

**Exponents (Level 2)****Question Paper, Set 04**

Time: 1 hour 30 minutes

Maximum marks: 50

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

1. Answer all 20 questions. The marks for each question are shown in brackets.
2. Show your working for the 2, 3 and 5 mark questions.
3. The key sheet and worked solutions are at the end of this paper.

1. Find the value of  $\left(\frac{2}{5}\right)^{-3}$ . [1 mark]
2. Find the value of  $\sqrt[3]{2^9} + \sqrt[4]{3^8}$ . [2 marks]
3. Simplify  $49^3 \times 7^{-4}$ . [1 mark]
4. Find the value of  $\frac{(49)^{\frac{3}{2}} \times (64)^{\frac{1}{6}}}{(343)^{\frac{2}{3}}}$ . [2 marks]
5. Find the value of  $\left(\frac{9}{4}\right)^{-\frac{3}{2}} + \frac{1}{\left(\frac{1}{8}\right)^{\frac{1}{3}}} + \frac{\sqrt{16}}{\sqrt[3]{8}}$ . [3 marks]
6. Simplify  $\frac{9^{-\frac{1}{2}} \times 8^{\frac{2}{3}} \times 25^{\frac{3}{4}}}{3^{\frac{1}{5}} \times 2^{\frac{8}{5}}} \div \frac{8^{\frac{1}{3}} \times 5^{\frac{3}{2}}}{12^{\frac{1}{5}} \times 27^{\frac{2}{3}}}$ . [5 marks]
7. Show that  $\left(x^{\frac{a}{a-b}}\right)^{\frac{1}{a-c}} \cdot \left(x^{\frac{b}{b-c}}\right)^{\frac{1}{b-a}} \cdot \left(x^{\frac{c}{c-a}}\right)^{\frac{1}{c-b}} = 1$ . [3 marks]
8. If  $a + b + c = 0$ , show that  $\left(x^{\frac{a}{b}}\right)^{\frac{a}{c}} \cdot \left(x^{\frac{b}{c}}\right)^{\frac{b}{a}} \cdot \left(x^{\frac{c}{a}}\right)^{\frac{c}{b}} = x^3$ . [5 marks]
9. Solve  $3^{2x-1} = 243$ . [1 mark]
10. Solve  $2^{2x-3} \times 5^x = 50$ . [2 marks]
11. If  $10^{2y} = 64$ , find  $10^{-y}$ . [1 mark]
12. Solve  $8^x = \frac{1}{16}$ . [2 marks]
13. Solve  $49^x = 7^{2x+1} - 294$ . [5 marks]
14. Solve  $9^x = \frac{243}{3^x}$ . [3 marks]
15. If  $2^x = 5$ ,  $5^y = 7$  and  $7^z = 32$ , find  $xyz$ . [2 marks]
16. If  $5^x = 2^y = 40^z$ , show that  $\frac{1}{z} = \frac{1}{x} + \frac{3}{y}$ . [5 marks]
17. Find the value of  $\sqrt[4]{16^{-3}}$ . [1 mark]
18. If  $x = 7^{\frac{1}{3}} + 7^{\frac{2}{3}}$ , find the value of  $x^3 - 21x$ . [3 marks]

19. Find the value of  $(16)^{0.35} \times (16)^{0.4}$ .

[2 marks]

20. If  $\left[ \left\{ \left( \frac{1}{2^3} \right)^{-2} \right\}^{-\frac{1}{3}} \right]^{\frac{1}{2}} = 2^m$ , find  $m$ .

[1 mark]

## Key sheet

| Q  | Answer                                           |
|----|--------------------------------------------------|
| 1  | $\frac{125}{8}$                                  |
| 2  | 17                                               |
| 3  | 49                                               |
| 4  | 14                                               |
| 5  | $\frac{116}{27}$                                 |
| 6  | 2                                                |
| 7  | Shown: $\text{LHS} = 1 = \text{RHS}$             |
| 8  | Shown: $\text{LHS} = x^3 = \text{RHS}$           |
| 9  | $x = 3$                                          |
| 10 | $x = 2$                                          |
| 11 | $\frac{1}{8}$                                    |
| 12 | $x = -\frac{4}{3}$                               |
| 13 | $x = 1$                                          |
| 14 | $x = \frac{5}{3}$                                |
| 15 | $xyz = 5$                                        |
| 16 | Shown: $\frac{1}{z} = \frac{1}{x} + \frac{3}{y}$ |
| 17 | $\frac{1}{8}$                                    |
| 18 | 56                                               |
| 19 | 8                                                |
| 20 | $m = -1$                                         |

