EXPRESSWALA ACADEMY

Exponents (Level 2)

Question Paper, Set 05

Time: 1 hour 30 minutes | Maximum marks: 50

Name: _____ Class: _____ Date: _____

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. If $x^5 = 3^{10}$, find x. [1 mark]

2. Simplify $\frac{12^5}{4^5} \times 3^{-3}$. [2 marks]

3. Simplify $32^2 \times 8^{-3}$. [1 mark]

4. Find the value of $\frac{(64)^{\frac{5}{6}} \times (9)^{\frac{3}{2}}}{(16)^{\frac{3}{4}}}$. [2 marks]

5. Find the value of $\left(\frac{1}{5}\right)^{-2} - 4 \times 8^{\frac{2}{3}} \times 9^0 + \left(\frac{16}{9}\right)^{-\frac{1}{2}}$. [3 marks]

6. Simplify $\frac{9 \times 2^{\frac{7}{5}} \times 5^{\frac{9}{5}}}{15^{\frac{4}{5}} \times 4^{\frac{3}{5}}} \div \frac{27^{\frac{2}{3}} \times 9^{\frac{1}{5}}}{12^{\frac{2}{5}} \times 3^{\frac{4}{5}}}$. [5 marks]

7. Show that $\left(\frac{x^a}{x^b}\right)^{a^2+ab+b^2} \cdot \left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2} \cdot \left(\frac{x^c}{x^a}\right)^{c^2+ca+a^2} = 1$. [3 marks]

8. If $a + b + c = 0$, show that $\frac{x^{\frac{a^2}{bc}} \cdot x^{\frac{b^2}{ca}}}{x^{-\frac{c^2}{ab}}} = x^3$. [5 marks]

9. Solve $2^{x-3} = \frac{1}{8}$. [1 mark]

10. Solve $3^{x+1} \times 2^{x-1} = 54$. [2 marks]

11. If $7^x = 98$, find 7^{x-2}. [1 mark]

12. Solve $\left(\sqrt{\frac{5}{2}}\right)^{x+3} = \frac{4}{25}$. [2 marks]

13. Solve $27^{x-1} = 3^{3x-1} - 216$. [5 marks]

14. If $25^{2x-1} = 125^{x+1}$, find 2^{x-2}. [3 marks]

15. If $a^x = b$, $b^y = c$ and $c^z = a^2$, find xyz. [2 marks]

16. If $4^x = 6^y = 24^z$, show that $z = \frac{xy}{x+y}$. [5 marks]

17. Find the value of $\sqrt[3]{0.343}$. [1 mark]

18. If $x = 5$, find the value of $(x^{\frac{1}{3}} + x^{-\frac{1}{3}})(x^{\frac{2}{3}} + x^{-\frac{2}{3}} - 1)$. [3 marks]

19. If $\frac{x}{x^{1.25}} = 8x^{-1}$ and $x > 0$, find x. [2 marks]

20. Find the value of $\left\{27^{-\frac{2}{3}} \div 3^{-4}\right\}^{\frac{1}{2}}$. [1 mark]

Key sheet

Q	Answer
1	$x = 9$
2	9
3	2
4	108
5	$\frac{39}{4}$
6	10
7	Shown: LHS $= 1 =$ RHS
8	Shown: LHS $= x^3 =$ RHS
9	$x = 0$
10	$x = 2$
11	2
12	$x = -7$
13	$x = 2$
14	8
15	$xyz = 2$
16	Shown: $z = \frac{xy}{x+y}$
17	0.7
18	$\frac{26}{5}$
19	$x = 16$
20	3

Solutions

1. If $x^5 = 3^{10}$, find x. [1]

$a^m = b^n \Rightarrow a = b^{\frac{n}{m}}$

$x = 3^{\frac{10}{5}} = 3^2 = 9$

Ans: $x = 9$

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2. Simplify $\frac{12^5}{4^5} \times 3^{-3}$. [2]

$\frac{12^5}{4^5} = \left(\frac{12}{4}\right)^5 = 3^5$

$3^5 \times 3^{-3} = 3^{5-3} = 3^2$

$= 9$

Ans: 9

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3. Simplify $32^2 \times 8^{-3}$. [1]

$= (2^5)^2 \times (2^3)^{-3}$

$= 2^{10} \times 2^{-9}$

$= 2^{10-9} = 2^1$

$= 2$

Ans: 2

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4. Find the value of $\frac{(64)^{\frac{5}{6}} \times (9)^{\frac{3}{2}}}{(16)^{\frac{3}{4}}}$. [2]

$= \frac{(2^6)^{\frac{5}{6}} \times (3^2)^{\frac{3}{2}}}{(2^4)^{\frac{3}{4}}}$

$= \frac{2^5 \times 3^3}{2^3}$

$= \frac{32 \times 27}{8}$

$= 108$

Ans: 108

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5. Find the value of $\left(\frac{1}{5}\right)^{-2} - 4 \times 8^{\frac{2}{3}} \times 9^0 + \left(\frac{16}{9}\right)^{-\frac{1}{2}}$. [3]

