

Exponents (Level 2)**Question Paper, Set 06**

Time: 1 hour 30 minutes

Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Write $6^{\frac{2}{7}}$ in root form. [1 mark]
2. Find the value of $\left(\frac{3}{2}\right)^{-2} + \sqrt[3]{8^2}$. [2 marks]
3. Simplify $81^2 \times 27^{-2}$. [1 mark]
4. Find the value of $\frac{(625)^{\frac{3}{4}} \times (4)^{\frac{5}{2}}}{(1000)^{\frac{2}{3}}}$. [2 marks]
5. Find the value of $\left(\frac{16}{81}\right)^{-\frac{3}{4}} + \frac{1}{\left(\frac{27}{64}\right)^{\frac{1}{3}}} - \frac{\sqrt{64}}{\sqrt[3]{64}}$. [3 marks]
6. Simplify $\frac{27^{\frac{1}{3}} \times 5^{\frac{2}{5}} \times 6^{\frac{6}{5}}}{25^{\frac{1}{5}} \times 9^{-\frac{1}{5}}} \div \frac{2^{\frac{6}{5}} \times 3^{\frac{7}{5}}}{4^{\frac{1}{2}} \times 25^{\frac{1}{5}}}$. [5 marks]
7. Show that $\left(\frac{x^{a+b}}{x^{2c}}\right)^{a-b} \cdot \left(\frac{x^{b+c}}{x^{2a}}\right)^{b-c} \cdot \left(\frac{x^{c+a}}{x^{2b}}\right)^{c-a} = 1$. [3 marks]
8. If $a + b + c = 0$, show that $9^{\frac{a^2}{bc}} \cdot 9^{\frac{b^2}{ca}} \cdot 9^{\frac{c^2}{ab}} = 729$. [5 marks]
9. Solve $6^{x+1} = 216$. [1 mark]
10. Solve $5^{2x-5} \times 3^{x-1} = 45$. [2 marks]
11. If $2^x = 7$, find 8^x . [1 mark]
12. Solve $\left(\frac{3}{4}\right)^{2x} \left(\frac{4}{3}\right)^x = \frac{27}{64}$. [2 marks]
13. Solve $36^{x-1} = 6^{2x-1} - 180$. [5 marks]
14. If $9^{x-1} = 27$, find 4^x . [3 marks]
15. If $3^x = 4$, $4^y = 5$ and $5^z = \frac{1}{9}$, find xyz . [2 marks]
16. If $9^x = 4^y = 6^z$, show that $z = \frac{2xy}{x+y}$. [5 marks]
17. Find the value of $\sqrt[5]{243^{-3}}$. [1 mark]
18. If $x = 3^{\frac{2}{3}} + 3^{\frac{4}{3}}$, show that $x^3 - 27x = 90$. [3 marks]
19. Find the value of $(1024)^{0.07} \times (1024)^{0.13}$. [2 marks]

20. Find the value of $\left\{ (7^2)^{-3} \right\}^{-\frac{1}{6}}$.

[1 mark]

Key sheet

Q	Answer
1	$\sqrt[7]{6^2} = \sqrt[7]{36}$
2	$\frac{40}{9}$
3	9
4	40
5	$\frac{65}{24}$
6	18
7	Shown: $\text{LHS} = 1 = \text{RHS}$
8	Shown: $\text{LHS} = 9^3 = 729 = \text{RHS}$
9	$x = 2$
10	$x = 3$
11	343
12	$x = 3$
13	$x = 2$
14	32
15	$xyz = -2$
16	Shown: $z = \frac{2xy}{x+y}$
17	$\frac{1}{27}$
18	Shown: $x^3 - 27x = 90$
19	4
20	7