## Solutions

---

1. Find the value of  $\left(\frac{2}{5}\right)^{-3}$ . [1]

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$
$$\left(\frac{2}{5}\right)^{-3} = \left(\frac{5}{2}\right)^3 = \frac{125}{8}$$

Ans:  $\frac{125}{8}$

---

2. Find the value of  $\sqrt[3]{2^9} + \sqrt[4]{3^8}$ . [2]

$$\sqrt[n]{a^m} = a^{\frac{m}{n}}$$
$$\sqrt[3]{2^9} = 2^{\frac{9}{3}} = 2^3 = 8$$
$$\sqrt[4]{3^8} = 3^{\frac{8}{4}} = 3^2 = 9$$
$$8 + 9 = 17$$

Ans: 17

---

3. Simplify  $49^3 \times 7^{-4}$ . [1]

$$= (7^2)^3 \times 7^{-4}$$
$$= 7^6 \times 7^{-4}$$
$$= 7^{6-4} = 7^2$$
$$= 49$$

Ans: 49

---

4. Find the value of  $\frac{(49)^{\frac{3}{2}} \times (64)^{\frac{1}{6}}}{(343)^{\frac{2}{3}}}$ . [2]

$$= \frac{(7^2)^{\frac{3}{2}} \times (2^6)^{\frac{1}{6}}}{(7^3)^{\frac{2}{3}}}$$
$$= \frac{7^3 \times 2}{7^2}$$
$$= \frac{343 \times 2}{49}$$
$$= 14$$

Ans: 14

---

5. Find the value of  $\left(\frac{9}{4}\right)^{-\frac{3}{2}} + \frac{1}{\left(\frac{1}{8}\right)^{\frac{1}{3}}} + \frac{\sqrt{16}}{\sqrt[3]{8}}$ . [3]

$$= \left[\left(\frac{3}{2}\right)^2\right]^{-\frac{3}{2}} + \frac{1}{\left[\left(\frac{1}{2}\right)^3\right]^{\frac{1}{3}}} + \frac{\sqrt{4^2}}{\sqrt[3]{2^3}}$$
$$= \left(\frac{2}{3}\right)^3 + \frac{1}{\frac{1}{2}} + \frac{4}{2}$$
$$= \frac{8}{27} + 2 + 2$$
$$= \frac{8+54+54}{27}$$
$$= \frac{116}{27}$$

Ans:  $\frac{116}{27}$

---

6. Simplify  $\frac{9^{-\frac{1}{2}} \times 8^{\frac{2}{3}} \times 25^{\frac{3}{4}}}{3^{\frac{9}{5}} \times 2^{\frac{8}{5}}} \div \frac{8^{\frac{1}{3}} \times 5^{\frac{3}{2}}}{12^{\frac{4}{5}} \times 27^{\frac{2}{3}}}$ . [5]

$$\begin{aligned}
 &= \frac{9^{-\frac{1}{2}} \times 8^{\frac{2}{3}} \times 25^{\frac{3}{4}}}{3^{\frac{9}{5}} \times 2^{\frac{8}{5}}} \times \frac{12^{\frac{4}{5}} \times 27^{\frac{2}{3}}}{8^{\frac{1}{3}} \times 5^{\frac{3}{2}}} \\
 &= \frac{(3^2)^{-\frac{1}{2}} \times (2^3)^{\frac{2}{3}} \times (5^2)^{\frac{3}{4}}}{3^{\frac{9}{5}} \times 2^{\frac{8}{5}}} \times \frac{(2^2 \times 3)^{\frac{4}{5}} \times (3^3)^{\frac{2}{3}}}{(2^3)^{\frac{1}{3}} \times 5^{\frac{3}{2}}} \\
 &= \frac{3^{-1} \times 2^2 \times 5^{\frac{3}{2}}}{3^{\frac{9}{5}} \times 2^{\frac{8}{5}}} \times \frac{2^{\frac{8}{5}} \times 3^{\frac{4}{5}} \times 3^2}{2 \times 5^{\frac{3}{2}}} \\
 &= 2^{2-\frac{8}{5}-1+\frac{8}{5}} \times 3^{-1-\frac{9}{5}+\frac{4}{5}+2} \times 5^{\frac{3}{2}-\frac{3}{2}} \\
 &= 2^1 \times 3^0 \times 5^0 \\
 &= 2
 \end{aligned}$$

