EXPRESSWALA ACADEMY

Exponents (Level 2)

Question Paper, Set 07

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Find the value of $\sqrt[4]{2^{12}}$. [1 mark]

2. Find the value of $\sqrt[5]{3^{15}} - \left(-\frac{1}{2}\right)^{-3}$. [2 marks]

3. Simplify $64^{2} \times 16^{-2}$. [1 mark]

4. Find the value of $\frac{(243)^{\frac{2}{5}} \times (8)^{\frac{4}{3}}}{(216)^{\frac{1}{3}}}$. [2 marks]

5. Find the value of $\left(\frac{1}{7}\right)^{-2} - 5 \times 32^{\frac{3}{5}} \times 3^{0} + \left(\frac{9}{25}\right)^{-\frac{1}{2}}$. [3 marks]

6. Simplify $\frac{12^{\frac{4}{5}} \times 25^{\frac{1}{5}} \times 2^{\frac{7}{5}}}{25 \times 27^{-\frac{1}{4}}} \div \frac{3^{\frac{7}{4}} \times 15^{-\frac{6}{5}}}{8^{-\frac{1}{3}} \times 5^{\frac{2}{5}}}$. [5 marks]

7. Show that $\left(\frac{x^{p}}{x^{q}}\right)^{p+q} \cdot \left(\frac{x^{q}}{x^{r}}\right)^{q+r} \cdot \left(\frac{x^{r}}{x^{t}}\right)^{r+t} \cdot \left(\frac{x^{t}}{x^{p}}\right)^{t+p} = 1$. [3 marks]

8. If $a + b + c = 0$, show that $\left(x^{\frac{a^2}{bc}} \cdot x^{\frac{b^2}{ca}} \cdot x^{\frac{c^2}{ab}}\right)^{\frac{1}{3}} = x$. [5 marks]

9. Solve $11^{3x} = 121$. [1 mark]

10. Solve $2^{x+2} \times 7^{x-3} = 448$. [2 marks]

11. If $4^{x} = 320$, find 4^{x-3}. [1 mark]

12. Solve $25^{3x} = \frac{1}{125}$. [2 marks]

13. Solve $25^{x} - 5^{2x-1} = 2500$. [5 marks]

14. Solve $16^{x} = \frac{32}{4^{x}}$. [3 marks]

15. If $a = b^{x}$, $b = c^{y}$ and $c = a^{z}$, find xyz. [2 marks]

16. If $a^{x} = b^{y} = c^{z}$ and $abc = 1$, show that $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$. [5 marks]

17. Find the value of $\sqrt[4]{0.0625}$. [1 mark]

18. If $x = 3^{\frac{1}{3}} + 3^{-\frac{1}{3}}$, show that $x^{3} - 3x = \frac{10}{3}$. [3 marks]

19. If $\frac{x^{2.5}}{x} = 125$ and $x > 0$, find x. [2 marks]

20. If $\left[\left\{\left(\frac{1}{3^2}\right)^3\right\}^{-\frac{1}{2}}\right]^{\frac{1}{3}} = 3^m$, find m. [1 mark]

Key sheet

Q	Answer
1	8
2	35
3	16
4	24
5	$\frac{32}{3}$
6	12
7	Shown: LHS $=1=$ RHS
8	Shown: LHS $=x=$ RHS
9	$x=\frac{2}{3}$
10	$x=4$
11	5
12	$x=-\frac{1}{2}$
13	$x=\frac{5}{2}$
14	$x=\frac{5}{6}$
15	$xyz=1$
16	Shown: $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=0$
17	0.5
18	Shown: $x^3-3x=\frac{10}{3}$
19	$x=25$
20	$m=1$

Solutions

1. Find the value of $\sqrt[4]{2^{12}}$. [1]

$\sqrt[n]{a^m} = a^{\frac{m}{n}}$

$\sqrt[4]{2^{12}} = 2^{\frac{12}{4}} = 2^3 = 8$

Ans: 8

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2. Find the value of $\sqrt[5]{3^{15}} - \left(-\frac{1}{2}\right)^{-3}$. [2]

$\sqrt[5]{3^{15}} = 3^{\frac{15}{5}} = 3^3 = 27$

$\left(-\frac{1}{2}\right)^{-3} = (-2)^3 = -8$

$27 - (-8) = 27 + 8 = 35$

Ans: 35

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3. Simplify $64^2 \times 16^{-2}$. [1]

$= (2^6)^2 \times (2^4)^{-2}$

$= 2^{12} \times 2^{-8}$

$= 2^{12-8} = 2^4$

$= 16$

Ans: 16

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4. Find the value of $\frac{(243)^{\frac{2}{5}} \times (8)^{\frac{4}{3}}}{(216)^{\frac{1}{3}}}$. [2]

