EXPRESSWALA ACADEMY

Exponents (Level 2)

Question Paper, Set 09

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Find the value of $(-1)^{37} + (-1)^{52} + 7^0$. [1 mark]

2. Simplify $\left(\frac{3}{5}\right)^4 \times \left(\frac{5}{3}\right)^6$. [2 marks]

3. Simplify $25^4 \times 125^{-2}$. [1 mark]

4. Find the value of $\frac{(25)^{\frac{3}{2}} \times (8)^{\frac{5}{3}}}{(10000)^{\frac{3}{4}}}$. [2 marks]

5. Find the value of $\left(\frac{1}{27}\right)^{-\frac{2}{3}} + \left(\frac{1}{2}\right)^{-4} - 6 \times 9^{\frac{1}{2}} \times 10^0$. [3 marks]

6. Simplify $\frac{15^{\frac{2}{5}} \times 5^{\frac{1}{5}} \times 25^{\frac{1}{5}}}{10^{\frac{1}{3}} \times 6^{\frac{1}{3}}} \div \frac{2^{\frac{1}{3}} \times 4^{-\frac{1}{2}}}{3^{\frac{3}{5}} \times 15^{\frac{1}{3}}}$. [5 marks]

7. Show that $\left(\frac{x^a}{x^b}\right)^{a+b} \div \left(\frac{x^a}{x^c}\right)^{a+c} \times \left(\frac{x^b}{x^c}\right)^{b+c} = 1$. [3 marks]

8. If $a + b + c = 0$, show that $x^{a^3} \cdot x^{b^3} \cdot x^{c^3} = \left(x^{abc}\right)^3$. [5 marks]

9. Solve $9^x = 27$. [1 mark]

10. Solve $2^{3x-5} \times 11^{x-1} = 22$. [2 marks]

11. If $6^x = 5$, find 6^{x+1}. [1 mark]

12. Solve $\left(\frac{5}{3}\right)^x \left(\frac{3}{5}\right)^{3x} = \frac{81}{625}$. [2 marks]

13. Solve $5^x - 5^{x-2} = 600$. [5 marks]

14. If $81^{x-1} = 27^{x+1}$, find 7^{x-5}. [3 marks]

15. If $2^x = 9$ and $9^y = 64$, find xy. [2 marks]

16. If $2^x = 3^y = 24^z$, show that $z = \frac{xy}{x+3y}$. [5 marks]

17. Find the value of $(0.000125)^{\frac{2}{3}}$. [1 mark]

18. If $x = 2^{\frac{2}{3}} + 2^{-\frac{2}{3}}$, show that $x^3 - 3x = \frac{17}{4}$. [3 marks]

19. Find the value of $(81)^{0.6} \div (81)^{0.35}$. [2 marks]

20. Find the value of $\left\{16^{\frac{3}{4}} \div 2^{5}\right\}^{-\frac{1}{2}}$. [1 mark]

Key sheet

Q	Answer
1	1
2	$\frac{25}{9}$
3	25
4	4
5	7
6	15
7	Shown: LHS $= 1 =$ RHS
8	Shown: both sides $= x^{3abc}$
9	$x = \frac{3}{2}$
10	$x = 2$
11	30
12	$x = 2$
13	$x = 4$
14	49
15	$xy = 6$
16	Shown: $z = \frac{xy}{x+3y}$
17	0.0025
18	Shown: $x^3 - 3x = \frac{17}{4}$
19	3
20	2

Solutions

1. Find the value of $(-1)^{37} + (-1)^{52} + 7^0$. [1]

$(-1)^{37} = -1$ (37 is odd), $(-1)^{52} = 1$ (52 is even), $7^0 = 1$

$-1 + 1 + 1 = 1$

Ans: 1

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2. Simplify $\left(\frac{3}{5}\right)^4 \times \left(\frac{5}{3}\right)^6$. [2]

$\left(\frac{3}{5}\right)^4 = \left(\frac{5}{3}\right)^{-4}$

$\left(\frac{5}{3}\right)^{-4} \times \left(\frac{5}{3}\right)^6 = \left(\frac{5}{3}\right)^2$

$= \frac{25}{9}$

Ans: $\frac{25}{9}$

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3. Simplify $25^4 \times 125^{-2}$. [1]

$= (5^2)^4 \times (5^3)^{-2}$

$= 5^8 \times 5^{-6}$

$= 5^{8-6} = 5^2$

$= 25$

Ans: 25

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4. Find the value of $\frac{(25)^{\frac{3}{2}} \times (8)^{\frac{5}{3}}}{(10000)^{\frac{3}{4}}}$. [2]