Ans: 2

7. Show that  $(x^{\frac{a}{a-b}})^{\frac{1}{a-c}} \cdot (x^{\frac{b}{b-c}})^{\frac{1}{b-a}} \cdot (x^{\frac{c}{c-a}})^{\frac{1}{c-b}} = 1$ . [3]

$$\begin{aligned}
 \text{LHS} &= x^{\frac{a}{(a-b)(a-c)}} \cdot x^{\frac{b}{(b-c)(b-a)}} \cdot x^{\frac{c}{(c-a)(c-b)}} \\
 \text{Power} &= \frac{a}{(a-b)(a-c)} + \frac{b}{(b-c)(b-a)} + \frac{c}{(c-a)(c-b)} \\
 &= \frac{-a(b-c)-b(c-a)-c(a-b)}{(a-b)(b-c)(c-a)} \\
 &= \frac{-ab+ac-bc+ab-ca+bc}{(a-b)(b-c)(c-a)} = 0 \\
 \text{LHS} &= x^0 = 1 = \text{RHS}
 \end{aligned}$$

Ans: Shown: LHS = 1 = RHS

8. If  $a + b + c = 0$ , show that  $(x^{\frac{a}{b}})^{\frac{a}{c}} \cdot (x^{\frac{b}{c}})^{\frac{b}{a}} \cdot (x^{\frac{c}{a}})^{\frac{c}{b}} = x^3$ . [5]

$$\begin{aligned}
 \text{Given } a + b + c = 0 &\Rightarrow a^3 + b^3 + c^3 = 3abc \\
 \text{LHS} &= x^{\frac{a}{b} \cdot \frac{a}{c}} \cdot x^{\frac{b}{c} \cdot \frac{b}{a}} \cdot x^{\frac{c}{a} \cdot \frac{c}{b}} \\
 &= x^{\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}} \\
 &= x^{\frac{a^3+b^3+c^3}{abc}} = x^{\frac{3abc}{abc}} = x^3 = \text{RHS}
 \end{aligned}$$

Ans: Shown: LHS =  $x^3$  = RHS

9. Solve  $3^{2x-1} = 243$ . [1]

$$\begin{aligned}
 3^{2x-1} &= 3^5 \\
 \Rightarrow 2x - 1 &= 5 \\
 \Rightarrow x &= 3
 \end{aligned}$$

Ans:  $x = 3$

10. Solve  $2^{2x-3} \times 5^x = 50$ . [2]

$$\begin{aligned}
 50 &= 2^1 \times 5^2 \\
 \Rightarrow 2^{2x-3} \times 5^x &= 2^1 \times 5^2 \\
 \Rightarrow 2x - 3 &= 1 \text{ and } x = 2 \\
 \Rightarrow x &= 2 \text{ and } x = 2 \\
 \therefore x &= 2
 \end{aligned}$$