Solutions

1. Write $6^{\frac{2}{7}}$ in root form. [1]

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

$$6^{\frac{2}{7}} = \sqrt[7]{6^2} = \sqrt[7]{36}$$

Ans: $\sqrt[7]{6^2} = \sqrt[7]{36}$

2. Find the value of $\left(\frac{3}{2}\right)^{-2} + \sqrt[3]{8^2}$. [2]

$$\left(\frac{3}{2}\right)^{-2} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

$$\sqrt[3]{8^2} = 8^{\frac{2}{3}} = (2^3)^{\frac{2}{3}} = 2^2 = 4$$

$$\frac{4}{9} + 4 = \frac{40}{9}$$

Ans: $\frac{40}{9}$

3. Simplify $81^2 \times 27^{-2}$. [1]

$$= (3^4)^2 \times (3^3)^{-2}$$

$$= 3^8 \times 3^{-6}$$

$$= 3^{8-6} = 3^2$$

$$= 9$$

Ans: 9

4. Find the value of $\frac{(625)^{\frac{3}{4}} \times (4)^{\frac{5}{2}}}{(1000)^{\frac{2}{3}}}$. [2]

$$= \frac{(5^4)^{\frac{3}{4}} \times (2^2)^{\frac{5}{2}}}{(10^3)^{\frac{2}{3}}}$$

$$= \frac{5^3 \times 2^5}{10^2}$$

$$= \frac{125 \times 32}{100}$$

$$= 40$$

Ans: 40

5. Find the value of $\left(\frac{16}{81}\right)^{-\frac{3}{4}} + \frac{1}{\left(\frac{27}{64}\right)^{\frac{1}{3}}} - \frac{\sqrt{64}}{\sqrt[3]{4^3}}$. [3]

$$= \left[\left(\frac{2}{3}\right)^4\right]^{-\frac{3}{4}} + \frac{1}{\left[\left(\frac{3}{4}\right)^3\right]^{\frac{1}{3}}} - \frac{\sqrt{8^2}}{\sqrt[3]{4^3}}$$

$$= \left(\frac{3}{2}\right)^3 + \frac{1}{\frac{3}{4}} - \frac{8}{4}$$

$$= \frac{27}{8} + \frac{4}{3} - 2$$

$$= \frac{81+32-48}{24}$$

$$= \frac{65}{24}$$

Ans: $\frac{65}{24}$

6. Simplify $\frac{27^{\frac{1}{3}} \times 5^{\frac{2}{5}} \times 6^{\frac{6}{5}}}{25^{\frac{4}{5}} \times 9^{-\frac{3}{5}}} \div \frac{2^{\frac{6}{5}} \times 3^{\frac{7}{5}}}{4^{\frac{1}{2}} \times 25^{\frac{3}{5}}}$. [5]

$$\begin{aligned}
 &= \frac{27^{\frac{1}{3}} \times 5^{\frac{2}{5}} \times 6^{\frac{6}{5}}}{25^{\frac{4}{5}} \times 9^{-\frac{3}{5}}} \times \frac{4^{\frac{1}{2}} \times 25^{\frac{3}{5}}}{2^{\frac{6}{5}} \times 3^{\frac{7}{5}}} \\
 &= \frac{(3^3)^{\frac{1}{3}} \times 5^{\frac{2}{5}} \times (2 \times 3)^{\frac{6}{5}}}{(5^2)^{\frac{4}{5}} \times (3^2)^{-\frac{3}{5}}} \times \frac{(2^2)^{\frac{1}{2}} \times (5^2)^{\frac{3}{5}}}{2^{\frac{6}{5}} \times 3^{\frac{7}{5}}} \\
 &= \frac{3 \times 5^{\frac{2}{5}} \times 2^{\frac{6}{5}} \times 3^{\frac{6}{5}}}{5^{\frac{8}{5}} \times 3^{-\frac{6}{5}}} \times \frac{2 \times 5^{\frac{6}{5}}}{2^{\frac{6}{5}} \times 3^{\frac{7}{5}}} \\
 &= 2^{\frac{6}{5} - \frac{6}{5} + 1} \times 3^{1 + \frac{6}{5} + \frac{6}{5} - \frac{7}{5}} \times 5^{\frac{2}{5} - \frac{8}{5} + \frac{6}{5}} \\
 &= 2^1 \times 3^2 \times 5^0 \\
 &= 18
 \end{aligned}$$

