EXPRESSWALA ACADEMY

# Exponents (Level 2)

## Question Paper, Set 08

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

**1.** If $a^4 = 5^8$ and $a > 0$, find $a$. [1 mark]

**2.** Find the value of $2^{-3} \times 5^{-3} \times 10^4$. [2 marks]

**3.** Simplify $243^2 \times 9^{-4}$. [1 mark]

**4.** Find the value of $\frac{(32)^{\frac{4}{5}} \times (27)^{\frac{4}{3}}}{(36)^{\frac{3}{2}}}$. [2 marks]

**5.** Find the value of $\left(\frac{16}{25}\right)^{-\frac{3}{2}} - \frac{1}{\left(\frac{16}{25}\right)^{\frac{1}{2}}} + \frac{\sqrt{81}}{\sqrt[3]{27}}$. [3 marks]

**6.** Simplify $\frac{6^{-\frac{3}{5}} \times 8^{\frac{2}{5}} \times 4^{\frac{4}{5}}}{2^{\frac{4}{3}} \times 9^{\frac{1}{5}}} \div \frac{12^{-\frac{2}{5}} \times 5^{\frac{1}{3}}}{10^{\frac{1}{3}} \times 3^{\frac{3}{5}}}$. [5 marks]

**7.** Show that $\left(\frac{x^{a^2}}{x^{b^2}}\right)^{\frac{1}{a+b}} \cdot \left(\frac{x^{b^2}}{x^{c^2}}\right)^{\frac{1}{b+c}} \cdot \left(\frac{x^{c^2}}{x^{a^2}}\right)^{\frac{1}{c+a}} = 1$. [3 marks]

**8.** If $a + b + c = 0$, show that $x^{\frac{a^2+bc}{bc}} \cdot x^{\frac{b^2+ca}{ca}} \cdot x^{\frac{c^2+ab}{ab}} = x^6$. [5 marks]

**9.** Solve $4^x = \frac{1}{64}$. [1 mark]

**10.** Solve $3^x \times 5^{2x-6} = 2025$. [2 marks]

**11.** If $3^{2y} = 49$, find $3^{-y}$. [1 mark]

**12.** Solve $\left(\sqrt[3]{\frac{3}{4}}\right)^{x-2} = \frac{16}{9}$. [2 marks]

**13.** Solve $2^{2x+1} - 4^{x-1} = 112$. [5 marks]

**14.** If $\sqrt[3]{2^n} = 64$, find $3^{\frac{n}{6}-1}$. [3 marks]

**15.** If $5^a = 7$ and $7^b = 125$, find $ab$. [2 marks]

**16.** If $a^x = b^y = c^z$ and $a = b^2c$, show that $\frac{1}{x} = \frac{2}{y} + \frac{1}{z}$. [5 marks]

**17.** Find the value of $\sqrt[3]{64^{-4}}$. [1 mark]

**18.** If $x = 9$, find the value of $(x^{\frac{1}{2}} + x^{-\frac{1}{2}})(x + x^{-1} - 1)$. [3 marks]

**19.** Find the value of $(49)^{0.3} \times (49)^{0.2}$. [2 marks]

**20.** Find the value of $\left\{\left(4^{-1}\right)^{3}\right\}^{-\frac{1}{2}}$. [1 mark]

## Key sheet

| Q | Answer |
|---|---|
| 1 | $a = 25$ |
| 2 | $10$ |
| 3 | $9$ |
| 4 | $6$ |
| 5 | $\frac{237}{64}$ |
| 6 | $4$ |
| 7 | Shown: LHS $= 1 =$ RHS |
| 8 | Shown: LHS $= x^6 =$ RHS |
| 9 | $x = -3$ |
| 10 | $x = 4$ |
| 11 | $\frac{1}{7}$ |
| 12 | $x = -4$ |
| 13 | $x = 3$ |
| 14 | $9$ |
| 15 | $ab = 3$ |
| 16 | Shown: $\frac{1}{x} = \frac{2}{y} + \frac{1}{z}$ |
| 17 | $\frac{1}{256}$ |
| 18 | $\frac{730}{27}$ |
| 19 | $7$ |
| 20 | $8$ |