Ans:  $x = 2$

11. If  $10^{2y} = 64$ , find  $10^{-y}$ . [1]

$$(10^y)^2 = 8^2$$

$$\Rightarrow 10^y = 8$$

$$10^{-y} = \frac{1}{10^y} = \frac{1}{8}$$

Ans:  $\frac{1}{8}$

---

12. Solve  $8^x = \frac{1}{16}$ . [2]

$$(2^3)^x = \frac{1}{2^4}$$

$$\Rightarrow 2^{3x} = 2^{-4}$$

$$\Rightarrow 3x = -4$$

$$\Rightarrow x = -\frac{4}{3}$$

Ans:  $x = -\frac{4}{3}$

---

13. Solve  $49^x = 7^{2x+1} - 294$ . [5]

$$(7^2)^x = 7^{2x+1} - 294$$

$$\Rightarrow 7^{2x} = 7^{2x+1} - 294$$

$$\Rightarrow 7^{2x} = 7 \times 7^{2x} - 294$$

$$\Rightarrow 7^{2x} (7 - 1) = 294$$

$$\Rightarrow 7^{2x} \times 6 = 294$$

$$\Rightarrow 7^{2x} = 294 \times \frac{1}{6} = 49 = 7^2$$

$$\Rightarrow 2x = 2 \Rightarrow x = 1$$

Ans:  $x = 1$

---

14. Solve  $9^x = \frac{243}{3^x}$ . [3]

$$(3^2)^x \cdot 3^x = 243$$

$$\Rightarrow 3^{2x} \cdot 3^x = 3^5$$

$$\Rightarrow 3^{3x} = 3^5$$

$$\Rightarrow 3x = 5 \Rightarrow x = \frac{5}{3}$$

Ans:  $x = \frac{5}{3}$

---

15. If  $2^x = 5$ ,  $5^y = 7$  and  $7^z = 32$ , find  $xyz$ . [2]

$$7 = 5^y = (2^x)^y = 2^{xy}$$

$$32 = 7^z = (2^{xy})^z = 2^{xyz}$$

$$2^{xyz} = 32 = 2^5$$

$$\Rightarrow xyz = 5$$

Ans:  $xyz = 5$

---

16. If  $5^x = 2^y = 40^z$ , show that  $\frac{1}{z} = \frac{1}{x} + \frac{3}{y}$ . [5]

Let  $5^x = 2^y = 40^z = k$  (say)

$$\Rightarrow 5 = k^{\frac{1}{x}}, 2 = k^{\frac{1}{y}}, 40 = k^{\frac{1}{z}}$$

$$40 = 5 \times 2^3$$

$$\Rightarrow k^{\frac{1}{z}} = k^{\frac{1}{x}} \times \left(k^{\frac{1}{y}}\right)^3$$

$$\Rightarrow k^{\frac{1}{z}} = k^{\frac{1}{x} + \frac{3}{y}}$$

$$\Rightarrow \frac{1}{z} = \frac{1}{x} + \frac{3}{y}$$

Ans: Shown:  $\frac{1}{z} = \frac{1}{x} + \frac{3}{y}$

---

17. Find the value of  $\sqrt[4]{16^{-3}}$ . [1]

$$\sqrt[n]{a^m} = a^{\frac{m}{n}}$$

$$\sqrt[4]{16^{-3}} = 16^{-\frac{3}{4}} = (2^4)^{-\frac{3}{4}}$$

$$= 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

Ans:  $\frac{1}{8}$

---

18. If  $x = 7^{\frac{1}{3}} + 7^{\frac{2}{3}}$ , find the value of  $x^3 - 21x$ . [3]

$$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$$

$$x^3 = \left(7^{\frac{1}{3}}\right)^3 + \left(7^{\frac{2}{3}}\right)^3 + 3 \cdot 7^{\frac{1}{3}} \cdot 7^{\frac{2}{3}} \left(7^{\frac{1}{3}} + 7^{\frac{2}{3}}\right)$$

$$= 7 + 49 + 3 \times 7 \times x$$

$$x^3 = 56 + 21x$$

$$\Rightarrow x^3 - 21x = 56$$

Ans: 56

---

19. Find the value of  $(16)^{0.35} \times (16)^{0.4}$ . [2]

$$= (16)^{0.35+0.4} = (16)^{0.75}$$

$$0.75 = \frac{3}{4}$$

$$= (2^4)^{\frac{3}{4}} = 8$$

Ans: 8

---

20. If  $\left[\left\{\left(\frac{1}{2^3}\right)^{-2}\right\}^{-\frac{1}{3}}\right]^{\frac{1}{2}} = 2^m$ , find  $m$ . [1]

$$\frac{1}{2^3} = 2^{-3}$$

$$= 2^{(-3) \times (-2) \times (-\frac{1}{3}) \times \frac{1}{2}}$$

$$= 2^{-1}$$

$$\Rightarrow m = -1$$

Ans:  $m = -1$

---