$= \frac{(5^2)^{\frac{3}{2}} \times (2^3)^{\frac{5}{3}}}{(10^4)^{\frac{3}{4}}}$

$= \frac{5^3 \times 2^5}{10^3}$

$= \frac{125 \times 32}{1000}$

$= 4$

Ans: 4

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5. Find the value of $\left(\frac{1}{27}\right)^{-\frac{2}{3}} + \left(\frac{1}{2}\right)^{-4} - 6 \times 9^{\frac{1}{2}} \times 10^0$. [3]

$= \left[\left(\frac{1}{3}\right)^3\right]^{-\frac{2}{3}} + 2^4 - 6 \times (3^2)^{\frac{1}{2}} \times 1$

$= (3)^2 + 16 - 6 \times 3$

$= 9 + 16 - 18$

$= 7$

Ans: 7

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6. Simplify $\frac{15^{\frac{2}{5}}\times 5^{\frac{1}{5}}\times 25^{\frac{1}{5}}}{10^{\frac{1}{3}}\times 6^{\frac{1}{3}}} \div \frac{2^{\frac{1}{3}}\times 4^{-\frac{1}{2}}}{3^{\frac{3}{5}}\times 15^{\frac{1}{3}}}$. [5]

$= \frac{15^{\frac{2}{5}}\times 5^{\frac{1}{5}}\times 25^{\frac{1}{5}}}{10^{\frac{1}{3}}\times 6^{\frac{1}{3}}} \times \frac{3^{\frac{3}{5}}\times 15^{\frac{1}{3}}}{2^{\frac{1}{3}}\times 4^{-\frac{1}{2}}}$

$= \frac{(3\times 5)^{\frac{2}{5}}\times 5^{\frac{1}{5}}\times (5^2)^{\frac{1}{5}}}{(2\times 5)^{\frac{1}{3}}\times (2\times 3)^{\frac{1}{3}}} \times \frac{3^{\frac{2}{5}}\times (3\times 5)^{\frac{1}{3}}}{2^{\frac{1}{3}}\times (2^2)^{-\frac{1}{2}}}$

$= \frac{3^{\frac{2}{5}}\times 5^{\frac{2}{5}}\times 5^{\frac{1}{5}}\times 5^{\frac{2}{5}}}{2^{\frac{1}{3}}\times 5^{\frac{1}{3}}\times 2^{\frac{1}{3}}\times 3^{\frac{1}{3}}} \times \frac{3^{\frac{3}{5}}\times 3^{\frac{1}{3}}\times 5^{\frac{1}{3}}}{2^{\frac{1}{3}}\times 2^{-1}}$

$= 2^{-\frac{1}{3}-\frac{1}{3}-\frac{1}{3}+1} \times 3^{\frac{2}{5}-\frac{1}{3}+\frac{3}{5}+\frac{1}{3}} \times 5^{\frac{2}{5}+\frac{1}{5}+\frac{2}{5}-\frac{1}{3}+\frac{1}{3}}$

$= 2^0 \times 3^1 \times 5^1$

$= 15$

Ans: 15

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7. Show that $\left(\frac{x^a}{x^b}\right)^{a+b} \div \left(\frac{x^a}{x^c}\right)^{a+c} \times \left(\frac{x^b}{x^c}\right)^{b+c} = 1$. [3]

LHS $= x^{(a-b)(a+b)} \div x^{(a-c)(a+c)} \times x^{(b-c)(b+c)}$

$= x^{a^2-b^2} \div x^{a^2-c^2} \times x^{b^2-c^2}$

$= x^{a^2-b^2-(a^2-c^2)+b^2-c^2}$

$= x^{a^2-b^2-a^2+c^2+b^2-c^2} = x^0 = 1 =$ RHS

Ans: Shown: LHS $= 1 =$ RHS

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8. If $a+b+c=0$, show that $x^{a^3}\cdot x^{b^3}\cdot x^{c^3} = \left(x^{abc}\right)^3$. [5]

Given $a+b+c=0 \Rightarrow a^3+b^3+c^3 = 3abc$

LHS $= x^{a^3+b^3+c^3} = x^{3abc}$

RHS $= \left(x^{abc}\right)^3 = x^{3abc}$

So LHS = RHS

Ans: Shown: both sides $= x^{3abc}$

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9. Solve $9^x = 27$. [1]

$(3^2)^x = 27$

$\Rightarrow 3^{2x} = 3^3$

$\Rightarrow 2x = 3$

$\Rightarrow x = \frac{3}{2}$

Ans: $x = \frac{3}{2}$

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10. Solve $2^{3x-5} \times 11^{x-1} = 22$. [2]

$22 = 2^1 \times 11^1$

$\Rightarrow 2^{3x-5} \times 11^{x-1} = 2^1 \times 11^1$

$\Rightarrow 3x-5 = 1$ and $x-1 = 1$

$\Rightarrow x = 2$ and $x = 2$

$\therefore x = 2$

Ans: $x = 2$

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11. If $6^x = 5$, find 6^{x+1}. [1]

