$$

13. Factorize $4p^2 + 9q^2 + 16r^2 - 12pq - 24qr + 16rp$. [2]

$$4p^2 + 9q^2 + 16r^2 - 12pq - 24qr + 16rp = (2p)^2 + (3q)^2 + (4r)^2 - 2(2p)(3q) - 2(3q)(4r) + 2(4r)(2p)$$

$$= (2p - 3q + 4r)^2$$

$$\text{using } a^2 + b^2 + c^2 - 2ab - 2bc + 2ca = (a - b + c)^2$$

$$\text{Ans: } (2p - 3q + 4r)^2$$

14. Factorize $t^3 + 12t^2 + 48t + 64$. [2]

$$t^3 + 12t^2 + 48t + 64 = t^3 + 4^3 + 3(t)^2(4) + 3(t)(4)^2$$

$$= (t + 4)^3$$

$$\text{using } (a + b)^3 = a^3 + b^3 + 3ab(a + b)$$

$$\text{Ans: } (t + 4)^3$$

15. Factorize $64p^6 - 125$. [2]

$$64p^6 - 125 = (4p^2)^3 - 5^3$$

$$= (4p^2 - 5)[(4p^2)^2 + (4p^2)(5) + 5^2]$$

$$= (4p^2 - 5)(16p^4 + 20p^2 + 25)$$

$$\text{using } a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$\text{Ans: } (4p^2 - 5)(16p^4 + 20p^2 + 25)$$

16. Factorize $y^6z^6 + 216$. [3]

$$y^6z^6 + 216 = (y^2z^2)^3 + 6^3$$

$$= (y^2z^2 + 6)[(y^2z^2)^2 - (y^2z^2)(6) + 6^2]$$

$$= (y^2z^2 + 6)(y^4z^4 - 6y^2z^2 + 36)$$

$$\text{using } a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$\text{Ans: } (y^2z^2 + 6)(y^4z^4 - 6y^2z^2 + 36)$$

17. Factorize $x^3 - 43x + 42$. [5]

$$x^3 - 43x + 42 = x^3 - x - 42x + 42$$

$$= x(x^2 - 1) - 42(x - 1)$$

$$= x(x - 1)(x + 1) - 42(x - 1)$$

$$= (x - 1)[x(x + 1) - 42]$$

$$= (x - 1)(x^2 + x - 42)$$

$$\text{Now } x^2 + x - 42: (-6) \times 7 = -42 \text{ and } -6 + 7 = 1$$

$$x^2 + x - 42 = x^2 - 6x + 7x - 42$$

$$= x(x - 6) + 7(x - 6)$$

$$= (x - 6)(x + 7)$$

$$\text{So } x^3 - 43x + 42 = (x - 1)(x - 6)(x + 7)$$

$$\text{Ans: } (x - 1)(x - 6)(x + 7)$$

18. Factorize $p^4 + 16p^2q^2 + 256q^4$. [5]

$$p^4 + 16p^2q^2 + 256q^4 = (p^2)^2 + 2(p^2)(16q^2) + (16q^2)^2 - 16p^2q^2$$

$$= (p^2 + 16q^2)^2 - (4pq)^2$$

$$= (p^2 + 16q^2 + 4pq)(p^2 + 16q^2 - 4pq)$$

$$= (p^2 + 4pq + 16q^2)(p^2 - 4pq + 16q^2)$$

$$\text{Ans: } (p^2 + 4pq + 16q^2)(p^2 - 4pq + 16q^2)$$

19. Factorize $(3a - b)^3 + (b - 4c)^3 + (4c - 3a)^3$. [2]

$$\text{Here } (3a - b) + (b - 4c) + (4c - 3a) = 0$$

$$\text{If } a + b + c = 0, \text{ then } a^3 + b^3 + c^3 = 3abc$$

$$\therefore (3a - b)^3 + (b - 4c)^3 + (4c - 3a)^3 = 3(3a - b)(b - 4c)(4c - 3a)$$

$$\text{Ans: } 3(3a - b)(b - 4c)(4c - 3a)$$

20. Factorize $48x^5 - 243xz^4$. [3]

$$48x^5 - 243xz^4 = 3x(16x^4 - 81z^4)$$

$$= 3x[(4x^2)^2 - (9z^2)^2]$$

$$= 3x(4x^2 + 9z^2)(4x^2 - 9z^2)$$

$$= 3x(4x^2 + 9z^2)[(2x)^2 - (3z)^2]$$

$$= 3x(4x^2 + 9z^2)(2x + 3z)(2x - 3z)$$

$$\text{Ans: } 3x(4x^2 + 9z^2)(2x + 3z)(2x - 3z)$$