$= \frac{(3^5)^{\frac{2}{5}} \times (2^3)^{\frac{4}{3}}}{(6^3)^{\frac{1}{3}}}$

$= \frac{3^2 \times 2^4}{6}$

$= \frac{9 \times 16}{6}$

$= 24$

Ans: 24

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5. Find the value of $\left(\frac{1}{7}\right)^{-2} - 5 \times 32^{\frac{3}{5}} \times 3^0 + \left(\frac{9}{25}\right)^{-\frac{1}{2}}$. [3]

$= 7^2 - 5 \times (2^5)^{\frac{3}{5}} \times 1 + \left[\left(\frac{3}{5}\right)^2\right]^{-\frac{1}{2}}$

$= 49 - 5 \times 2^3 + \frac{5}{3}$

$= 49 - 40 + \frac{5}{3}$

$= \frac{147 - 120 + 5}{3}$

$= \frac{32}{3}$

Ans: $\frac{32}{3}$

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6. Simplify $\frac{12^{\frac{4}{5}}\times 25^{\frac{1}{5}}\times 2^{\frac{7}{5}}}{25\times 27^{-\frac{1}{4}}} \div \frac{3^{\frac{7}{4}}\times 15^{-\frac{6}{5}}}{8^{-\frac{1}{3}}\times 5^{\frac{2}{5}}}$. [5]

$= \frac{12^{\frac{4}{5}}\times 25^{\frac{1}{5}}\times 2^{\frac{7}{5}}}{25\times 27^{-\frac{1}{4}}} \times \frac{8^{-\frac{1}{3}}\times 5^{\frac{2}{5}}}{3^{\frac{7}{4}}\times 15^{-\frac{6}{5}}}$

$= \frac{(2^2\times 3)^{\frac{4}{5}}\times (5^2)^{\frac{1}{5}}\times 2^{\frac{7}{5}}}{(5^2)\times (3^3)^{-\frac{1}{4}}} \times \frac{(2^3)^{-\frac{1}{3}}\times 5^{\frac{2}{5}}}{3^{\frac{7}{4}}\times (3\times 5)^{-\frac{6}{5}}}$

$= \frac{2^{\frac{8}{5}}\times 3^{\frac{4}{5}}\times 5^{\frac{2}{5}}\times 2^{\frac{7}{5}}}{5^2\times 3^{-\frac{3}{4}}} \times \frac{2^{-1}\times 5^{\frac{2}{5}}}{3^{\frac{7}{4}}\times 3^{-\frac{6}{5}}\times 5^{-\frac{6}{5}}}$

$= 2^{\frac{8}{5}+\frac{7}{5}-1} \times 3^{\frac{4}{5}+\frac{3}{4}-\frac{7}{4}+\frac{6}{5}} \times 5^{\frac{2}{5}-2+\frac{6}{5}+\frac{2}{5}}$

$= 2^2 \times 3^1 \times 5^0$

$= 12$

Ans: 12

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7. Show that $\left(\frac{x^p}{x^q}\right)^{p+q} \cdot \left(\frac{x^q}{x^r}\right)^{q+r} \cdot \left(\frac{x^r}{x^t}\right)^{r+t} \cdot \left(\frac{x^t}{x^p}\right)^{t+p} = 1$. [3]

LHS $= (x^{p-q})^{p+q} \cdot (x^{q-r})^{q+r} \cdot (x^{r-t})^{r+t} \cdot (x^{t-p})^{t+p}$

$= x^{p^2-q^2} \cdot x^{q^2-r^2} \cdot x^{r^2-t^2} \cdot x^{t^2-p^2}$

$= x^{p^2-q^2+q^2-r^2+r^2-t^2+t^2-p^2}$

$= x^0 = 1 =$ RHS

Ans: Shown: LHS $= 1 =$ RHS

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8. If $a+b+c=0$, show that $\left(x^{\frac{a^2}{bc}} \cdot x^{\frac{b^2}{ca}} \cdot x^{\frac{c^2}{ab}}\right)^{\frac{1}{3}} = x$. [5]

Given $a+b+c=0 \Rightarrow a^3+b^3+c^3 = 3abc$

LHS $= \left(x^{\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}}\right)^{\frac{1}{3}}$

$= \left(x^{\frac{a^3+b^3+c^3}{abc}}\right)^{\frac{1}{3}} = \left(x^{\frac{3abc}{abc}}\right)^{\frac{1}{3}}$

$= \left(x^3\right)^{\frac{1}{3}} = x =$ RHS

Ans: Shown: LHS $= x =$ RHS

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9. Solve $11^{3x} = 121$. [1]

$11^{3x} = 11^2$

$\Rightarrow 3x = 2$

$\Rightarrow x = \frac{2}{3}$

Ans: $x = \frac{2}{3}$

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10. Solve $2^{x+2} \times 7^{x-3} = 448$. [2]