$= 5^2 - 4 \times (2^3)^{\frac{2}{3}} \times 1 + \left[\left(\frac{4}{3}\right)^2\right]^{-\frac{1}{2}}$

$= 25 - 4 \times 2^2 + \frac{3}{4}$

$= 25 - 16 + \frac{3}{4}$

$= \frac{100-64+3}{4}$

$= \frac{39}{4}$

Ans: $\frac{39}{4}$

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6. Simplify $\frac{9\times 2^{\frac{7}{5}}\times 5^{\frac{9}{5}}}{15^{\frac{4}{5}}\times 4^{\frac{3}{5}}} \div \frac{27^{\frac{2}{3}}\times 9^{\frac{1}{5}}}{12^{\frac{2}{5}}\times 3^{\frac{4}{5}}}$. [5]

$= \frac{9\times 2^{\frac{7}{5}}\times 5^{\frac{9}{5}}}{15^{\frac{4}{5}}\times 4^{\frac{3}{5}}} \times \frac{12^{\frac{2}{5}}\times 3^{\frac{4}{5}}}{27^{\frac{2}{3}}\times 9^{\frac{1}{5}}}$

$= \frac{(3^2)\times 2^{\frac{7}{5}}\times 5^{\frac{9}{5}}}{(3\times 5)^{\frac{4}{5}}\times (2^2)^{\frac{3}{5}}} \times \frac{(2^2\times 3)^{\frac{2}{5}}\times 3^{\frac{4}{5}}}{(3^3)^{\frac{2}{3}}\times (3^2)^{\frac{1}{5}}}$

$= \frac{3^2\times 2^{\frac{7}{5}}\times 5^{\frac{9}{5}}}{3^{\frac{4}{5}}\times 5^{\frac{4}{5}}\times 2^{\frac{6}{5}}} \times \frac{2^{\frac{4}{5}}\times 3^{\frac{2}{5}}\times 3^{\frac{4}{5}}}{3^2\times 3^{\frac{2}{5}}}$

$= 2^{\frac{7}{5}-\frac{6}{5}+\frac{4}{5}} \times 3^{2-\frac{4}{5}-2-\frac{2}{5}+\frac{2}{5}+\frac{4}{5}} \times 5^{\frac{9}{5}-\frac{4}{5}}$

$= 2^1 \times 3^0 \times 5^1$

$= 10$

Ans: 10

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7. Show that $\left(\frac{x^a}{x^b}\right)^{a^2+ab+b^2} \cdot \left(\frac{x^b}{x^c}\right)^{b^2+bc+c^2} \cdot \left(\frac{x^c}{x^a}\right)^{c^2+ca+a^2} = 1$. [3]

LHS $= x^{(a-b)(a^2+ab+b^2)} \cdot x^{(b-c)(b^2+bc+c^2)} \cdot x^{(c-a)(c^2+ca+a^2)}$

$(a-b)(a^2+ab+b^2) = a^3+a^2b+ab^2-a^2b-ab^2-b^3 = a^3-b^3$

In the same way the other powers are b^3-c^3 and c^3-a^3

LHS $= x^{a^3-b^3+b^3-c^3+c^3-a^3} = x^0 = 1 =$ RHS

Ans: Shown: LHS $= 1 =$ RHS

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8. If $a+b+c=0$, show that $\frac{x^{\frac{a^2}{bc}}\cdot x^{\frac{b^2}{ca}}}{x^{-\frac{c^2}{ab}}} = x^3$. [5]

Given $a+b+c=0 \Rightarrow a^3+b^3+c^3=3abc$

LHS $= x^{\frac{a^2}{bc}+\frac{b^2}{ca}-\left(-\frac{c^2}{ab}\right)}$

$= x^{\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}}$

$= x^{\frac{a^3+b^3+c^3}{abc}} = x^{\frac{3abc}{abc}} = x^3 =$ RHS

Ans: Shown: LHS $= x^3 =$ RHS

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9. Solve $2^{x-3} = \frac{1}{8}$. [1]

$2^{x-3} = \frac{1}{8} = 2^{-3}$

$\Rightarrow x-3 = -3$

$\Rightarrow x = 0$

Ans: $x=0$

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10. Solve $3^{x+1} \times 2^{x-1} = 54$. [2]

$54 = 3^3 \times 2^1$

$\Rightarrow 3^{x+1} \times 2^{x-1} = 3^3 \times 2^1$

$\Rightarrow x+1 = 3$ and $x-1 = 1$

$\Rightarrow x = 2$ and $x = 2$

$\therefore x = 2$

Ans: $x=2$

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11. If $7^x = 98$, find 7^{x-2}. [1]

$7^{x-2} = \frac{7^x}{7^2}$

$= \frac{98}{49} = 2$

Ans: 2

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12. Solve $\left(\sqrt{\frac{5}{2}}\right)^{x+3} = \frac{4}{25}$. [2]