## Solutions

**1.** If $a^4 = 5^8$ and $a > 0$, find $a$. [1]

$a^m = b^n \Rightarrow a = b^{\frac{n}{m}}$

$a = 5^{\frac{8}{4}} = 5^2 = 25$

**Ans:** $a = 25$

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**2.** Find the value of $2^{-3} \times 5^{-3} \times 10^4$. [2]

$2^{-3} \times 5^{-3} = (2 \times 5)^{-3} = 10^{-3}$

$10^{-3} \times 10^4 = 10^{-3+4} = 10^1$

$= 10$

**Ans:** $10$

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**3.** Simplify $243^2 \times 9^{-4}$. [1]

$= (3^5)^2 \times (3^2)^{-4}$

$= 3^{10} \times 3^{-8}$

$= 3^{10-8} = 3^2$

$= 9$

**Ans:** $9$

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**4.** Find the value of $\frac{(32)^{\frac{4}{5}} \times (27)^{\frac{4}{3}}}{(36)^{\frac{3}{2}}}$. [2]

$= \frac{(2^5)^{\frac{4}{5}} \times (3^3)^{\frac{4}{3}}}{(6^2)^{\frac{3}{2}}}$

$= \frac{2^4 \times 3^4}{6^3}$

$= \frac{16 \times 81}{216}$

$= 6$

**Ans:** $6$

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**5.** Find the value of $\left(\frac{16}{25}\right)^{-\frac{3}{2}} - \frac{1}{\left(\frac{16}{25}\right)^{\frac{1}{2}}} + \frac{\sqrt{81}}{\sqrt[3]{27}}$. [3]

$= \left[\left(\frac{4}{5}\right)^2\right]^{-\frac{3}{2}} - \frac{1}{\left[\left(\frac{4}{5}\right)^2\right]^{\frac{1}{2}}} + \frac{\sqrt{9^2}}{\sqrt[3]{3^3}}$

$= \left(\frac{5}{4}\right)^3 - \frac{1}{\frac{4}{5}} + \frac{9}{3}$

$= \frac{125}{64} - \frac{5}{4} + 3$

$= \frac{125 - 80 + 192}{64}$

$= \frac{237}{64}$

**Ans:** $\frac{237}{64}$

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**6.** Simplify $\frac{6^{-\frac{3}{5}}\times 8^{\frac{2}{5}}\times 4^{\frac{4}{5}}}{2^{\frac{4}{3}}\times 9^{\frac{1}{5}}} \div \frac{12^{-\frac{2}{5}}\times 5^{\frac{1}{3}}}{10^{\frac{1}{3}}\times 3^{\frac{3}{5}}}$. [5]

$= \frac{6^{-\frac{3}{5}}\times 8^{\frac{2}{5}}\times 4^{\frac{4}{5}}}{2^{\frac{4}{3}}\times 9^{\frac{1}{5}}} \times \frac{10^{\frac{1}{3}}\times 3^{\frac{3}{5}}}{12^{-\frac{2}{5}}\times 5^{\frac{1}{3}}}$

$= \frac{(2\times 3)^{-\frac{3}{5}}\times (2^3)^{\frac{2}{5}}\times (2^2)^{\frac{4}{5}}}{2^{\frac{4}{3}}\times (3^2)^{\frac{1}{5}}} \times \frac{(2\times 5)^{\frac{1}{3}}\times 3^{\frac{3}{5}}}{(2^2\times 3)^{-\frac{2}{5}}\times 5^{\frac{1}{3}}}$