$448 = 2^6 \times 7^1$

$\Rightarrow 2^{x+2} \times 7^{x-3} = 2^6 \times 7^1$

$\Rightarrow x+2 = 6$ and $x-3 = 1$

$\Rightarrow x = 4$ and $x = 4$

$\therefore x = 4$

Ans: $x = 4$

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11. If $4^x = 320$, find 4^{x-3}. [1]

$4^{x-3} = \frac{4^x}{4^3}$

$= \frac{320}{64} = 5$

Ans: 5

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12. Solve $25^{3x} = \frac{1}{125}$. [2]

$(5^2)^{3x} = \frac{1}{5^3}$

$\Rightarrow 5^{6x} = 5^{-3}$

$\Rightarrow 6x = -3$

$\Rightarrow x = -\frac{1}{2}$

Ans: $x = -\frac{1}{2}$

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13. Solve $25^x - 5^{2x-1} = 2500$. [5]

$(5^2)^x - 5^{2x-1} = 2500$

$\Rightarrow 5^{2x} - 5^{2x-1} = 2500$

$\Rightarrow 5^{2x} - \frac{5^{2x}}{5} = 2500$

$\Rightarrow 5^{2x}\left(1 - \frac{1}{5}\right) = 2500$

$\Rightarrow 5^{2x} \times \frac{4}{5} = 2500$

$\Rightarrow 5^{2x} = 2500 \times \frac{5}{4} = 3125 = 5^5$

$\Rightarrow 2x = 5 \Rightarrow x = \frac{5}{2}$

Ans: $x = \frac{5}{2}$

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14. Solve $16^x = \frac{32}{4^x}$. [3]

$(2^4)^x \cdot (2^2)^x = 32$

$\Rightarrow 2^{4x} \cdot 2^{2x} = 2^5$

$\Rightarrow 2^{6x} = 2^5$

$\Rightarrow 6x = 5 \Rightarrow x = \frac{5}{6}$

Ans: $x = \frac{5}{6}$

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15. If $a = b^x$, $b = c^y$ and $c = a^z$, find xyz. [2]

$a = b^x = (c^y)^x = c^{xy}$

$= (a^z)^{xy} = a^{xyz}$

$a^1 = a^{xyz} \Rightarrow xyz = 1$

Ans: $xyz = 1$

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16. If $a^x = b^y = c^z$ and $abc = 1$, show that $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$. [5]

Let $a^x = b^y = c^z = k$ (say)

$\Rightarrow a = k^{\frac{1}{x}}, b = k^{\frac{1}{y}}, c = k^{\frac{1}{z}}$

Given $abc = 1$

$\Rightarrow k^{\frac{1}{x}} \cdot k^{\frac{1}{y}} \cdot k^{\frac{1}{z}} = 1 = k^0$

$\Rightarrow k^{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}} = k^0$

$\Rightarrow \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$

Ans: Shown: $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$

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17. Find the value of $\sqrt[4]{0.0625}$. [1]

$0.0625 = \frac{625}{10000} = \left(\frac{5}{10}\right)^4 = (0.5)^4$

$\sqrt[4]{0.0625} = (0.0625)^{\frac{1}{4}} = \left[(0.5)^4\right]^{\frac{1}{4}} = 0.5$

Ans: 0.5

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18. If $x = 3^{\frac{1}{3}} + 3^{-\frac{1}{3}}$, show that $x^3 - 3x = \frac{10}{3}$. [3]

$(a+b)^3 = a^3 + b^3 + 3ab(a+b)$

$x^3 = \left(3^{\frac{1}{3}}\right)^3 + \left(3^{-\frac{1}{3}}\right)^3 + 3 \cdot 3^{\frac{1}{3}} \cdot 3^{-\frac{1}{3}} \left(3^{\frac{1}{3}} + 3^{-\frac{1}{3}}\right)$

$= 3 + \frac{1}{3} + 3 \times 1 \times x$

$x^3 = \frac{10}{3} + 3x$

$\Rightarrow x^3 - 3x = \frac{10}{3}$

Ans: Shown: $x^3 - 3x = \frac{10}{3}$

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19. If $\frac{x^{2.5}}{x} = 125$ and $x > 0$, find x. [2]

$2.5 = \frac{5}{2}$

$x^{\frac{5}{2}-1} = 125 \Rightarrow x^{\frac{3}{2}} = 125$

$x = 125^{\frac{2}{3}} = (5^3)^{\frac{2}{3}} = 5^2 = 25$

Ans: $x = 25$

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20. If $\left[\left\{\left(\frac{1}{3^2}\right)^3\right\}^{-\frac{1}{2}}\right]^{\frac{1}{3}} = 3^m$, find m. [1]

$\frac{1}{3^2} = 3^{-2}$

$= 3^{(-2)\times 3\times(-\frac{1}{2})\times\frac{1}{3}}$

$= 3^1$

$\Rightarrow m = 1$

Ans: $m = 1$