Ans: 18

7. Show that $\left(\frac{x^{a+b}}{x^{2c}}\right)^{a-b} \cdot \left(\frac{x^{b+c}}{x^{2a}}\right)^{b-c} \cdot \left(\frac{x^{c+a}}{x^{2b}}\right)^{c-a} = 1$. [3]

$$\begin{aligned}
 \text{LHS} &= \frac{x^{(a+b)(a-b)}}{x^{2c(a-b)}} \cdot \frac{x^{(b+c)(b-c)}}{x^{2a(b-c)}} \cdot \frac{x^{(c+a)(c-a)}}{x^{2b(c-a)}} \\
 &= \frac{x^{a^2-b^2} \cdot x^{b^2-c^2} \cdot x^{c^2-a^2}}{x^{2ca-2cb} \cdot x^{2ab-2ac} \cdot x^{2bc-2ba}} \\
 &= \frac{x^{a^2-b^2+b^2-c^2+c^2-a^2}}{x^{2ca-2cb+2ab-2ac+2bc-2ba}} \\
 &= \frac{x^0}{x^0} = 1 = \text{RHS}
 \end{aligned}$$

Ans: Shown: LHS = 1 = RHS

8. If $a + b + c = 0$, show that $9^{\frac{a^2}{bc}} \cdot 9^{\frac{b^2}{ca}} \cdot 9^{\frac{c^2}{ab}} = 729$. [5]

Given $a + b + c = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc$

$$\begin{aligned}
 \text{LHS} &= 9^{\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}} \\
 &= 9^{\frac{a^3+b^3+c^3}{abc}} \\
 &= 9^{\frac{3abc}{abc}} = 9^3 = 729 = \text{RHS}
 \end{aligned}$$

Ans: Shown: LHS = $9^3 = 729 = \text{RHS}$

9. Solve $6^{x+1} = 216$. [1]

$$\begin{aligned}
 6^{x+1} &= 6^3 \\
 \Rightarrow x+1 &= 3 \\
 \Rightarrow x &= 2
 \end{aligned}$$

Ans: $x = 2$

10. Solve $5^{2x-5} \times 3^{x-1} = 45$. [2]

$$\begin{aligned}
 45 &= 5^1 \times 3^2 \\
 \Rightarrow 5^{2x-5} \times 3^{x-1} &= 5^1 \times 3^2 \\
 \Rightarrow 2x-5 &= 1 \text{ and } x-1 = 2 \\
 \Rightarrow x &= 3 \text{ and } x = 3 \\
 \therefore x &= 3
 \end{aligned}$$

Ans: $x = 3$

11. If $2^x = 7$, find 8^x . [1]

$$\begin{aligned}
 8^x &= (2^3)^x = (2^x)^3 \\
 &= 7^3 = 343
 \end{aligned}$$

Ans: 343

12. Solve $\left(\frac{3}{4}\right)^{2x} \left(\frac{4}{3}\right)^x = \frac{27}{64}$. [2]

$$\begin{aligned}\left(\frac{4}{3}\right)^x &= \left(\frac{3}{4}\right)^{-x} \\ \Rightarrow \left(\frac{3}{4}\right)^{2x} \cdot \left(\frac{3}{4}\right)^{-x} &= \left(\frac{3}{4}\right)^3 \\ \Rightarrow \left(\frac{3}{4}\right)^x &= \left(\frac{3}{4}\right)^3 \\ \Rightarrow x &= 3\end{aligned}$$

Ans: $x = 3$

13. Solve $36^{x-1} = 6^{2x-1} - 180$. [5]

$$\begin{aligned}(6^2)^{x-1} &= 6^{2x-1} - 180 \\ \Rightarrow 6^{2x-2} &= 6^{2x-1} - 180 \\ \Rightarrow \frac{6^{2x}}{36} &= \frac{6^{2x}}{6} - 180 \\ \Rightarrow 6^{2x} \left(\frac{1}{6} - \frac{1}{36}\right) &= 180 \\ \Rightarrow 6^{2x} \times \frac{5}{36} &= 180 \\ \Rightarrow 6^{2x} &= 180 \times \frac{36}{5} = 1296 = 6^4 \\ \Rightarrow 2x &= 4 \Rightarrow x = 2\end{aligned}$$