$= \frac{2^{-\frac{3}{5}}\times 3^{-\frac{3}{5}}\times 2^{\frac{6}{5}}\times 2^{\frac{8}{5}}}{2^{\frac{4}{3}}\times 3^{\frac{2}{5}}} \times \frac{2^{\frac{1}{3}}\times 5^{\frac{1}{3}}\times 3^{\frac{3}{5}}}{2^{-\frac{4}{5}}\times 3^{-\frac{2}{5}}\times 5^{\frac{1}{3}}}$

$= 2^{-\frac{3}{5}+\frac{6}{5}+\frac{8}{5}-\frac{4}{3}+\frac{4}{5}+\frac{1}{3}} \times 3^{-\frac{3}{5}-\frac{2}{5}+\frac{2}{5}+\frac{3}{5}} \times 5^{-\frac{1}{3}+\frac{1}{3}}$

$= 2^2 \times 3^0 \times 5^0$

$= 4$

**Ans:** $4$

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**7.** Show that $\left(\frac{x^{a^2}}{x^{b^2}}\right)^{\frac{1}{a+b}} \cdot \left(\frac{x^{b^2}}{x^{c^2}}\right)^{\frac{1}{b+c}} \cdot \left(\frac{x^{c^2}}{x^{a^2}}\right)^{\frac{1}{c+a}} = 1$. [3]

LHS $= x^{\frac{a^2-b^2}{a+b}} \cdot x^{\frac{b^2-c^2}{b+c}} \cdot x^{\frac{c^2-a^2}{c+a}}$

$\frac{a^2-b^2}{a+b} = \frac{(a+b)(a-b)}{a+b} = a-b$, and so on

$= x^{a-b} \cdot x^{b-c} \cdot x^{c-a}$

$= x^{a-b+b-c+c-a} = x^0 = 1 =$ RHS

**Ans: Shown: LHS $= 1 =$ RHS**

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**8.** If $a+b+c=0$, show that $x^{\frac{a^2+bc}{bc}} \cdot x^{\frac{b^2+ca}{ca}} \cdot x^{\frac{c^2+ab}{ab}} = x^6$. [5]

Given $a+b+c=0 \Rightarrow a^3+b^3+c^3 = 3abc$

$\frac{a^2+bc}{bc} = \frac{a^2}{bc} + 1$, and the same for the other two powers

LHS $= x^{\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}+3}$

$= x^{\frac{a^3+b^3+c^3}{abc}+3} = x^{3+3} = x^6 =$ RHS

**Ans: Shown: LHS $= x^6 =$ RHS**

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**9.** Solve $4^x = \frac{1}{64}$. [1]

$4^x = \frac{1}{64} = 4^{-3}$

$\Rightarrow x = -3$

$\Rightarrow x = -3$

**Ans:** $x = -3$

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**10.** Solve $3^x \times 5^{2x-6} = 2025$. [2]

$2025 = 3^4 \times 5^2$

$\Rightarrow 3^x \times 5^{2x-6} = 3^4 \times 5^2$

$\Rightarrow x = 4$ and $2x-6 = 2$

$\Rightarrow x = 4$ and $x = 4$

$\therefore x = 4$

**Ans:** $x = 4$

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**11.** If $3^{2y} = 49$, find $3^{-y}$. [1]

$(3^y)^2 = 7^2$

$\Rightarrow 3^y = 7$

$3^{-y} = \frac{1}{3^y} = \frac{1}{7}$

**Ans:** $\frac{1}{7}$

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**12.** Solve $\left(\sqrt[3]{\frac{3}{4}}\right)^{x-2} = \frac{16}{9}$. [2]

$\left[\left(\frac{3}{4}\right)^{\frac{1}{3}}\right]^{x-2} = \left(\frac{4}{3}\right)^2$

$\Rightarrow \left(\frac{3}{4}\right)^{\frac{x-2}{3}} = \left(\frac{3}{4}\right)^{-2}$

$\Rightarrow \frac{x-2}{3} = -2$

$\Rightarrow x - 2 = -6$

$\Rightarrow x = -4$

**Ans:** $x = -4$

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**13.** Solve $2^{2x+1} - 4^{x-1} = 112$. [5]