$\left[\left(\frac{5}{2}\right)^{\frac{1}{2}}\right]^{x+3} = \left(\frac{2}{5}\right)^2$

$\Rightarrow \left(\frac{5}{2}\right)^{\frac{x+3}{2}} = \left(\frac{5}{2}\right)^{-2}$

$\Rightarrow \frac{x+3}{2} = -2$

$\Rightarrow x + 3 = -4$

$\Rightarrow x = -7$

Ans: $x = -7$

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13. Solve $27^{x-1} = 3^{3x-1} - 216$. [5]

$(3^3)^{x-1} = 3^{3x-1} - 216$

$\Rightarrow 3^{3x-3} = 3^{3x-1} - 216$

$\Rightarrow \frac{3^{3x}}{27} = \frac{3^{3x}}{3} - 216$

$\Rightarrow 3^{3x}\left(\frac{1}{3} - \frac{1}{27}\right) = 216$

$\Rightarrow 3^{3x} \times \frac{8}{27} = 216$

$\Rightarrow 3^{3x} = 216 \times \frac{27}{8} = 729 = 3^6$

$\Rightarrow 3x = 6 \Rightarrow x = 2$

Ans: $x = 2$

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14. If $25^{2x-1} = 125^{x+1}$, find 2^{x-2}. [3]

$(5^2)^{2x-1} = (5^3)^{x+1}$

$\Rightarrow 5^{4x-2} = 5^{3x+3}$

$\Rightarrow 4x - 2 = 3x + 3 \Rightarrow x = 5$

$2^{x-2} = 2^{5-2} = 2^3 = 8$

Ans: 8

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15. If $a^x = b$, $b^y = c$ and $c^z = a^2$, find xyz. [2]

$c = b^y = (a^x)^y = a^{xy}$

$a^2 = c^z = (a^{xy})^z = a^{xyz}$

$\Rightarrow xyz = 2$

Ans: $xyz = 2$

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16. If $4^x = 6^y = 24^z$, show that $z = \frac{xy}{x+y}$. [5]

Let $4^x = 6^y = 24^z = k$ (say)

$\Rightarrow 4 = k^{\frac{1}{x}}$, $6 = k^{\frac{1}{y}}$, $24 = k^{\frac{1}{z}}$

$24 = 4 \times 6$

$\Rightarrow k^{\frac{1}{z}} = k^{\frac{1}{x}} \times k^{\frac{1}{y}}$

$\Rightarrow k^{\frac{1}{z}} = k^{\frac{1}{x}+\frac{1}{y}}$

$\Rightarrow \frac{1}{z} = \frac{1}{x} + \frac{1}{y}$

$\Rightarrow \frac{1}{z} = \frac{y+x}{xy}$

$\Rightarrow z = \frac{xy}{x+y}$

Ans: Shown: $z = \frac{xy}{x+y}$

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17. Find the value of $\sqrt[3]{0.343}$. [1]

$0.343 = \frac{343}{1000} = \left(\frac{7}{10}\right)^3 = (0.7)^3$

$\sqrt[3]{0.343} = (0.343)^{\frac{1}{3}} = \left[(0.7)^3\right]^{\frac{1}{3}} = 0.7$

Ans: 0.7

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18. If $x = 5$, find the value of $(x^{\frac{1}{3}} + x^{-\frac{1}{3}})(x^{\frac{2}{3}} + x^{-\frac{2}{3}} - 1)$. [3]

$(a+b)(a^2+b^2-ab) = a^3+b^3$ with $a = x^{\frac{1}{3}}$, $b = x^{-\frac{1}{3}}$

$ab = x^{\frac{1}{3}} \cdot x^{-\frac{1}{3}} = x^0 = 1$

So the value $= (x^{\frac{1}{3}})^3 + (x^{-\frac{1}{3}})^3 = x + \frac{1}{x}$

$= 5 + \frac{1}{5} = \frac{26}{5}$

Ans: $\frac{26}{5}$

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19. If $\frac{x}{x^{1.25}} = 8x^{-1}$ and $x > 0$, find x. [2]

$1.25 = \frac{5}{4}$

$\frac{x}{x^{\frac{5}{4}}} = \frac{8}{x} \Rightarrow \frac{x^2}{x^{\frac{5}{4}}} = 8$

$\Rightarrow x^{2-\frac{5}{4}} = 8 \Rightarrow x^{\frac{3}{4}} = 8$

$\Rightarrow x = 8^{\frac{4}{3}} = (2^3)^{\frac{4}{3}} = 2^4 = 16$

Ans: $x = 16$

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20. Find the value of $\left\{27^{-\frac{2}{3}} \div 3^{-4}\right\}^{\frac{1}{2}}$. [1]

$= \left\{\frac{(3^3)^{-\frac{2}{3}}}{3^{-4}}\right\}^{\frac{1}{2}}$

$= \left\{\frac{3^{-2}}{3^{-4}}\right\}^{\frac{1}{2}} = \left\{3^2\right\}^{\frac{1}{2}}$

$= 3^1 = 3$

Ans: 3