Ans: $x = 2$

14. If $9^{x-1} = 27$, find 4^x . [3]

$$\begin{aligned}(3^2)^{x-1} &= 3^3 \\ \Rightarrow 3^{2x-2} &= 3^3 \Rightarrow 2x - 2 = 3 \Rightarrow x = \frac{5}{2} \\ 4^x &= 4^{\frac{5}{2}} = (2^2)^{\frac{5}{2}} = 2^5 = 32\end{aligned}$$

Ans: 32

15. If $3^x = 4$, $4^y = 5$ and $5^z = \frac{1}{9}$, find xyz . [2]

$$\begin{aligned}5 &= 4^y = (3^x)^y = 3^{xy} \\ \frac{1}{9} &= 5^z = (3^{xy})^z = 3^{xyz} \\ 3^{xyz} &= \frac{1}{9} = 3^{-2} \\ \Rightarrow xyz &= -2\end{aligned}$$

Ans: $xyz = -2$

16. If $9^x = 4^y = 6^z$, show that $z = \frac{2xy}{x+y}$. [5]

$$\begin{aligned}\text{Let } 9^x &= 4^y = 6^z = k \text{ (say)} \\ \Rightarrow 9 &= k^{\frac{1}{x}}, 4 = k^{\frac{1}{y}}, 6 = k^{\frac{1}{z}} \\ 6 &= 3 \times 2 = 9^{\frac{1}{2}} \times 4^{\frac{1}{2}} \\ \Rightarrow k^{\frac{1}{z}} &= \left(k^{\frac{1}{x}}\right)^{\frac{1}{2}} \times \left(k^{\frac{1}{y}}\right)^{\frac{1}{2}} = k^{\frac{1}{2x} + \frac{1}{2y}} \\ \Rightarrow \frac{1}{z} &= \frac{1}{2x} + \frac{1}{2y} = \frac{y+x}{2xy} \\ \Rightarrow z &= \frac{2xy}{x+y}\end{aligned}$$

Ans: Shown: $z = \frac{2xy}{x+y}$

17. Find the value of $\sqrt[5]{243^{-3}}$. [1]

$$\sqrt[n]{a^m} = a^{\frac{m}{n}}$$

$$\sqrt[5]{243^{-3}} = 243^{-\frac{3}{5}} = (3^5)^{-\frac{3}{5}}$$

$$= 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$$

Ans: $\frac{1}{27}$

18. If $x = 3^{\frac{2}{3}} + 3^{\frac{4}{3}}$, show that $x^3 - 27x = 90$. [3]

$$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$$

$$x^3 = \left(3^{\frac{2}{3}}\right)^3 + \left(3^{\frac{4}{3}}\right)^3 + 3 \cdot 3^{\frac{2}{3}} \cdot 3^{\frac{4}{3}} \left(3^{\frac{2}{3}} + 3^{\frac{4}{3}}\right)$$

$$= 9 + 81 + 3 \times 9 \times x$$

$$x^3 = 90 + 27x$$

$$\Rightarrow x^3 - 27x = 90$$

Ans: Shown: $x^3 - 27x = 90$

19. Find the value of $(1024)^{0.07} \times (1024)^{0.13}$. [2]

$$= (1024)^{0.07+0.13} = (1024)^{0.2}$$

$$0.2 = \frac{1}{5}$$

$$= (2^{10})^{\frac{1}{5}} = 4$$

Ans: 4

20. Find the value of $\left\{(7^2)^{-3}\right\}^{-\frac{1}{6}}$. [1]

$$= 7^{2 \times (-3) \times (-\frac{1}{6})}$$

$$= 7^1$$

$$= 7$$

Ans: 7