$2^{2x+1} - (2^2)^{x-1} = 112$

$\Rightarrow 2^{2x+1} - 2^{2x-2} = 112$

$\Rightarrow 2 \times 2^{2x} - \frac{2^{2x}}{4} = 112$

$\Rightarrow 2^{2x}\left(2 - \frac{1}{4}\right) = 112$

$\Rightarrow 2^{2x} \times \frac{7}{4} = 112$

$\Rightarrow 2^{2x} = 112 \times \frac{4}{7} = 64 = 2^6$

$\Rightarrow 2x = 6 \Rightarrow x = 3$

**Ans:** $x = 3$

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**14.** If $\sqrt[3]{2^n} = 64$, find $3^{\frac{n}{6}-1}$. [3]

$2^{\frac{n}{3}} = 64 = 2^6$

$\Rightarrow \frac{n}{3} = 6 \Rightarrow n = 18$

$3^{\frac{n}{6}-1} = 3^{3-1} = 3^2 = 9$

**Ans:** $9$

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**15.** If $5^a = 7$ and $7^b = 125$, find $ab$. [2]

$125 = 7^b = (5^a)^b = 5^{ab}$

$5^{ab} = 125 = 5^3$

$\Rightarrow ab = 3$

**Ans:** $ab = 3$

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**16.** If $a^x = b^y = c^z$ and $a = b^2c$, show that $\frac{1}{x} = \frac{2}{y} + \frac{1}{z}$. [5]

Let $a^x = b^y = c^z = k$ (say)

$\Rightarrow a = k^{\frac{1}{x}}, b = k^{\frac{1}{y}}, c = k^{\frac{1}{z}}$

Given $a = b^2c$

$\Rightarrow k^{\frac{1}{x}} = \left(k^{\frac{1}{y}}\right)^2 \cdot k^{\frac{1}{z}} = k^{\frac{2}{y}+\frac{1}{z}}$

$\Rightarrow \frac{1}{x} = \frac{2}{y} + \frac{1}{z}$

**Ans: Shown:** $\frac{1}{x} = \frac{2}{y} + \frac{1}{z}$

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**17.** Find the value of $\sqrt[3]{64^{-4}}$. [1]

$\sqrt[n]{a^m} = a^{\frac{m}{n}}$

$\sqrt[3]{64^{-4}} = 64^{-\frac{4}{3}} = (2^6)^{-\frac{4}{3}}$

$= 2^{-8} = \frac{1}{2^8} = \frac{1}{256}$

**Ans:** $\frac{1}{256}$

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**18.** If $x = 9$, find the value of $(x^{\frac{1}{2}} + x^{-\frac{1}{2}})(x + x^{-1} - 1)$. [3]

$(a+b)(a^2+b^2-ab) = a^3 + b^3$ with $a = x^{\frac{1}{2}}, b = x^{-\frac{1}{2}}$

$ab = x^{\frac{1}{2}} \cdot x^{-\frac{1}{2}} = x^0 = 1$

So the value $= (x^{\frac{1}{2}})^3 + (x^{-\frac{1}{2}})^3 = x^{\frac{3}{2}} + x^{-\frac{3}{2}}$

$= 9^{\frac{3}{2}} + 9^{-\frac{3}{2}} = \frac{730}{27}$

**Ans:** $\frac{730}{27}$

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**19.** Find the value of $(49)^{0.3} \times (49)^{0.2}$. [2]

$= (49)^{0.3+0.2} = (49)^{0.5}$

$0.5 = \frac{1}{2}$

$= (7^2)^{\frac{1}{2}} = 7$

**Ans:** $7$

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**20.** Find the value of $\left\{\left(4^{-1}\right)^3\right\}^{-\frac{1}{2}}$. [1]

$= 4^{(-1)\times 3\times(-\frac{1}{2})}$

$= 4^{\frac{3}{2}}$

$= 8$

**Ans:** $8$

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