$6^{x+1} = 6^x \times 6^1$

$= 5 \times 6 = 30$

Ans: 30

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12. Solve $\left(\frac{5}{3}\right)^x \left(\frac{3}{5}\right)^{3x} = \frac{81}{625}$. [2]

$\left(\frac{5}{3}\right)^x = \left(\frac{3}{5}\right)^{-x}$

$\Rightarrow \left(\frac{3}{5}\right)^{-x} \cdot \left(\frac{3}{5}\right)^{3x} = \left(\frac{3}{5}\right)^4$

$\Rightarrow \left(\frac{3}{5}\right)^{2x} = \left(\frac{3}{5}\right)^4$

$\Rightarrow 2x = 4 \Rightarrow x = 2$

Ans: $x = 2$

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13. Solve $5^x - 5^{x-2} = 600$. [5]

$5^x - \frac{5^x}{25} = 600$

$\Rightarrow 5^x \left(1 - \frac{1}{25}\right) = 600$

$\Rightarrow 5^x \times \frac{24}{25} = 600$

$\Rightarrow 5^x = 600 \times \frac{25}{24} = 625 = 5^4$

$\Rightarrow x = 4$

Ans: $x = 4$

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14. If $81^{x-1} = 27^{x+1}$, find 7^{x-5}. [3]

$(3^4)^{x-1} = (3^3)^{x+1}$

$\Rightarrow 3^{4x-4} = 3^{3x+3}$

$\Rightarrow 4x - 4 = 3x + 3 \Rightarrow x = 7$

$7^{x-5} = 7^{7-5} = 7^2 = 49$

Ans: 49

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15. If $2^x = 9$ and $9^y = 64$, find xy. [2]

$64 = 9^y = (2^x)^y = 2^{xy}$

$2^{xy} = 64 = 2^6$

$\Rightarrow xy = 6$

Ans: $xy = 6$

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16. If $2^x = 3^y = 24^z$, show that $z = \frac{xy}{x+3y}$. [5]

Let $2^x = 3^y = 24^z = k$ (say)

$\Rightarrow 2 = k^{\frac{1}{x}}, 3 = k^{\frac{1}{y}}, 24 = k^{\frac{1}{z}}$

$24 = 2^3 \times 3$

$\Rightarrow k^{\frac{1}{z}} = \left(k^{\frac{1}{x}}\right)^3 \times k^{\frac{1}{y}}$

$\Rightarrow k^{\frac{1}{z}} = k^{\frac{3}{x}+\frac{1}{y}}$

$\Rightarrow \frac{1}{z} = \frac{3}{x} + \frac{1}{y}$

$\Rightarrow \frac{1}{z} = \frac{3y+x}{xy}$

$\Rightarrow z = \frac{xy}{x+3y}$

Ans: Shown: $z = \frac{xy}{x+3y}$

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17. Find the value of $(0.000125)^{\frac{2}{3}}$. [1]

$0.000125 = \frac{125}{1000000} = \left(\frac{5}{100}\right)^3 = (0.05)^3$

$(0.000125)^{\frac{2}{3}} = \left[(0.05)^3\right]^{\frac{2}{3}} = (0.05)^2 = 0.0025$

Ans: 0.0025

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18. If $x = 2^{\frac{2}{3}} + 2^{-\frac{2}{3}}$, show that $x^3 - 3x = \frac{17}{4}$. [3]

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$x^3 = \left(2^{\frac{2}{3}}\right)^3 + \left(2^{-\frac{2}{3}}\right)^3 + 3 \cdot 2^{\frac{2}{3}} \cdot 2^{-\frac{2}{3}} \left(2^{\frac{2}{3}} + 2^{-\frac{2}{3}}\right)$

$= 4 + \frac{1}{4} + 3 \times 1 \times x$

$x^3 = \frac{17}{4} + 3x$

$\Rightarrow x^3 - 3x = \frac{17}{4}$

Ans: Shown: $x^3 - 3x = \frac{17}{4}$

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19. Find the value of $(81)^{0.6} \div (81)^{0.35}$. [2]

$= (81)^{0.6-0.35} = (81)^{0.25}$

$0.25 = \frac{1}{4}$

$= (3^4)^{\frac{1}{4}} = 3$

Ans: 3

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20. Find the value of $\left\{16^{\frac{3}{4}} \div 2^5\right\}^{-\frac{1}{2}}$. [1]

$= \left\{\frac{(2^4)^{\frac{3}{4}}}{2^5}\right\}^{-\frac{1}{2}}$

$= \left\{\frac{2^3}{2^5}\right\}^{-\frac{1}{2}} = \left\{2^{-2}\right\}^{-\frac{1}{2}}$

$= 2^1 = 2$

Ans: 2